

THE
ESSENTIALS OF ARITHMETIC

ORAL AND WRITTEN

BOOK II
FOR UPPER GRADES

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THE ESSENTIALS OF ARITHMETIC. Book I. For
Lower Grades.

THE ESSENTIALS OF ARITHMETIC. Book II. For
Higher Grades.

Both books are published with and without answers.

Key to Book II. for use of Teachers only.

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PREFACE.

Book II. of the present series follows its predecessor after a considerable interval of time. It is for upper grammar grades, or for all grades above the primary when but one text-book in arithmetic is required. The manner of treating elementary subjects provides for this double adaptation.

Much that has been deemed not to belong among the "essentials," though commonly found in arithmetics, has been omitted, or left accessible in a subordinate form in the appendix. The order of presentation is in the main the usual one, though previous acquaintance with the rudiments of a subject has often been assumed, and some subjects have been introduced in a preparatory way a few steps in advance of the full and formal treatment, which thus becomes far easier to comprehend.

It is of high importance to be quick with figures, and long practice is needed: exercises specified as oral, written, for dictation, etc., are accordingly given in abundance, alternately upon the subject in hand, or as constantly recurring reviews.

But the methods suggested call for effort and study, and look to the mathematical training of older children in something more than mechanical figuring by imitation. What ought to be perceived or discovered by thinking and reasoning is not first stated outright in print, though often led up to by stimulating questions. Such teaching should develop habits of correct and ready expression, with intelligent and permanent grasp of simple principles and processes.

As to division of time between solving problems and analyzing them, the teachers must decide; but it has been shown that princi-

ples and methods cannot be securely fixed by mere repetition, or the working of many examples. Fewer problems, if solved independently and logically analyzed, will do most toward attaining the highest purpose of arithmetical work.

Many a principle is made conspicuous upon the page; definitions are collected in five groups, and arranged alphabetically for reference; set rules are given only as a summary, also for convenient reference, in the appendix.

An introduction to the study of algebra is included in the appendix; and throughout the book letters are conveniently used to represent unknown quantities.

The contents of each section are given in side headings, to which the following index furnishes a complete guide.

JUNE, 1895.

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THE ESSENTIALS OF ARITHMETIC.

BOOK II.

1.—Use of Numbers. *What need of numbers has:* 1. A merchant? 2. A carpenter? 3. A farmer?
4. A tailor? 5. A shipmaster? 6. A surveyor?

7. Speaking generally, for what are numbers used?

2.—Their Names. 1. Explain the meaning of their names from thirteen to nineteen. 2. From thirty to ninety. 3. The syllable *-teen* means what? What does *-ty* mean?
4. What does *twenty* mean?

5. What do we call ten tens? 6. Ten hundreds? 7. A thousand thousands?

8. Mention thirty numbers, each named by a single word.
9. Show how other numbers are named.

10. How many of these thirty words are used in telling the number of feet in a mile?

3.—Numbers 1. When we write numbers by the Arabic
Expressed in Figures. system, how many different figures are used?
2. What people first used these figures?
3. What did the Romans use instead?

4. How is it that so many different numbers can be written with only ten figures?

4. — A Decimal
System.

1. In 5, 50, 500, 5000, how does the 5 change in value? 2. What value has the zero? Why is it used? 3. The value of a figure depends on what two things?

4. In 505,050 name the orders of units. 5. Compare the value of each 5 with the value of the one next to it.

In a decimal system ten units of any order make one unit of the next higher order.

6. In place of the wavy line supply one or more words; in place of an x , y , or z supply a number:

100 ones = 1 \sim ; 50 tens = x hundreds;
 x hundreds = 10 thousands; 10 \sim = 1 million; 10 \sim = a hundred thousand.

7. The money of the United States and of Canada has a *decimal* system of values. Explain what this means, by referring to dollars, cents, and dimes. Remember that *decem* is Latin for "ten."

5. — Reading Num-
bers.

407,826,903,531.

1. Why are the preceding figures grouped in threes? 2. Name each period beginning with the lowest. 3. Read each period so as to show its value.

4. Read the whole number. 5. What order of units does the 9 represent? The 6? The 7? The 4?

6. Explain the use of the ciphers. 7. Compare the 3's in value.

8. Mention something counted by millions. 9. Can you think of any use for billions, trillions, quadrillions, or larger numbers?

6. — 1st: Oral Read-
ing. 2d: Writing from
Dictation.

Read without using the word and:

1.	4,705	6,137,008	42,200,020
2.	27,003	3,000,975	34,000,739
3.	195,006	600,001	93,040,760
4.	70,590	17,080,005	349,000,672,084
5.	104,509	3,864,219,461	16,080,372,094

7.—Writing Numbers.

At sight or from dictation.

1. Write the largest possible number, using these six figures only : 0, 0, 2, 7, 3, 1.

Write in figures, putting a comma after each period before filling another : —

2. 101 thousand.
3. 1 thousand 1 hundred 1.
4. 3 billion 108 thousand 11.
5. 828 million 7 thousand 9.
6. Two hundred million, seventy-six.
7. Ten billion, two million, sixty.
8. One less than a billion.
9. Ten thousand hundreds.
10. The sum of — 18000, 200000, 520, 6.

8.—Test Questions. 1. What part of 4000 remains when we drop the last cipher? 2. When we annex a cipher we add ∞ , and make the 4000 x times larger.

3. Read 708 without the zero. How is the value of each figure changed? 4. Putting the cipher after the 8 would change its value how?

5. What is the effect of moving a figure to the left? 6. To the right?

7. How would you increase the value of 478 a hundred times? 8. A thousandth part of 375000 is what?

9. Each cipher annexed to a number changes its value how? 10. Each cipher removed from the right?

9.—Fractions.

1. What do integers contain. — whole ones or parts and fragments of whole ones? What kind of units have they? 2. What is a unit?

3. Divide an object — an apple, a circle, a stick — into two equal parts. Is each part a *unit*? Why? 4. Is each part a whole unit? A whole 1? A whole half?

5. How many fractional units are made by cutting into thirds? Into fourths? Which are larger?

6. Units of any size smaller than 1 are called \sim . 7. One or more fractional units make \sim . 8. Give the largest possible fractional unit, and explain. 9. Give a very small one. How many of these make 1?

**10.—The Terms of
a Fraction.**

1. Can you illustrate in the divisions of the window sash, or otherwise, these or any other fractions?— $\frac{1}{4}$, $\frac{1}{10}$, $\frac{1}{2}$, $\frac{1}{5}$, $\frac{1}{3}$, $\frac{1}{12}$, $\frac{1}{16}$, $\frac{1}{6}$.
2. Read them in the order of their size. 3. How many of each would make 1? 4. Upon what does their size depend?

5. How many fractional units in $\frac{2}{3}$, $\frac{7}{10}$, $\frac{14}{5}$? 6. How many of each size make 1? 7. Which fraction is nearest to 1 in value?

8. A fraction is expressed in what two terms? 9. Which shows *how many* units the fraction contains? 10. Which *names* them according to their size?

11. Give numerator and denominator and the use of each:— $\frac{2}{6}$; $\frac{4}{5}$; $\$ \frac{6}{10}$; $\frac{3}{12}$ yr.; $\frac{1}{7}$ wk.; $5\frac{1}{2}$; $7\frac{2}{3}$; $8\frac{9}{16}$ oz.

12. An integer with a fraction added is called a \sim number.

13. How many units in $\$ 5\frac{7}{10}$? Which are integral, which fractional?

**11.—Decimal
Fractions.**

1. In the decimal system how may the figure 1 have a certain value and then a tenth of that value?

Supply the omissions below:—

2. $10.0 = \frac{1}{10}$ of \sim

6. $0.001 = \frac{1}{10}$ of $\sim = \frac{1}{1000}$

3. $1.0 = \frac{1}{10}$ of \sim

7. $0.10 = \text{ten } \sim$

4. $0.1 = \frac{1}{10}$ of $\sim = \frac{1}{10}$

8. $0.100 = \sim$ thousandths

5. $0.01 = \frac{1}{10}$ of $\sim = \frac{1}{100}$

9. $0.010 = \text{ten } \sim$

10. What denominators must fractions have that they may be written decimally? 11. If not written, how is the denominator of a decimal known? 12. The decimal point is used for what?

**12.—Reading
Decimals.**

I. *Read the following.* II. *Give numerator and denominator.* III. *Give the value of each figure separately.*

1. $3\frac{6}{10}$; 3.6

4. $\frac{3}{1000}$; 0.003

7. $\frac{249}{1000}$; 0.249

2. $\frac{65}{100}$; 0.54

5. $2\frac{43}{100}$; 4.053

8. 0.1478; 0.090

3. $2\frac{9}{100}$; 2.09

6. 0.219; 0.765

9. 16.47; 18.476

10. Compare number of decimal places with number of ciphers in denominator. 11. What are mixed decimals? Where is *and* used in reading them?

**13.—Abstract and
Concrete Numbers.**

1. Compare 7 and 5 with 7 days and 5 days. Which are easier to add? 2. Calling a number *abstract* when used by itself, and *concrete* when associated with something, describe the numbers in section 10.

3. Classify: \$ 275; 362 lb.; 873; one thousand; ten feet; a million people; a bushel and a half.

4. What is it that you really multiply and add,—figures in ink that represent numbers, or numbers themselves? 5. In oral work do you add names of numbers or numbers themselves?

**14.—United States
Money.**

At sight.—1. Why may 1.23 be read as dollars, dimes, and cents, but not as yards, feet, and inches? 2. What is meant by a decimal system? (§ 4.)

3. How many dimes are represented in \$ 12.625? 4. How are they usually read? 5. How else may the 5 mills be read?

6. Read, and explain the use of ciphers: \$ 7.77; \$ 7.07; \$ 7.7; \$ 7.70.

Explain the meanings of dime, cent, and mill, as shown below:—

7. *Decem* means ~; dime = a tenth of ~.

8. *Century* means a hundred; cent = a hundredth of ~.

9. *Million* means a thousand; mill = a thousandth of ~.

NUMBERS IN REVIEW.

From dictation. — 10. What is a double eagle? A quarter eagle?
11. Name four silver coins. 12. What is a nickel? 13. What
other metal is coined? 14. What is a mint? 15. What is bullion?
16. Why are not mills coined? 17. Of what use are they?

18. What is counterfeit money? 19. What gives value to paper
money? 20. Are U. S. coins made of pure silver and of pure gold?
21. Why is an alloy used? 22. Try to find what 18-carat gold is.

15.—To be Read; I. *Read as dollars, cents, and mills.* II. *As*
then Written from *dollars and cents.* III. *As dollars and thou-*
Dictation. *sandths.*

- | | | |
|--------------------------|--------------|------------------------------|
| 1. \$38.19 | 4. \$0.625 | 7. \$64 $\frac{16}{100}$ |
| 2. \$5.19 $\frac{1}{2}$ | 5. \$309.083 | 8. \$2908 $\frac{113}{1000}$ |
| 3. \$24.07 $\frac{1}{2}$ | 6. \$400.040 | 9. \$80,076.95 |
| 10. \$1,014,806.09 | | |

16.—Like Numbers. *Mention one number of each kind:—* 1. Inte-
gers. 2. Common fractions. 3. Decimals.
4. Mixed numbers. 5. Abstract numbers. 6. Concrete numbers.
7. Concrete integers. 8. Abstract fractions. 9. Concrete mixed
decimals.

10. Tell what kind of number, and the unit of each:—
4 ft.; \$2; $\frac{2}{3}$ yd.; $\frac{2}{3}$; \$0.10; 18; 0.36; 16 ft.
11. Select two or more with *like* units; that is, of the same size and
kind. 12. Define like numbers.
13. Mention other numbers having the same units as those in 10.
14. Give the integral unit and the fractional unit of—
4 $\frac{1}{3}$ dozen 21 $\frac{1}{3}$ 9 $\frac{1}{2}$ quires 2.1 seconds
15. Which of the following numbers have the same integral unit?—
 $\frac{1}{4}$ lb. $\frac{1}{2}$ oz. $\frac{1}{4}$ ton 2,000 lb. 1 ton $\frac{1}{2}$ cwt.
16. Change the unit without changing the value:—
36 in. 6 ft. 1 $\frac{1}{2}$ ft. 120 sec. $\frac{1}{3}$ h. $\frac{1}{2}$ wk.

17. DEFINITIONS AND SIGNS.

[FOR REFERENCE.]

To the Teacher. — Having learned to understand and use technical terms, the student should be led to formulate his own definitions of them. Such as are given here may aid in securing exactness and brevity.

Arabic System of Notation. So called because it came into Europe from Arabia, and was brought by Arabs from India.

Decimal System of Numbers. A system in which *ten* units of any order make one unit of the next higher order.

Decimal Fraction. One or more tenths, hundredths, thousandths, etc., of an integral unit.

Decimals. Decimal fractions written after the decimal point, without a denominator.

Decimal Point. A period used after ones and before tenths.

Digits. The numbers for which the nine Arabic figures stand.

Denominator. The lower term of a fraction. It *names* the fractional units according to their size and shows into how many equal parts the integral unit is divided.

Fraction. One or more of the equal parts of an integral unit.

Integer. A whole number of which the lowest unit is *one*, not any part of one.

Mixed Number. An integer and a fraction taken together.

Notation. The writing of numbers in figures or letters.

Number. That which answers the question "How many?" ; one or more units.

Numeration. The reading of numbers expressed in figures.

Numerator. The upper term of a fraction. It *numbers* the fractional units contained.

Period. One of the groups, of three figures each, counting from the units' place.

Roman System of Notation. So called because invented and used by the Romans.

Terms of a Fraction. The two numbers used to express it.

Unit. One ; a single thing.

+ Plus ; and ; the sign of addition.

— Minus ; less ; the sign of subtraction.

× Times ; multiplied by ; the sign of multiplication.

÷ or : Divided by ; signs of division.

) In ; a sign of division.

—, /, (as in $\frac{1}{2}$, $\frac{2}{3}$) Signs of division.

= Equals or equal ; the sign of equality.

\$ Dollar or dollars.

% Hundredths ; per cent.

Ct., c., or ¢ Cent or cents.

@ At (the rate of).

∴ Therefore.

[FOR REFERENCE.]

12 things = 1 dozen (doz.)
12 dozen = 1 gross (gro.)
12 gross = 1 great gross (g. gr.)
20 things = 1 score

$$\begin{array}{lcl} 24 \text{ sheets (paper)} & = & 1 \text{ quire} \\ 20 \text{ quires or} & \} & \\ 480 \text{ sheets} & & = 1 \text{ ream} \end{array}$$

60 seconds (sec.)	= 1 minute (min.)
60 minutes	= 1 hour (h.)
24 hours	= 1 day (d.)
7 days	= 1 week (wk.)
2 weeks	= 1 fortnight
30 (31, 28, 29) days	= 1 month (mo.)
3 months or 13 weeks	} = 1 quarter
12 months or 365 days	
365 d. 5 h. 48 min. 49.7 sec.	} = 1 true or solar year
366 days	
10 years	= 1 decade
100 years	= 1 century (C.)

10 mills (mi.)	= 1 ct. (ct., c., or ¢)
10 cents	= 1 dime (di.)
100 cents or 10 dimes	} = 1 dollar (\$)
10 dollars	
	= 1 eagle

$$\begin{aligned} 12 \text{ pence (d.)} &= 1 \text{ shilling (s.)} = \pounds 0.243 + \\ 20 \text{ shillings} &= 1 \text{ pound (£)} = \pounds 4.8665 \end{aligned}$$

100 pfennigs = 1 mark (M.) = \$0.238

4 gills (gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)

2 pints	= 1 quart
8 quarts	= 1 peck (pk.)
4 pecks	= 1 bushel (bu.)
10 pecks	} = 1 barrel (bbl.)
2½ bushels	

16 ounces (oz.)	= 1 pound (lb.)
100 pounds	= { 1 hundred-
	weight (cwt.)
2000 pounds or	} = { 1 ton (T.)
20 hundredweight	
2240 pounds	= 1 long ton

*60 pounds	= 1 bushel	{ wheat or
		{ potatoes
*50 "	= 1 "	corn or rye
*32 "	= 1 "	oats
196 "	= 1 barrel	flour
200 "	= 1 "	beef or pork

* In most States.

Troy Weight.

(For precious metals, jewels, etc.)

24 grains	{ = 1 pennyweight
	{ (pwt.)
20 pennyweights	= 1 ounce
12 ounces	= 1 pound

437½ grains	= 1 ounce	} Av.
7000 "	= 1 pound	
480 "	= 1 ounce	} Troy
5760 "	= 1 pound	

Apothecaries' Weight.

20 grains	= 1 scruple (sc. or ℥)
3 scruples	= 1 dram (dr. or ℥)
8 drams	= 1 ounce (oz. or ℥)
12 ounces	} = 1 pound (lb. or lb.)
5760 grains	

Length.

12 inches (in.)	= 1 foot (ft.)
3 feet	= 1 yard (yd.)
16½ feet or	} = 1 rod (rd.)
5½ yards	
320 rods	} = 1 mile (m.)
5280 feet	
63,360 inches	
4 inches	= 1 hand

6 feet	= 1 fathom
6086.7 feet or	{ = 1 knot
1.15 + com-	
mon miles	
	{ = 1 nautical mile
	{ = 1 geographic mile
3 knots	
	= 1 league

Circular Measure.

60 seconds (")	= 1 minute (')
60 minutes	= 1 degree (°)
360 degrees	= 1 circumference
69½ miles or	{ = 1° of latitude; or
60 geographic	
miles	
	{ = 1° of longitude
	{ on the equator

Surface or Square.

144 square inches	{ = 1 square foot
(sq. in.)	
9 square feet	= 1 square yard
	(sq. yd.)
30½ square yards	{ = 1 square rod
272½ square feet	
	(sq. rd.)
160 square rods	{ = 1 acre (A.)
43,560 square feet	
640 acres	{ = 1 square mile
1 mile square	= 1 section
36 square miles	= 1 township
100 square feet	{ = 1 square
	(in roofs,
	floors, etc.)

Solid or Cubic.

1728 cubic inches	{ = 1 cubic foot
(cu. in.)	
27 cubic feet	{ = 1 cubic yard
	(cu. yd.)

Wood Measures.

16 cubic feet	= 1 cord foot (cd. ft.)
128 cubic feet	{ = 1 cord (cd.)
8 cord feet	

Addition.

19. — Combining

Like Numbers into

One Sum.

At sight.

1. Give the sum of 7 and 8; then prove by counting. 2. If you had not learned the sum of 8 bu. and 6 bu., how could you find it? 3. Find and explain a quick way of adding 6, 4, 7, 3, 1, 9, 2, 8, 5. 4. Which do you think more important: to add rapidly or accurately?

Explain what change you make before adding —

5. 3 wk. and 14 da. 6. 3 yd., 7 ft., and 24 in. 7. \$13 and \$0.85.

8. In number 6 why not change to inches? 9. In number 7 change to cents and add.

10. 2 gal. + 6 pt. = x qt. 11. 48 oz. + 2 lb. = x lb.

12. 7 yr. + 96 mo. = x yr. 13. $\frac{7}{12} + \frac{3}{12} = \frac{x}{12}$.

14. 9 ft. + 108 in. ? 15. 6 yd. + 36 ft. ?

16. 1 h. + 300 min. ? 17. Why not add $\frac{2}{3}$ and $\frac{1}{4}$? What change must be made?

Before unlike numbers can be combined into one sum their units must be made alike.

20. — Practice in

Rapid Adding.

*At sight.**Give in thirty seconds 48 sums of two digits: —*

1. By columns of two each.

2. In pairs along the line.

3. Add the four digits in each square: —

A.	1	3	3	1	3	6	3	5	3	6	2	6	8	3	1	2	2	5	1	2	2	7	2	3
	1	5	1	6	4	6	6	9	2	8	2	7	8	8	5	7	9	5	8	6	5	8	4	9
B.	7	4	7	5	4	9	4	2	4	4	5	8	4	2	4	5	3	3	1	6	1	7	2	4
	7	6	9	6	8	9	1	1	9	5	8	9	7	8	4	7	3	7	7	9	9	3	8	6

4. Add by 10's: 8 6 2 4 7 3 2 1 7 6 3 4 7 2 8 9

21.—Rapid Adding. *Practise until you can give each sum At sight. instantly.*

1. 46, 34	10. 127, 123	19. 3000, 1798, 2000
2. 19, 71	11. 900, 140	20. 1300, 2000, 175
3. 53, 47	12. 560, 240	21. 4080, 1507, 6000
4. 86, 32	13. 767, 232	22. 85, 300, 9000
5. 38, 69	14. 808, 191	23. $\frac{2}{15}, \frac{7}{15}, \frac{3}{15}, \frac{1}{15}$
6. 47, 46	15. 346, 509	24. 0.06, 0.18, 0.24
7. 65, 25	16. 888, 212	25. 25%, 8%, 30%
8. 32, 99	17. 694, 106	26. 0.41, 0.19, 0.40
9. 72, 88	18. 333, 766	27. \$4.75, \$3.25, \$7.87

22.—Written *Give directions for five steps in adding 378, 492, 864, 793, 956, 309.*

Addition: the Process Described. I. Arranging the numbers. 378

II. Beginning to add. 492

421 III. Setting down the sum. 864

635 IV. "Carrying." V. Testing. 793

442 The separate sums of four long columns are 956

340 set down at the left. Read each to show its 309

390971 value. Why are they kept separate? 3792

23.—Written Exercise. *Without copying, first add vertically; then horizontally.*

1.	2.	3.	4.	5.
6. \$ 3.47	\$ 14.69	\$ 193.67	\$ 4769.83	\$ 6483.47
7. 8.62	48.96	846.84	4392.16	8432.97
8. 9.46	37.81	932.71	8437.66	6432.98
9. 6.58	47.94	683.77	6989.84	8469.32
10. 7.39	82.66	865.75	4329.41	9396.48
11. <u>9.88</u>	<u>68.43</u>	<u>392.50</u>	<u>6832.47</u>	<u>9375.58</u>

24.—Rapid Adding. *Try by practice*
Written. to do one example
a minute. Without copying add 1st, in A;
2d, in B:—

1. From *c* to *l* inclusive.

2. *p* to *y* 7. *d* to *q* 12. *f* to *v*

3. *d* to *m* 8. *c* to *r* 13. *e* to *w*

4. *e* to *n* 9. *i* to *s* 14. *d* to *x*

5. *f* to *o* 10. *h* to *t* 15. *c* to *y*

6. *e* to *p* 11. *g* to *u*

Subtraction.

25.—Taking a 1. One part of
 Part away. 17 eggs is 8 eggs.

How would you
 find the other part if you had forgotten
 that 8 from 17 leaves 9?

2. Take out 10 stormy days in January: *x* remain. 31 less 10, or -10 , or diminished by $10 = x$.

3. Make a problem in subtraction, using concrete numbers. Which is subtrahend, which minuend? 4. The other part is \sim . The largest part is \sim .

5. In subtraction, which terms must be like numbers?

26.—Finding the
 Difference.

Subtract and say whether the result is remainder or difference:—

1. You have \$12 and spend \$7. 2. You have \$12, and I have \$7. 3. You have \$9, and will earn enough to make it \$15.

A.	B.
c. \$475.21	\$ 648.90
d. 649.85	938.27
e. 837.64	642.85
f. 246.89	937.63
g. 937.63	846.75
h. 821.74	324.93
i. 658.55	698.79
j. 327.83	128.93
k. 469.75	648.72
l. 984.96	562.37
m. 678.94	689.85
n. 627.34	283.97
o. 234.56	135.42
p. 789.12	698.57
q. 346.89	569.38
r. 764.83	783.92
s. 758.75	964.83
t. 839.65	385.75
u. 387.93	978.59
v. 648.77	628.32
w. 849.64	759.67
x. 376.86	314.11
y. 978.35	629.55

4. How do you find the third term when you have the difference and the subtrahend? 5. The minuend and the difference?

27.—The Terms 1. Which is the larger number, 3 ft. or 24 in.? 2. Which is the larger quantity?
in Subtraction must be Like Numbers. 3. How can one be subtracted from the other?
At sight.

4. A boulder weighs 7000 lb., a stone block $\frac{1}{2}$ a ton. The difference in weight is x . Explain the process.

Where you can, give two values to x , first like the minuend, then like the subtrahend:—

5. 4 lb. — 32 oz. = x

9. 10 h. — 240 min. = x

6. 60 mo. — 2 yr. = x

10. $\frac{7}{8} - \frac{1}{4} = x$

7. $0.7 - 0.03 = x$

11. 2 T. — 2 lb. = x

8. $\$250 + x = \525

12. $x - \$166 = \34

28.—Rapid
Subtraction.
Oral.

Give in one minute or less the difference between each number and the one below it; between each number and the one at the right of it.

A.	11	9	13	7	10	13	12	7	15	9	18	14
	2	5	7	3	9	8	6	5	8	4	8	5
B.	9	9	10	16	11	6	14	11	15	11	8	16
	3	6	3	8	6	3	9	7	9	8	4	9
C.	15	12	7	10	15	17	14	12	11	8	13	10
	7	3	4	5	6	9	8	7	5	5	4	7
D.	12	10	17	12	11	13	16	13	18	12	8	12
	8	4	8	9	3	5	7	9	9	5	3	4

29. — Rapid**Subtraction.***Sight or dictation.*

1. Give the difference between 100 and each of the following numbers. 2. Between each number and the one at its right. 3. Between each number and the one below it.

a.	11	88	44	74	52	70	36	13	60	67	37	48	31.	87
b.	35	61	82	14	91	33	22	65	42	53	15	59	69	47
c.	83	30	23	57	89	43	95	29	68	32	84	26	79	16
d.	55	81	73	72	21	94	56	99	17	93	41	66	58	51
e.	92	19	63	45	96	18	86	46	76	49	71	62	27	85
f.	64	77	24	28	34	39	78	38	80	25	97	54	98	40

4. From 1000 take 120 175 225 350 760 807 901

5. What remains when each of the numbers in the table is taken out of 129? 151? 173?

30. — At Sight.

1.	2.	3.	4.	5.	6.
700	3000	60503	25000	35111	36459
<u>-325</u>	<u>800</u>	<u>40402</u>	<u>37892</u>	<u>46221</u>	<u>47560</u>

7. $34 + x = 48$ $x + 27 = 80$ $x - 79 = 23$ $x + 24 = 150$.

8. Replace x with the proper number. Think quickly.

Minuend	48	62	x	84	27	x	80	36	x	80
Subtrahend	16	x	19	16	x	46	x	17	24	x
Remainder	x	40	26	x	19	15	19	x	17	21

31. — Oral Problems.

1. If you sleep 8 h. and spend $5\frac{1}{4}$ at school, how many of the 24 remain?

For dictation.

2. Out of \$2 I spend $37\frac{1}{2}$, a half-dollar, and a dime. What have I left?

3. What is the hundredth day in 1897?

4. What is the difference in latitude between a city 35° north of the equator and one 34° south of it?

5. If a person is 69 yr. old to-day; when was he born? 6. A man who died in 1879 would have been 100 yr. old if he had lived 13 yr. longer. When was he born? 7. In what year was a house built that now lacks 12 yr. of being 150 yr. old?

8. What number is 16 less than $100 - 59$?

9. At 72 min. after half-past three, what time is it?

10. Find what remains in counting backward by 13's from 100.

32. — Oral

Exercise.

At sight.

1. From 97 count backward rapidly by 6's; 8's; 9's; 12's.

2. Count up to 200 by 17's.

3. $13 + x = 120$ $250 - x = 120$ $x + 2\frac{1}{2} = 17$

What change from a \$5.00 bill that pays for—

4. Oysters, \$0.75

5. Gloves, \$1.25

6. Pens, \$0.35

Crackers, 0.38

Scarf, 0.75

Ink, 0.15

Cheese, 0.62

Pin, 2.50

Paper, $0.87\frac{1}{2}$

7. Add the difference between 38 and 67 to the subtrahend.

Find what remains after receiving and paying as shown below:—

Received.	Paid.	Received.	Paid.	Received.	Paid.
8. \$1.16	\$0.93	9. \$45.00	\$28.00	10. \$2.25	\$1.75
0.24	0.17	95.00	19.00	3.75	2.30
0.60	0.25	70.00	23.00	1.25	1.25

33. — Written

Subtraction:

the Process.

1. Try subtracting one order at a time, in the work at the left, giving each figure its real value. What is the first difficulty?

457 from 683.

Minuend 683

Subtrahend 457

Remainder 226

2. If you had 83 sticks in bundles of 10 each, with 3 sticks over, how would you subtract 7 sticks? How many bundles remain?

3. At the left, 5 tens are to be taken from x tens.

4. What was added to the 3? 5. What may be added to test the work?

6. Give directions for each separate step in the process.

34. — Written Exercises.

Without copying find quickly the sum of the four differences between —

- | | | | | |
|--------------------------|--------------------------|--------------------------|--------------------------|---------------------------|
| 1. <i>e</i> and <i>f</i> | 3. <i>g</i> and <i>h</i> | 5. <i>i</i> and <i>j</i> | 7. <i>k</i> and <i>l</i> | 9. <i>m</i> and <i>n</i> |
| 2. <i>f</i> and <i>g</i> | 4. <i>h</i> and <i>i</i> | 6. <i>j</i> and <i>k</i> | 8. <i>l</i> and <i>m</i> | 10. <i>n</i> and <i>e</i> |

Find the sum of the ten differences between —

- | | | |
|---------------------------|---------------------------|---------------------------|
| 11. <i>A</i> and <i>B</i> | 13. <i>C</i> and <i>D</i> | 15. <i>A</i> and <i>C</i> |
| 12. <i>B</i> and <i>C</i> | 14. <i>D</i> and <i>A</i> | 16. <i>B</i> and <i>D</i> |

	<i>A.</i>	<i>B.</i>	<i>C.</i>	<i>D.</i>
<i>e.</i>	\$ 3764.82	\$ 4769.31	\$ 5000.37	\$ 9000.15
<i>f.</i>	927.35	3468.97	689.82	794.38
<i>g.</i>	860.83	385.68	1348.75	1866.75
<i>h.</i>	1527.96	2487.32	946.	2889.43
<i>i.</i>	3784.98	694.39	37.89	648.95
<i>j.</i>	2876.45	1748.64	9586.34	1864.37
<i>k.</i>	825.35	4839.87	829.85	624.94
<i>l.</i>	96.47	658.34	1472.98	1739.41
<i>m.</i>	849.33	1987.62	468.32	866.
<i>n.</i>	3276.41	594.83	5500.31	49.75

35. — Oral

Problems.

At sight.

- Four parts of 75 are 18, 9, 13, and 22. The fifth part is *x*.
- 37 gallons are in a tank. Add 17 while 23 run out. What remains?
- An engine goes forward 25 rd., back 38 rd., forward 60 rd. How far is it from the starting-point?
- How much farther is it round a 17-foot square than round a square 13 ft. wide?
- By annexing to 57 the figure 6 how much is added?
- Taking the 5 from 275 leaves how much?
- Bought a pony and phaeton for \$ 500. Sold the pony for \$ 175, losing \$ 50. What did the phaeton cost?

8. Having \$400 in bank a person draws \$25, deposits \$150, draws \$75 and \$50. How much remains?

9. One horse is worth \$50 more than a second and \$150 more than a third. If the highest priced one is worth \$200, what are they all worth?

10. If you find 5 eggs one day and 6 the next, how many dozens will you get at that rate in a week?

36. — Problems.

Written work.

1. How much remained in bank to Mr. Rich's credit Saturday night, what he put in and took out being as follows for the week:—

Deposits: \$26.95, \$793.82, \$427.96, \$839.64, \$500, \$387.28.

Withdrawals: \$18.56, \$689.37, \$419.28, \$649.39, \$600, \$125.82.

2. A merchant's assets are as follows:—

Merchandise in store, \$24876.39; cash on deposit, \$1489.38; due from customers, \$4897.64; real estate, \$28649.27. He owes for merchandise, \$16483.56; for real estate, \$6498.27; on promissory notes, \$6483.75. How much will his estate be worth if he closes out his business and pays his debts?

3. I have on hand at the opening of business cash to the amount of \$846.95. I pay out \$84.92, \$64.87, and have on hand at night \$837.69. What have I received?

4. I received during the day \$249.85, and I paid out \$521.75. I had on hand at night \$37.62; in the morning x .

5. Thomas Bond begins business January 1 with cash \$478.37 and merchandise \$1875.28. At the close of the year he has \$1487.63 worth of merchandise and \$738.29 in cash. How much has he gained, or lost, during the year?

6. The sum of two numbers is 346301. The smaller is 89795, the larger x .

7. What number must be subtracted from one million to leave the difference between 347689 and 486931?

8. The distance from A to B is 628 feet, from A to C 1426 feet, and from B to D 1648 feet, all in a straight line. How far is it from C to D? Draw a line and mark off the distances.

9. How many days of 1897 have passed before Aug. 15?

10. What is the difference between the sum of column *A*, page 12, and that of column *B*?

**37.— Cash
Accounts.**

NOTE.—An account with "Cash" is, as it were, an account with one's cash-box or pocket-book. Cash is debtor for all that is put in, and cash is credited with all that is taken out.

<i>Dr.</i>		CASH.		<i>Cr.</i>	
1896			1896		
Apr. 1	On hand	100 00	May 3	By Mdse. bought	450 00
5	To Rent rec'd	50 00	4	" Piano "	350 00
7	" Mdse. sold	25 00	8	" Clothing "	25 00
10	" Land "	725 00	11	" Balance	75 00
		900 00			900 00
May 22	On hand	75 00			

1. Cash is charged with having received four amounts which it owes me and for which it is my debtor. How much was there at the beginning?

2. What was added from sales of merchandise? 3. Cash is debtor for the price received for land. Why is the income from rents charged to Cash? 4. What is the total amount my cash has received if I wish to draw upon it? 5. How much does Cash pay back to me for the piano purchase?

6. Why do I credit Cash with my clothing expenses? 7. What are the total outgoes for the month?

8. What is the footing of the debit side? 9. What more might I have spent so as to balance the footings? 10. How is the balance found?

38. — Written

Exercise.

1. Balance the cash account of Charles Watson. He has on hand \$4.21. He receives at various times \$6.24, \$7.36, \$8.49, \$7.34, \$6.75. He pays out \$8.75, \$9.81, \$3.26, \$8.39.

2. Monday morning a merchant begins business with \$247.84 on hand. He receives \$24.75, \$86.91, \$84.28, \$97.25, \$164.29. He pays out \$18.99, \$37.49, \$64.91, \$83.15. Find the balance on hand.

Find the balance of each of the following accounts: —

3.		4.		5.	
Dr.	Cr.	Dr.	Cr.	Dr.	Cr.
\$ 987.65	\$ 629.55	\$ 4768.82	\$ 468.34	\$ 649.81	\$ 82.46
1839.76	83.74	947.61	984.59	8439.87	981.32
6482.91	968.71	847.77	1483.22	648.38	641.25
478.85	28.46	3998.64		91.76	239.86
698.47	318.93	8372.91			728.41

Multiplication.

39. — Numbers

A: Unequal numbers. *B*: Equal numbers.

Combined. $9+8+7+4=28$. $7+7+7+7$, or $4 \times 7 = 28$.

At sight.

1. In *A*, the combining process is ~.

2. Can more than two numbers be added at a time?

3. Under *B*, the first process is ~; the shortened process is ~.

4. Do you know the product of 4 7's by counting or from memory?

5. Which number is to be multiplied? Which is the multiplier?

6. Why not get the result in *A* by multiplying? 7. Compare addition and multiplication.

40. — Rapid Factoring. What numbers multiplied together, i.e.

Oral.

what *factors* (smaller than 14), produce —

1. 28, 32, 33, 35

4. 65, 66, 72, 77

7. 108, 110, 117

2. 36, 39, 42, 45

5. 78, 81, 84, 88

8. 121, 130, 132

3. 48, 49, 52, 54

6. 91, 96, 99, 104

9. 143, 156, 169

10. The sign $\sqrt{\quad}$ means: separate into two equal factors, or find the square root. $\sqrt{25}$; $\sqrt{81}$; $\sqrt{36}$; $\sqrt{49}$. 11. $\sqrt{144} = x \times y$; $\sqrt{121} = x$; $\sqrt{64} = x$; $\sqrt{4 \times 25} = x$.

41. — Principles in $x \times 88 = 824$.

Multiplying. $4 \times x = 8$ sq. ft.

At sight. — 1. Say

which is multiplier and which multiplicand, giving values. 2. What are the factors (makers) of 824? Of 8 sq. ft.?

For dictation. — 3. What is an addend? A subtrahend? A multiplicand?

4. Make two examples: the multiplicand concrete, in one, abstract in the other.

5. Try multiplying by 5 stones or any concrete number.

6. Show with objects that 3×4 of a kind are 12 of the same kind. 7. Give the factors of 821; 49 m.; 18 cases.

8. Compare 4×5 bu. and 5×4 bu.

42. — Rapid

Multiplying.

Oral.

For dictation. — 1. 9 and 12 are factors of

what? 13 and 5? 19 and 3? 11 and 12?

88 multiplied by 4? $4\frac{1}{2}$? $4\frac{1}{4}$?

Give the product: —

3. 3, 6¢; 10¢; 2; 12, 13.

4. 13×83 ; $5 \times 8\frac{1}{4}$; 2×37 .

5. $37\frac{1}{2}$; 2; 5; $12\frac{1}{2}$; $81\frac{1}{4}$; 4.

Multiply by 9 and add 9: —

6. 7 8 4 6 12 5 11 13

Give two factors making —

7. 63 72 48 91 ft. 78 56 d.

8. Multiply the following by 8; by 9; by 12: —

7 bales 70 bales 80 rods 30 fathoms 90 feet

At sight. — 9. Give each product quickly, stating which factor is multiplicand: —

8800	600	700 rd.	900	800 m.	400	6000
<u>9</u>	<u>12 yd.</u>	<u>8</u>	<u>9 m.</u>	<u>12</u>	<u>16 m.</u>	<u>13</u>

I. Only one factor can be concrete. Both may be abstract.

II. The product and the concrete factor will be 'like' numbers.

III. The order in which the factors are used will not affect the product.

10. Give two factors of 132 sec.; 125 in.; 144° ; 108 h.
 11. Take 4×7 from 9×7 . 13. Add 18×13 and 2×13 .
 12. Take 6×8 from 8×9 . 14. 3×9 in. + 6×9 in. = x .
 15. $3^2 = 3 \times 3 = 9$; 4^2 ; 6^2 ; 8^2 ; 7^2 ; 9^2 ; 12^2 ; 20^2 ; 50^2 .

43.—1st: Written Work; 2d: Oral Explanation.

Arrange as shown at the right, then complete the equations.

1. Find 8×858 .

2. $7 \times 679 =$

3. $12 \times 543 =$

4. $s \times s =$

Think what is required and write equations from the following data: $3 \times 9^{\circ} = 27^{\circ}$.

4. At 2 for a quarter what will 14 baskets cost? 8. 103½ lb. of 6¢ sugar.
5. 387 poles at \$4 each. 9. 8 h. 20 min. a day, 6 days.
6. 6 doz. barrels, \$2.50 each. 10. \$1 a week for 2 yr.
7. 12 men, 10 d., \$2 a day.

44.—Written
Multiplication.
For oral analysis.

<i>A.</i>	<i>B.</i>
183	183
7	70
<hr/>	<hr/>
1281	12,810

Every cipher annexed to an integer multiplies by 10.

<i>By an integer of one figure.—</i>	<i>C.</i>
1. Under <i>C</i> what are added to get	489
the result? 2. Explain their position	6
and real value. 3. Show how the re-	<hr/> 54
sult may be got without setting down	48
the partial products.	91

By 10, 100, 1000, etc.—4. Annex a cipher to 28, and give the values of the 8 and 2 before and after the change. 5. How would you multiply by 10,000? 6. Compare the work under *A* and *B*, and make a rule for multiplying by any number of 10's, 100's, etc.

7. Read these numbers multiplied by 10, by 100, by 1000:—

8. Compare 90×100 and 100×90 . 9. $120 \times 300 = \dots$

45.—For Rapid Fig- *Find the product:—*

uring.

1. $6 \times 4763 = x$. 2. $x = 8 \times 4931$. 3. 9 lb. and 6789. 4. 5987 by 7. 5. 84,965 by 80.
 6. $8 \times \$12,039$. 7. 843,794 and 300. 8. $1,203,900 \times 50$. 9. 1 ft. 8 in. $\times 3100$. 10. 34,000 and 90,000.

46.— Multiplying
by any Integer.*For oral analysis.*

1. In the work at the right, read the multiplier. 2. What three partial multipliers are used? 3. Read the 2d partial product. 4. 4×578 would be what? 5. Explain how the 3d partial product is got. 6. Would it affect the result if we should multiply first by 300? 7. In ordinary work how much of this may be omitted? 8. Where is the lowest figure of a partial product to be written?
- | |
|----------------------------|
| 578 |
| <u>346</u> |
| $6 \times 578 = 3,468$ |
| $40 \times 578 = 23,120$ |
| $300 \times 578 = 173,400$ |
| $346 \times 578 = 199,988$ |

47.—The Process
Described.

Choosing a Multiplier.

Oral.

$\$7$ multiplied by
 $\frac{2378}{\$16646}$ must equal

For $\frac{2378}{7}$
 multiplied by $\frac{7}{16646}$
 equals

Give directions for six steps in multiplying:

1. Arranging the factors.
 2. Beginning to multiply.
 3. Setting down and carrying.
 4. Arranging partial products.
 5. Finding entire product.
 6. Testing the work.
- a. In finding the cost of 2378 bbl. flour, at \$7, what is the true multiplicand?
 b. To shorten the process, why must we use abstract numbers as shown at the left?

48.—For Rapid Figuring.

- | | |
|-------------------------------|------------------------|
| 3. $37\frac{1}{2} \times 432$ | 7. 64 oz. $\times 976$ |
| 4. 427×83 | 8. 217×319 |
| 1. 536×846 | 5. 329×347 |
| 2. 3976×597 | 9. 846×9372 |
| | 6. $\$687 \times 4395$ |
| | 10. 387 T. @ \$8 |

- 49.—Examples *Oral.*—1. Perform the work at \$4.37
 with Decimals. the right aloud, giving each figure 19
 its value in dollars or cents. 2. Ex- \$39.33
 plain the position of the decimal point in the product. \$43.7
 \$83.03

Written.—3. 479 cd. @ \$12.00? Write the product in cents;

4. 4868 T. hay @ \$27? then in dollars:—

5. 25,789 bbl. flour @ \$5.00? 9. 239 men get \$5.75 each.

6. 787 M. brick @ \$16? 10. 2958 lb. tea @ \$0.67.

7. 793×7 cents? 11. 234 million half-dimes.

8. $\$0.07 \times 793$? 12. $\$5.62\frac{1}{2} \times 8 \times 3$.

50.—Oral
 Analysis.

Explain the process that you use:—

1. Take 16 in. from $\frac{3}{4}$ yd. 2. Add $\frac{1}{3}$ yd.
 and $\frac{1}{3}$ ft. To what may both be reduced?

3. $7 \times \frac{1}{8}$ of 120 = x .

4. $\frac{1}{10}$ of \$57 = x

5. $6 \times 800 = \sim$

6. $x = 200 \times 84$

$\frac{1}{100}$ of \$57 = y

$\frac{7 \times 800}{100} = \sim$

$y = \frac{1}{100}$ of 2375

$10 \times \$0.57 = z$

$13 \times 800 = x$

$z = 101 \times 80$

7. 19 qt. = x gal.

8. 2 yd. @ 18¢

9. $3\frac{1}{2}$ lb. @ 20¢

108 in. = x yd.

$\frac{1}{2}$ doz. @ 16¢

$\frac{3}{4}$ lb. @ 16¢

$\frac{1}{8}$ lb. = x oz.

$\frac{1}{4}$ yd. @ 32¢

2 qt. @ $12\frac{1}{2}$ ¢

$\frac{7}{8}$ lb. = x oz.

10. $x \times y = 64$; $8^2 = z$; $\sqrt{64} = x$.

51.—Analysis of

Problems.

1. Make a problem in which the equation

$4 \times 16\frac{2}{3}\text{¢} = x$ will indicate the work to be

Oral.

done. What are the two equal quantities?

2. Which is easier to do: reason about a problem so as to show how it may be solved; or figure out the result after being told how to do it? From which do you learn more? 3. Explain the maxim:

“Well understood is half done.” 4. Define an equation.

- 52.—Problems to be Stated. 1. \$2.50 was the expressage on 19 tables @ \$12.74, and 28 chairs @ \$2.58; x is the total cost.
For written work.

Statement. — $\$2.50 + 19 \times \$12.74 + 28 \times \$2.58 = x$.

Make an equation showing all that must be done to find the value of x ; then find it.

2. 130 men @ \$2 a day, 47 @ \$1, and 8 @ \$3.50 receive x dollars in one day. If paid weekly, they receive y .

- | | |
|------------------------------|--------------------------------|
| 3. x is what is paid for — | 4. The amount received = x . |
| 2½ and 6½ lb. tea @ \$1.19 | 217 A. wheat; |
| 59 bu. potatoes @ 75¢ | 27 bu. to an acre; |
| 30 lb. coffee @ 34¢ | sold for 83¢ a bushel. |

5. A nursery contains 1000 trees; 75 are dead; the rest are to be sold @ \$2 each. They will bring \$ x .

6. 3 house lots cost \$1260.80 each and sell for \$1500 each. The total gain is x .

Use a short method in finding —

7. The sum of $827 \times \$9.28$ and $183 \times \$9.28$.
8. The difference between $649 \times \$12.84$ and $149 \times \$12.84$.

Division.

- 53.—Finding an Unknown Factor. For dictation. — 1. The product of two factors is 48. One is 6; the other, x .
2. How many 9's in 54? What goes 9 times in 63? 3. 12 is the multiplicand; how many times is it taken to make the product 84? 4. What multiplicand, repeated 12 times, makes the product 108?

5. Suppose one factor and the product are known; how is the other factor found? Illustrate, using $\$6 \times x = \42 ; and $\$x \times 5 = \45 . 6. Why is the process called *Division*?

At sight. — 7. Show by the examples in 5 that —

- (a) The *product* becomes the *dividend* (something to be divided);
- (b) The *known factor* becomes the *divisor*;
- (c) The *unknown factor*, when found, becomes the *quotient* (showing how many times, or the size of each part).

8. Give the quotients: $x \times 15 = 30$ d.; $\$9 \times x = 108$;

$$\begin{array}{r} 6 \overline{)120} \\ x \end{array}; 96 \div 24 = x; 72 : x = 4; \frac{360}{18} = x; \frac{56}{x} = 8.$$

9. Describe the four different ways of indicating division shown in the preceding line.

10. Find two factors of 96 leagues; 91 d.; 168 h.; 182¢.

54. — The Process of Division.

1. Division is the reverse of \times . 2. How may the multiplication table help to find a quotient? 3. Without that table how might one find the number of 12's in 60? 4. Find by subtraction the number of 24's in 96.

5. How many 12's in 1740?

A.

$$\begin{array}{r} 12 \overline{)1740} \\ 1200 = 100 \text{ 12's} \\ \hline 540 \\ 480 = 40 \text{ 12's} \\ \hline 60 \\ 60 = 5 \text{ 12's} \\ \hline \text{Total} = 145 \text{ 12's} \end{array}$$

B.

$$\begin{array}{r} 145 \\ \hline 12 \overline{)1740} \\ 12 \\ \hline 54 \\ 48 \\ \hline 60 \\ 60 \end{array}$$

C.

$$\begin{array}{r} 12 \overline{)1740} \\ 145 \end{array}$$

6. Are there 200 12's in 1740? 7. Are there 100? 8. Subtract them: what remains? 9. How many 12's in 540?

10. Subtract 40 12's: what remains? 11. $60 = x$ 12's; subtract them: what remains?

12. How many 12's in all have been taken from 1740 by the three subtractions? 13. Explain the changes under B. 14. Perform aloud the work of C.

15. What is the difference between long and short division ?

A.

$$\begin{array}{r} 235 \overline{)48674} \\ \underline{47000} = 200 \text{ 235's} \\ 1674 \\ \underline{1645} = 7 \text{ 235's} \\ 29 \end{array}$$

B.

$$\begin{array}{r} 207 \overline{)48674} \\ \underline{470} \\ 1674 \\ \underline{1645} \\ 29 \end{array}$$

16. Explain process

A of finding how many 235's in 48674.

17. Explain process B.

18. Why are there no tens in the quotient?

19. What part of another 235 is found in the remainder, 29?

20. By how much should the dividend be increased to give 208 for a quotient?

55.—Examples.

Written.

1. How many 9's in 4752 ? 2. 8's in

9896 ? 3. 12's in 3300 ? 4. 15's in 4650 ?

5. $25 \overline{)16325}$. 6. $17784 \div 312$. 7. $\frac{19908}{144}$.

8. $27 \times x = 40527$. 9. Product = 9672; quotient = 372; divisor = x .

10. 96 and 75 are the factors of what dividend? Give proof.

11. Show that $33810 \div 245 = 138$. 12. Multiply 245 by 138 and find the partial products in the work of Example 11.

56.—For Dictation.

Oral.

1. How many 99's in 500 ? 1000 ? 10,000 ?

2. Count by 99's to 500. By 98's to 500.

3. Define *dividend*; *divisor*; *remainder*.

4. $\frac{1}{2}$ of 12 is $\frac{1}{3}$ of what? 5. $\$12 \div 24 = \x . 6. What kind of number is the quotient when the divisor exceeds the dividend?

7. If 6 shillings make \$1, one shilling is worth x cents.

8. Knowing one factor of 48, how can you be sure of the other?

9. What are the three factors of 18? Give 4 divisors of 18. 10. The largest divisor of 72 is x . Of 48? The greatest divisor of both is what?

57.—The Given Fac-

tor Like the Dividend.

At sight.

1. How many \$10 bills make \$300?

2. One factor of 60 yd. is 6 yd.; the other is \sim . 3. In multiplying two factors to

make a product, which factor is always

abstract? Which may be concrete?

4. If $7 \text{ ft.} \times 6 = 42 \text{ ft.}$, $42 \text{ ft.} \div 7 \text{ ft.} = x$, and $\frac{1}{6}$ of $42 \text{ ft.} = y$.

5. Show by the last example that when the *dividend* (or *product*) and the *given factor* are alike the factor to be found must be abstract. 6. Show that the quotient might be found by repeated subtractions.

A divisor that is like the dividend is one of its equal parts. The quotient tells their number, showing how many times the divisor can be subtracted from the dividend.

7. $90 \text{ in.} \div 6 \text{ in.} = x$.

9. $400 \text{ ft.}) 8000 \text{ ft.} (x$

8. 28 qt. in 84 qt. x times.

10. $45\% \div 15\% = x$.

58. — The Given Factor Abstract; Dividend Concrete.

For dictation.

1. 8 hats cost \$ 40, 1 costs x . $x \times 8 = \$ 40$; $\frac{1}{8}$ of \$ 40 = x . 2. When a product or dividend is concrete, are the factors like or unlike?

3. Which factor shows —

(a) the number of equal parts united?

(b) the fractional part of the dividend to be found?

(c) the size of the parts?

4. Each woman gets $\frac{1}{12}$ of \$ 60. The number of women is x .

5. When 20 books cost \$ 200, what part of it will one cost?

An abstract divisor of a concrete dividend shows how many equal numbers make the dividend.

The quotient is one of these numbers.

59. — Examples.

Written.

1. $6432 \text{ ft.} \div 90 \text{ ft.} = x$.

2. One factor of \$ 475,000 is \$ 250, the other is x .

3. 84 equal numbers make 6300 yd. Find one.

4. $360 \text{ m.} =$ multiplicand; $25,520 \text{ m.} =$ product; $x =$ multiplier.

5. Multiplier 125; product 100,000 bales; multiplicand x .

6. Divisor \times quotient = \$ 16,750; divisor = \$ 670, quotient = x .

7. Dividend = $197 \times$ \$ 461; quotient = x .

8. After subtracting 220 x times from 44022, what is the least that must remain?

Find cost of one acre, when—

9. 64 cost \$ 367 \times 28.

10. 128 cost \$ 1000.

60. — Division: the
Process Described.

	A.	60	B.	\$ 7.07
1. A box of pens contains a gross. Make a problem for the work under A.	144 pens)	10,000 pens	25)	\$ 176.95
2. Under B.		8640 pens		\$ 175.
		1360 pens		\$ 1.95
		1296 pens		\$ 1.75
		64 pens		\$ 0.20

3. Explain why in each case all the numbers but one are like numbers. 4. What numbers when added make the dividend? 5. Why not divide the \$ 0.20?

6. Give directions for six steps in division.

- Arranging the numbers.
- Choosing 1st partial dividend.
- Writing quotient figure.
- Finding product to subtract.
- Completing 2d partial dividend.
- Finishing the process.

61. — Rapid
Division.
Oral.

First column (p. 29). 1. Divide *quickly* by 2; 3; 4; 5; 6. Give integral quotient and remainder. 2. Find $\frac{1}{4}$; $\frac{1}{5}$; $\frac{1}{6}$; $\frac{1}{11}$; $\frac{1}{12}$. Give the exact size of the equal parts.

Second column. 3. In each number how many times will 10 go? 100? 200? 300? 40? 4. Use as divisor 50; 60; 80; 90; 70. Give remainders.

Third column. 5. Give quotient and remainder in cents after dividing by \$ 0.50; \$ 0.25; \$ 0.80; \$ 1.10; \$ 1.20.

Fourth column. 6. Give results in dollars and cents of each dividend \div 1,000; \div 2,000; \div 200; \div 4,000; \div 3,000.

	1.	2.	3.	4.	5.	6.
a.	21	450	\$1.50	\$4261	\$567.82	347,694
b.	32	520	1.25	8937	739.75	932,976
c.	43	636	1.75	6425	947.50	843,207
d.	54	724	1.38	8034	842.90	600,398
e.	65	837	2.75	6481	838.38	347,291
f.	76	964	3.25	8972	496.81	468,394
g.	87	523	4.50	4729	149.85	729,831
h.	98	649	5.40	6834	328.74	476,984
i.	89	732	9.60	9287	692.48	294,765
j.	78	807	7.23	9799	728.47	300,041

62. — Practice in Division. *First column.* 1. Divisors: 13, 14, 15, 16, 17. Use short division.

Written. 2. Divide by 18, 19, 20, 21, 22.

Second column. 3. By short division find the other factor when one is 99, 98, 97, 96, 95.

Third and Fourth columns. 4. Change both numbers to cents, then use the larger as dividend.

Second and Sixth columns. 5. Divide numbers in 6 by numbers in 2 and give remainders.

6. Col. 4 \div (col. 3 + col. 5). 7. Col. 6 \div col. 1 \times col. 2.

63. — Oral Problems. 1. $\frac{1}{12}$ of 10 min. = x sec. 2. 3 sec. in 10

At sight. min. x times. 3. x repeated 125 times = 750 min. 4. Three persons share \$12,630 un-

equally. How much may each receive? If they share equally, what *must* each have?

5. \$62,500 is separated into x packages containing \$125 each.

6. One of the equal parts of 16,250 is 250. How many *more* such equal parts are there?

7. If 208 tickets are distributed one at a time to each of eight persons, how many will each have when the tickets are half distributed?

8. How many bags will hold a million dollars if there are 100 twenty-dollar gold pieces in each bag?

9. Take $\frac{1}{5}$ of \$2.10 from $\frac{1}{4}$ of it.

10. Make two examples: one with quotient abstract; one concrete.

64. — The Fundamental Processes.

For dictation.

1. Which of them have to do with combining several numbers into one? 2. When is the shorter process used? 3. Contrast subtraction and division. How are they alike?
4. Is the number which equals 10 8's a sum or a product?

5. If you pay \$2.38 with a \$10 bill, what is left? 6. Four numbers make 87; 12, 16, and 5 are three; the fourth is what? 7. $37 - 15 = 2 \times$ what? 8. 9 is what part of 12? 9. $\frac{2}{3}$ wk. = how many hours? 10. Why pay 13¢ for $\frac{1}{2}$ yd. at 25¢? For $\frac{3}{4}$ yd. I pay ~.

65. — Short

Examples.

Written.

Without copying, find the difference: —

1.

2.

3.

\$478.36

\$548.79

379.64

\$1399.78

\$693.78

1633.99

4. \$847.21 — \$368.27 5. \$2000 — \$367.41 6. 932.61 — 878.95

7. Take the sum of the last three subtrahends from the sum of the last three minuends.

8. If 6 bbl. oil cost \$47.70, 29 bbl. cost x .

9. Multiply 648 by $8\frac{1}{4}$. 10. $\$24.84 \times 9\frac{1}{2} = x$.

66. — Oral Exercise.

At sight.

Supply values of x and y .

1.

2.

3.

4.

$$27 + 43 = x$$

$$16 \times 9 = x$$

$$\frac{1}{x} \text{ of } 630 = 9$$

$$x = 7 - 3\frac{1}{2}$$

$$26 + x = 44$$

$$7 \times x = 91$$

$$375 = 3 \times x$$

$$\frac{1}{2} \text{ of } 7 = x$$

$$58 - 19 = x$$

$$4x = 144$$

$$15 = \frac{1}{7} \text{ of } x$$

$$2 \times 3\frac{1}{2} = x$$

$$60 - x = 32$$

$$x = \frac{1}{18} \text{ of } 72$$

$$6 = \frac{1}{x} \text{ of } 144$$

$$x = 7 \div 3\frac{1}{2}$$

$$x + y = 100$$

$$\frac{1}{9} \text{ of } x = 90$$

$$720 \div x = 180$$

$$3\frac{1}{2} \times x = 7$$

5. The divisor is 7, the quotient is 346. How many 7's are subtracted from the dividend in finding the quotient?

6. In dividing 9000 into 4's, how many 4's do we *at first* subtract from the dividend? 7. What is $\frac{1}{4}$ of what remains?

8. Compare 6 lb. and 1 lb.; the cost of 6 lb. and the cost of 1 lb. 6 lb. cost 84¢, 1 lb. costs $\frac{1}{6}$ of 84¢, or x .

9. 9 is $\frac{1}{3}$ of x . 27 lb. cost \$1.80, 9 lb. cost y .

10. If 14 lb. cost 84¢, what will 10 lb. cost? $10 \times \frac{1}{14}$ of 84 = x .

67. — **Statement of Problems.** 1. If 16 cd. wood cost \$120, 24 cd. cost what? In solving such a problem which of these suggestions seem most important? —
Oral and written.

I. *What is to be found out?* (Cost of 24 cd.)

II. *Facts that help to find it.* (16 cd. cost \$120.)

III. *Process, by steps, briefly set down.* ($24 \times \frac{1}{16}$ of \$120 = cost of 24 cd.)

IV. *Indicated work performed.* ($24 \times \frac{1}{16}$ of \$120 = \$180.)

V. *Whether the result is reasonable.* (24 cd. should cost $1\frac{1}{2}$ times, as much as 16 cd.)

Apply the preceding suggestions, and explain orally: —

2. Bought 12 lb. tea @ 75¢, and 20 lb. coffee @ 40¢. How much butter at 30¢ would cost the same?

Statement. —
$$\frac{12 \times \$0.75 + 20 \times \$0.40}{\$0.30} = x.$$

3. Exchanged a 60-acre farm worth \$2400 for 200 acres of woodland valued at \$13.75 an acre. Gain?

4. Gave 3000 sq. ft. of 20-cent land for a span of horses and \$75. What were the horses valued at?

5. Sixty-four men are employed 25 days in digging a sewer. The contract price was \$1200. Nothing was gained or lost. What were the men paid each per day?

6. A train runs 280 miles in 11 hours. Seven 3-minute stops are made, and a hot axle makes a detention of 39 minutes. The rate per hour was x miles.

7. Six men buy 640 A. @ \$125, and sell for \$95,000. Each man gains x . [$\frac{1}{6}$ of $(\$95,000 - 640 \times \$125) =$ each man's gain.] In the statement what represents the cost of the land? The proceeds of the sale? The whole gain?

8. Bought 39 bbl. flour @ \$4.75; sold 15 bbl. @ \$5, and the remainder @ \$5.25. Required, my gain.

9. Three 1-lb. packages will go by mail each for 1¢ an oz. plus registration; by express for 25¢ each. Which way is cheaper?

10. A peck, 2 bushels, and 5 quarts are to be divided equally among seven persons. Any two receive x quarts.

68.—Product of
Mixed Numbers.

Oral.—1. Explain the process used in each example: $3 \times 12 = x$; $\frac{1}{4}$ of $12 = y$; $3\frac{1}{4} \times 12 = z$.

2. Give results: $3\frac{1}{4} \times 12$; $8\frac{1}{2} \times 10$; $5\frac{1}{3} \times 15$; $7 \times 8\frac{1}{8}$.

3. Give results: $\frac{2}{3}$ of 9; $\frac{3}{4}$ of 16; $\frac{5}{6}$ of 20; $\frac{1}{7}$ of 28.

4. Give results: $\frac{2}{5}$ of 45; $\frac{1}{3}$ of 56; $\frac{8}{9}$ of 72; $\frac{5}{7}$ of 63.

5. Give results: 0.6 of 20; 0.8 of 60; 0.9 of 70; 0.08 of 400.

576 \times

$8\frac{1}{2}$

$72 = \frac{1}{8}$ of 576

$504 = \frac{7}{8}$ of 576

$4608 = \sim \times \sim$

$5112 = 8\frac{1}{2} \times \sim$

6. Supply omissions in the work at the left. Show what might be omitted in ordinary work.

7. Give directions for each step in multiplying by a mixed number.

Written.—Carefully arrange partial products and results.

8. $9\frac{3}{7} \times 280$

11. $780 \times 19\frac{9}{10}$

14. $13\frac{5}{8} \times 280$

9. $18\frac{5}{6} \times 942$

12. $603\frac{3}{100} \times 2000$

15. $14\frac{5}{8}$ ft. \times 784

10. $110\frac{7}{12} \times 144$

13. $18\frac{7}{8} \times 1728$

16. $9\frac{1}{2}$ lb. \times 1080

17. Compare $3\frac{1}{3}$ with 10. Show quick ways of multiplying by $3\frac{1}{3}$, $33\frac{1}{3}$, and $333\frac{1}{3}$.

- 69.—**Oral Review.** 1. Count by $12\frac{1}{2}$'s from 100 to 0. 2. Count to 300 by $37\frac{1}{2}$'s. 3. To 500 by $62\frac{1}{2}$'s.

For dictation.

4. What two numbers larger than 1 give 7 as product? 5. Give the sum of $6\frac{1}{2}$, $3\frac{1}{2}$, 6, 4, $5\frac{1}{4}$, $4\frac{3}{4}$, 2, 3, $8\frac{1}{2}$, $1\frac{1}{2}$, 5, and 7. 6. Compare the time 6 men need to mend a road with the time required by 2 men. By 15 men.

7. If I spend $\frac{5}{8}$ of my money and give away \$8, I shall have nothing left. What have I now?

8. What is 5 mo. rent of a house hired for \$300 a year? 9. One year's interest is \$40; $2\frac{1}{2}$ years' will be what?

10. Count from 180 to 0 by 18's.

70.—**Review.**

1. $\frac{1}{8}$ of 4864 = x 3. 300 is $\frac{1}{7}$ of x

At sight.

2. $x = \frac{1}{9}$ of 54180

4. \sim minutes = $2\frac{1}{2}$ h.

5. $16\frac{1}{2}$ ft. = 1 rd.; 10 rd. = x ft. 6. 14 reams = x quires.

7. Give rapidly the following fractional parts of 100:—

$\frac{1}{8}$, $\frac{1}{4}$, $\frac{3}{8}$, $\frac{1}{2}$, $\frac{5}{8}$, $\frac{3}{4}$, $\frac{7}{8}$, $\frac{1}{6}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{6}$.

8. If I divide an integer by 356, the largest possible remainder is what?

9. $\frac{5}{6}$ of 24 is x more than $\frac{3}{4}$ of it. $(\frac{3}{4}$ of 24) + $x = \frac{5}{6}$ of 24.

10. $x = 10 \times 0.034$. 11. \$2000 is contained x times in \$80,000.

$x = 100 \times 0.034$. 12. x is $\frac{1}{25}$ of 100 thousand.

71.—**A Rule of Three** 1. If 5 lb. cost 42¢, 10 lb. cost what? Why Applied. is it needless to find the cost of 1 lb.? What Oral: at sight. would $2\frac{1}{2}$ lb. cost?

2. When 21 lb. cost \$3.21, 7 lb. cost x , and 3 lb. cost y .

3. 5 for \$1.70 makes 35 cost x . 4. 9 for \$1, 6 for x .

5. 18%, or $\frac{18}{100}$, of \$600 is profit. \$36 is what part of the profit?

6. \$7070 is the value of the crop of a 56-acre market garden. 16 A. at that rate yield \$ x . 40 A. ?

7. What 42 men can do in a week 7 men could do in ~~, and 28 men in ~.

8. Supplies for a regiment of 1000 men would maintain 100 men ~ or 600 men ~.

9. 10 papers a week, or x a year. 10. 1000 ft. in 12 sec.; x an hour.

72. — Problems for Study.

Written.

A. B. C.

$$257.36 - 129.28 = x$$

$$385.91 - 236.99 = y$$

$$536.84 - 327.45 = z$$

1. Compare 8000 and 2000. Find a short way of multiplying 599 by 8000 and dividing the product by 2000. What is $\frac{1}{2000}$ of 8000 times a number?

2. Copy the equations at the left, giving values to x, y, z . Add A; add B. Compare the difference of their sums with the sum of C, and explain.

3. In its circuit round the sun the earth traverses about 567 millions of miles in a year of 365 days. How many miles a day?

4. Compare 52 and 364. If 52 bbl. apples cost \$175, what will 364 bbl. cost?

5. Find the average weight of 6 men weighing respectively 135, 176, 180, 138, 207, and 156 lbs.

6. What would be the duty at 20%, or $\frac{1}{5}$, on 325 bu. beans at \$1.69 a bushel?

7. Mr. Fisk leased an office for 3 yr. @ \$37 $\frac{1}{2}$ a month. What had the use of it cost him at the end of 2 $\frac{1}{3}$ yr.?

8. A barrel of flour fills 8 bags, and costs \$4.50. What is the gain on 3 bbl. sold at \$0.62 $\frac{1}{2}$ a bag?

9. Hans can haul as much sand in 15 d. as Knut can haul in 20 d. Which should receive higher day wages? 10. Knut, working with a cart and horse, got \$60 for 20 days' work. If Hans had taken the job for that amount, what could he have earned a day?

73. — The Use of *Express in words, giving values to x:—*

Signs. 1. $3 \times 5^2 + 7 = x$ $27 \div 3 - \sqrt[3]{8} = x$

Oral. 2. $(9+6) \times (8-5) = x$ $x = \overline{18-3} \div \sqrt{25}$

[See pp. 7 and 47.] 3. $5^2 - 5 \times 2 = 3^2 + x$

4. Give name and meaning of each sign just used.

Why are the results unlike in the upper and lower lines:—

5. $3 + 4 \times 5 = 3 + 20$ or 23 6. $6 \times 12 - 8 \div 2 = 72 - 4$ or 68

$(3 + 4) \times 5 = 12 \times 5$ or 60 $6 \times \overline{12-8} \div 2 = 6 \times 4 \div 2$ or 12

7. When a number has \times or \div on one side and $+$ or $-$ on the other, which process must be performed first?

8. Compare in value $13 - 5 \times 2 + 8$, and $(13 - 5) \times (2 + 8)$.

Prove that —

9. $36 \div 4 - 45 \div 9 = \overline{12 \times 3 + 8} \div 11$.

10. $2 + 5 \times 6 = \text{twice } (2 \times 5 + 6)$.

11. $\frac{1}{3}$ of $(7 + 14) - \frac{1}{4}$ of $(37 - 9) = 8 \times (9 - 3) - 48 = (5 + 4)^2 - (36 \div 4)^2$.

74. — To Write from 1. $96 \div 12$ 2. Sq. root of 36

Dictation. mult. by mult. by 9
 $200 \div 25$ less $(7 - 2)$
 $= 4 \times 4^2$ $= 7^2$

3. $\sqrt{36}$	4. 3 mult. by	5. The sq. root
$\times (9 - 5)$	$(6 + 5)$	of $\frac{3}{4}$ of 48
$= \text{sq. root of}$	less $\frac{1}{5}$ of	added to
$(36 - 11)$	$(37 - 10)$	$\frac{5}{7}$ of 35
$+ 19$	$= 30$	$= 31$

75. — At Sight. 1. What expressions are here marked to be treated as one number?

$8 \times 16 \div 8 + 8 \times (16 - 9) = \sqrt{3 \times 9 \times 3} \times 108 \div 12 - 10 \div 1$

2. What signs are used to show that two or more numbers are to be treated as one number?

3. What number is to be divided by 11? —

$$(6 \times 5 + 4 \times 9) \div 11 = [(6 + 5) \times (10 - 4)] \div 11$$

Show why it was better to use brackets [] than curves ().

4. $x = (12^2 \div 24) \times \sqrt{54 - 5}$ 5. $(12 \times 4 + 6 \times 12) \div \sqrt{100} = x$

6. $(4800 \div 100) \div 0.01$ of 600 = x 7. $x = [(7 + 3) \times 2 - \frac{1}{3} \text{ of } 39]^2$

8. $x = [6 \times \overline{8 - 4} \times (14 + 4) + 60] \div 100$

9. $\left[\left(\frac{17 + 39}{7} \right)^2 \div \sqrt{64} \right]^2 = x$ 10. $[(\sqrt{25})^2 - \sqrt{6^2} \times \frac{1}{6}] \div \frac{1}{3} \text{ of } 72 = x$

76. — Substitution
of Numerical for Lit-
eral Quantities.

The first letters of the alphabet are often used to represent quantities whose value is known. $6a = 6 \times a$; $abe = a \times b \times e$.

Oral.

Suppose $a = 4$, $b = 5$, $c = 8$,
 $d = 12$, $e = 10$, $f = 24$.

Find the value of—

- | | | | | |
|-------------------|---------------|---------------------|-----------------------|--------------|
| 1. $6a$ | 6. $a + e$ | 11. $a \times b$ or | 16. $db \div e$ | 21. a^2 |
| 2. $7e$ | 7. $f - c$ | 12. ab | 17. $\frac{f}{e}$ | 22. e^2 |
| 3. $2f$ | 8. $2a + 3b$ | 13. cd | 18. $\frac{a + d}{c}$ | 23. d^2 |
| 4. $12c$ | 9. $d - 2a$ | 14. ef | 19. $\frac{ef}{cb}$ | 24. de^2 |
| 5. $\frac{1}{2}d$ | 10. $2f - 3c$ | 15. abc | 20. $abce$ | 25. b^2e^2 |

77. — Written.

Substitute the following values for the letters in the problems, and solve them: —

$a = 144$; $b = 9$; $c = 16$; $d = 128$; $e = 24$; $f = 216$.

- If b pounds cost a dollars, one pound will cost x .
- What will e yards cost if c yards cost \$ d ?
- d yards = x inches. 4. c miles = x feet. 5. $bc \div a + ef = x$.
- Add a hours and e days.

7. $\frac{cde}{a}$

8. $\frac{fbc}{d}$

9. $\frac{a + f + d}{c + e}$

78.—Equations.

Oral.

Use addition or subtraction in finding the unknown number or quantity represented by x , y , or z . Explain how you find it.

1. $x = 12 + 25$
2. $38 + 12 = x$
3. $44 - 19 = x$
4. $x = 100 - 72$
5. $x - 24 = 48$
6. $42 - x = 19$
7. $x + 17 = 32$
8. $28 + x = 50$
9. $\frac{7}{8} - x = \frac{3}{8}$
10. 1 lb. $- x = 12$ oz.
11. $19 + 11 + x = 50$
12. $72 + x + 14 = 96$
13. $40 + 20 - x = 50$
14. $\$2.75 + x = \4.50
15. $x - \$7.30 = \2.34

16. I am x years old; in 8 years my age will be 36 years.

$$(x + 8 \text{ yrs.} = 36 \text{ yrs.})$$

17. After taking \$14, \$16, and \$12 out of a sum of money \$3.75 remained. There were \$ x at first. $x - \$14 - \$16 - \$12 = \3.75 .

18. A prize cup contains 23 oz. of gold, 10 oz. of silver, and x oz. of alloy. The cup weighs 42 ounces. (Make an equation.)

19. 25 gallons run into a tank, and 46 run out. When the faucets were closed, 80 gallons remained. There were x gallons in the tank when the faucets were opened. (Equation.)

20. Make a problem about the weather in March to suit this equation: $31 \text{ d.} = 12 \text{ d.} + x \text{ d.} + 10 \text{ d.}$

79.—Equations.

Oral.

Use multiplication or division in finding the value of x , y , or z . Explain your method.

NOTE. — $3x$ is the same as 3 times x . $5y = 5 \times y$.

1. $4 \times x = 60$
2. $17 \times 5 = x$
3. $\frac{1}{8} \text{ of } x = 16$
4. $80 \div x = 4$
5. $144 \div x = 16$
6. $8x = 400$
7. $7y = 91$
8. $14z = 700$
9. $25x = 625$
10. $4\frac{1}{2}x = 45$
11. $\frac{x}{4} = 8$
12. $\frac{32}{x} = 4$
13. $\frac{x}{12} = 15$
14. $\frac{96}{x} = 4$
15. $\frac{x}{24} = 7$
16. $4x + 5 = 21$
17. $17y - 4 = 30$
18. $18y \times 10 = 180$
19. $42y \div 21 = 70$
20. $\frac{1}{2} \text{ of } 16x = 120$

21. At \$3 a day a mason earns \$ x in a week. ($x = 6 \times \$3$.)
22. At 2 cents a mile I can ride x miles for \$4. ($x = \frac{\$4.00}{2}$.)
23. My brother is 8 years my senior. This is $\frac{1}{4}$ of my age. How old am I? My brother is x years old. ($x = 4 \times 8 + 8$.)
24. $\frac{x}{16} = 8$ m. The distance to the city is required.
25. 3 times a number added to 7 times it = 280. ($3x + 7x = 280$.)
26. $\frac{1}{2}$ of my money taken from 6 times it leaves \$55. (Make an equation.)
27. $30x = \$150$; $\frac{x}{15} = 15$. (Make problems for these equations.)

80. — Oral

*Explain the process; or give the reason.***Exercises.**

1. How many 5's in half a million?

*For analysis.*2. Divide 80,000 by $\frac{1}{8}$ of itself, and the quotient is x .

3. Compare 2 men's work with 6 men's in quantity; in cost.
4. A board bill for 8 days @ \$10.50 a week is \$ x .
5. $\$11\frac{1}{2} = \sim \$\frac{1}{4}$'s. 6. $\frac{7}{8}$ bu. = x qt.
7. Compare $6\frac{2}{3}$ and 20. If 20 gal. cost \$15, $6\frac{2}{3}$ gal. costs $\frac{1}{7}$ as much or x .
8. 900 miles of railroad cost \$18,000,000 at the rate of \$ x a mile. \$900 is $x\%$ of \$3600.
9. Compare the interest of \$480 and that of \$160. [$\$211$.]
10. Compare 8 mo. interest with $1\frac{1}{3}$ yr. interest.
11. If you know the cost of 7 articles, how will you find the cost of 9 of the same kind?
12. Find the cost of housing 14,000 lb. coal @ 25¢ a ton.
13. At \$40 per M., what do I pay for 25,000 ft. of lumber?

81.—For Dictation.

1. Give 6 multiples of 25
2. $373 + x = 1000$
3. How much fencing is required for a 30 ft. square?
4. A man leaving his office at 8 A.M. is absent 20 h. At what o'clock does he return?
5. Square 2; 3; 4; 5; 6
6. Square 7; 8; 9; 11; 12
7. What number multiplied by itself equals 900?
8. What is $\frac{6}{100}$ or 6% of \$300?
9. If $3x = 30$, $1x$ or $x =$ what?
10. $12y = 72$, $y =$ what?
11. Give two exact divisors of 52; 51; 57; 58
12. Divide the cube of 17 by its square.
13. Divide a million by ten thousand.
14. $\sqrt{36}$; $\sqrt{49}$; $\sqrt{121}$; $\sqrt{81}$
15. Give three multiples of—11; 13; 15; 17
16. Give 12 divisors of 144
17. Make an example in which the result is an amount; a remainder; a product.
18. The factors of $10x$ are

82.—At Sight.

1. $\frac{2}{3}$ yd. of 75¢ satin cost x
2. At \$0.25, $3\frac{1}{4}$ doz. eggs cost x
3. 33 yd. cloth @ $16\frac{2}{3}$ ¢
4. Beef @ \$0.25: 72 lb. cost x
5. \$0.12 $\frac{1}{2}$ raisins: 48 lbs.
6. Coffee, 93 lbs., @ \$0.33 $\frac{1}{3}$
7. If I strike out from 39 its largest factor, I divide it by x and get y for quotient.
8. $25^2 = x$ $\sqrt{625} = y$
9. $12^3 = 1728$. $\sqrt[3]{1728} = x$
What should be paid for—
10. $6\frac{1}{4}$ yd. of 25¢ ribbon?
11. $2\frac{1}{2}$ doz. buttons @ 15¢?
12. At \$1.00 a yard, $\frac{2}{3}$ yd. silk?
13. \$2 less $\frac{1}{6}$ of it = x
14. Explain the difference between 8^2 , $\sqrt[3]{8}$, $8 + 8$
15. Which represents the cube of $3: 3 \times 3$, or $3 \div 3 + 3$, or $3 \times 3 \times 3$?
16. Find a common factor in 28^2 , 63^2 , and 91^2
17. Which is larger, subtrahend, or remainder in $2148 \text{ m.} - 1075 \text{ m.}$?
18. Express more simply.
 $\frac{\$0.75}{0.25}$; $\frac{\$3}{\$4}$

CHICAGO, Aug. 1, 1895.

Bought of JOHN V. FARWELL & CO.

May 18	23 yd. Brussels Carpeting,	\$ 1.50	34	50
June 22	16½ yd. Black Silk,	1.75	18	98
July 6	38 yd. Wamatta Cotton,	0.12½	4	75
		Less 1/10		
				58 23
				5 82
				52 41
		Cr.		
June 2	By Cash,	\$ 25.00		
" 22	" "	20.00		
			45	00
	Received payment,		7	41

By Smith.

Make out bills in proper form. Supply dates and names.

- | | | | | |
|----|--------------------------|------|-------|----------|
| 2. | 13 tons Franklin Coal | @ \$ | 7.25 | |
| | 6½ tons Lackawanna | @ | 5.50 | |
| | 1 Cord Hard Wood | @ | 11.00 | |
| | 3½ bbl. Cement | @ | 3.25 | |
| 3. | 2500 ft. Spruce Flooring | @ \$ | 13.75 | [per M.] |
| | 2500 ft. Western Pine | @ | 46.50 | " |
| | 1700 ft. Whitewood | @ | 30.00 | " |
| | Freight, | | 8.48 | |

4. A grocer's receipts for one week were \$365.18; \$193.75; \$96.48; \$89.24; \$198.65; \$479.83. The average of his daily expenses was \$128.00. What was the cash increase?

5. Monday, Jan. 1, 1894, Sam'l Chase had \$32.76 to his credit in a bank. If he deposited \$25 every week-day during the month, and \$100 extra every Saturday, what amount could he draw against Feb. 1?

6. In buying 785 music books @ 85¢ a discount of $\frac{1}{5}$ or 20% is allowed on cash payments. The net cost is x .

7. Bill $6\frac{3}{4}$ lb. Formosa Oolong Tea @ 60¢
 30 lb. Maracaibo Coffee @ $24\frac{1}{2}$ ¢
 2 bbl. "Bridal Veil" Flour @ \$5.25
 Discount 2% [for cash].

8. Bill $37\frac{1}{2}$ yd. Dwight Cotton @ 11¢
 $42\frac{1}{4}$ yd. Scotch Gingham @ 23¢
 $11\frac{3}{4}$ yd. India Silk @ \$1.75
 Credit mdse. returned, \$8.75.

9. A firm buys goods billed or invoiced at \$1837, less three discounts. 30% is allowed "to the trade." After deducting these $\frac{3}{10}$, 5% is allowed on "large lots." The amount due is then lessened by 2% "for cash." The net charge is x .

10. Invoice $3\frac{3}{4}$ gro. No. 314 Eagle Pencils @ \$4.20
 54 gro. No. 404 Gillott's Pens @ \$0.37 $\frac{1}{2}$
 $\frac{1}{12}$ gro. 4to Blank Books @ \$3.66

84. — Oral Review. 1. 2×16 , or \sim is $\frac{1}{2}$ of x , $\frac{1}{3}$ of y , $\frac{1}{10}$ of z .
 At sight. 2. A gross of pens at $\frac{1}{2}$ ¢ each costs the same as $\frac{1}{2}$ doz. pins at \sim ¢ each.

3. If 12 yd. silk cost \$36, $7\frac{2}{3}$ yd. cost \sim . $7\frac{2}{3} \times \frac{1}{12}$ of \$36 = $\frac{1}{2}$ of x .
 If $\frac{x}{2} = 6$, $x =$ what?

4. $9x$, or $9 \times x$, = the sum of the 9 digits.

5. $9^2 = \sim$; $\sqrt{144} = \sim$. 6. The cube of 3 is \sim . $\sqrt[3]{125} = x$.

7. Add 64, 36, 47, 53, 39, 61, 54, 46, 17, 83.

8. $(6 + 8) \div 2 = \sqrt{x}$. 9. $\sqrt{x} = 6 + 8 \div 2$.

10. $\frac{1}{5}$ of the sum of 5 numbers is their average. Find the average of 2, 3, 4, 5, 6, 7, 8.

85. — Problems.*For written work.*

1. Make an example to show whether it is necessary to begin multiplying with the ones' figure of the multiplier.
2. What is the value of 795 books at \$1.38 each?
3. Write in words the second partial product.
4. If a young man earns \$36 a month, in how many years would he earn \$5670?
5. An importer is charged \$45 on a hundred at the custom-house on a shipment of goods worth \$18,000. What duty does he pay?
6. In one minute divide 200,000 by 1728.
7. Write the second partial dividend in words.
8. How many such divisors may be subtracted from the dividend, leaving a remainder of 9920?
9. A cubic foot of hickory cord-wood weighs $49\frac{1}{8}$ lb.; what will a cord weigh?
10. How many feet in a mile? If 10 feet of wire weigh 9 lb., what will 2 miles weigh?
11. At 15¢, how much more than 5000 lb. of cotton can be bought for \$800.10?

86. — Oral Review.1. $\frac{1}{5}$ of 15 is $\frac{1}{x}$ of 27.*At sight.* 2. $\frac{1}{2}$ of $x = 12\frac{1}{2} = \frac{1}{y}$ of y .

3. The interest on \$2 for a year is 12¢ or 6%. The interest for a year on \$1 at 6% is what? On \$5? \$30? \$100? \$1000?

4. A year's interest on \$1000 is \$60 at 6%; at 7% it is x .5. Give the factors of 28, of 70. Their *common* factors are ____.6. $\sqrt{64} - \sqrt[3]{64} = \sqrt{x}$. 7. $x^2 = 64$; $\sqrt[3]{x} = 2$.8. 25% of \$800 = x . $\frac{1}{5} =$ what %? 80% of a ton = x lb.9. What fractional part of a dollar is \$0.25? \$0.33 $\frac{1}{3}$? 12 $\frac{1}{2}$ ¢? 75% of a dollar? 16 $\frac{2}{3}$ % of a dollar?

10. Find the largest common factor of 24, 60, and 100.

11. $(9 + 5) \times 3 = 6x$. 12. $6x = 9 + 5 \times 3$. 13. $x = \sqrt{6\frac{1}{4} \times 16}$.

- 87.—Denominate Numbers. 1. Among the tradesmen in your town a “shilling” is what part of a dollar? What other value is given to a shilling?

Oral.

2. The last decade of the XIXth century includes the years ~ to ~. The first decade of the XXth century includes the years 1901 to ~. To which century do the years 1899 and 1900 belong?
3. How many fortnights in a year? Days in a quarter?
4. How many cents = a double eagle?
5. At 4 for 7 cents, 2 doz. cost x , and $\frac{1}{2}$ a gross cost y .
6. $\frac{1}{2}$ a dozen, $\frac{1}{2}$ a score, and $\frac{1}{2}$ a gross make how many?
7. If there are 25 envelopes in a package, how many packages are needed for a ream of note paper? For 5 quires?
8. Ten pencils to a box, how many boxes will a gross fill? A great gross of pens fills x boxes.
9. 5 quires of paper, 96 sheets to a pound, weigh how much?
10. A 5-franc piece is worth about \$ x , or y marks.

- 88.—Denominate Numbers changed to Larger Units. 1. Change 120000 min. to days.
2. A newspaper prints an edition of 420000 copies of two sheets. x reams are used.
3. A factory puts up 15000 pint cans of tomatoes, or x bushels. (Allow $\frac{1}{2}$ more.)

Written.

[See pp. 8-9.]

4. In 100000 square inches there are x square feet.
5. 50000 cubic feet of wood equal x cords.
6. A number of capitalists control 3,600,000 acres of western land. This is the same as x square miles.
7. 200000 cubic inches make how many cubic feet?
8. 100000 ties 3 feet apart will extend how many miles on a doubled-tracked railroad?
9. 5 million screws are put up in boxes holding a great gross. How many boxes are needed? 10. The average weight is 15 oz. to the gross; the entire lot will weigh x tons.

- 89.—Denominate Numbers changed to Smaller Units.**
1. Thirteen tons of oatmeal will make how many one-pound packages?
 2. 296 bushels = x pints.
 3. x pint-bottles may be filled from 728 gallons of vanilla extract.
- Written.*
- [See pp. 8-9.]
4. 6389 tons of baking-powder would cost x dollars, if sold at 42 cents per pound package.
 5. A school uses 125 reams, or x quires, or y sheets, of paper in a year.
 6. One heart-beat a second, or x in a day.
 7. In 7 miles of chain how many inch links?
 8. 750 rods measure x feet.
 9. 1 acre = 160 sq. rds., or x sq. yd., or y sq. ft., or z sq. in.
 10. 15 cords of stone weigh the same as x cubic inches.

90.—Problems.

- Written.*
1. January 1, my gas metre read 67500; March 31, it read 91500. At \$1.60 per thousand my quarter's gas bill is \$ x .
 2. A week's sales of wheat in bushels: 2137, 3476, 972, 3041, 6732, 1849. Valued at $62\frac{3}{4}$ cents.
 3. What did my house cost me as shown by these items:—
Cellar, 18 days @ \$14.75; mason's contract, \$4375.86; carpenter, 137 days @ \$2.15 and 96 at \$3; materials, \$576.84; painting, etc., \$397.68.
 4. At an auction sale of land the following prices were obtained:—
3648 ft. at 23¢; 2894 at $31\frac{1}{2}$ ¢; 7642 at $19\frac{1}{2}$ ¢; 8641 at 25¢. The auctioneer's commission was 2 cents on the dollar, and advertising, etc., cost \$37.50. Required, the net proceeds.
 5. Bought a 100-acre wood lot for \$800. Paid 23 men \$1.75 for 18 days' work at cutting. Sold 175 cords at \$2.37, 215 at \$4.25, and the remainder with the lot for \$800. What did I gain?

6. A farmer wintered 17 horses from December 1 to April 1 at \$12 a month. He paid \$23 a ton for 22 tons of hay, and 42 cents each for 280 bushels of oats. He had \$14 worth of provender left. He made \$ x a month.

7. I can buy of one firm 732 tons of coal at \$4.20 and 75 cords of wood at \$8.16. Another firm bids \$4.16 for the coal and \$8.35 for the wood. Shall I buy of the first or of the second, and save how much?

8. What is an avoirdupois ton of silver worth at 54 cents an ounce?

9. A city paper prints for a week as follows:—

Sunday . . .	148,917	Thursday . . .	126,839
Monday . . .	127,832	Friday . . .	128,461
Tuesday . . .	132,947	Saturday . . .	147,219
Wednesday . .	147,842		

All but 175,000 daily and 14,760 Sunday papers are sold, the first at 2¢ and the others at 5¢ a copy. What is received?

10. There are 2741 operatives on a corporation. 12 overseers get \$3.50 a day, 25 second hands get \$2.50, 1305 earn \$1.50, 215 men and 731 women earn \$1.25 each, and the remainder on the average receive 96 cents. What is the weekly pay-roll?

11. Soundings from a ship's side are taken every 5 miles over a certain course, showing the depth in fathoms as follows: 6, 8, 7, 12, 20, 21, 19, 30, 40, 80. What is the average depth in feet?

12. Find the average receipts of 8 street-car trips as follows: \$3.75, \$3.80, \$4.20, \$2.70, \$1.55, \$3.90, \$2.55, \$1.55. How many more or how many fewer passengers per trip were carried than when \$110.55 was received from 33 trips?

13. The gates at a crossing are lowered 16 times on Sunday and 61 times on other days. The keeper's pay for 3 mo., beginning Sunday, March 1, is \$99.64, or at the rate of x cents for each lowering.

14. If 3 half-dollars make a pile $\frac{1}{4}$ in. high, how high a pile will 100,000 make?

91. DEFINITIONS AND SIGNS.

[FOR REFERENCE.]

Addition. The process of combining numbers, two by two, into one sum.

Addend. A number to be added to another.

Aliquot Part. The quotient of any number divided by an integer.

Amount. The result of addition.

Composite Number. The product of integral factors, 1 not included.

Cube (Number). The product of three equal numbers; the third power of a number.

Cube Root. One of the three equal factors forming a third power. $3 \times 3 \times 3 = 3^3$ or 27.

Common Factor of two or more numbers. A number that is a factor of each of them.

Difference. What must be added to the smaller of two numbers to make the larger.

Dividend. A number to be divided.

Division. The process of separating a number into equal parts, or of finding how many times one number is contained in another.

Divisor. A number to divide by; it shows *how large* or *how many* the equal parts of the dividend are to be.

Equation. Two quantities expressed as being equal.

Exponent or Index. One or more figures written above and at the right of a number to show how many times the number is taken as a factor.

Exact Divisor. One that gives an integral quotient, without a remainder.

Factors. Numbers multiplied together in making a product; commonly used as meaning integral factors.

Greatest Common Factor, Divisor, or Measure. The largest factor found in each of two or more numbers.

Least Common Multiple of two or more numbers. The smallest number of which each is a factor.

Like Numbers have units of the same size and kind.

Minuend. A number to be lessened.

Multiplicand. One of the equal numbers to be combined by multiplication; the factor to be repeated in making a product.

Multiplication. The process of combining equal numbers, by repetition, into one product. It repeats one number "many fold."

Multiplier. The factor that shows how many equal numbers are to be combined in the product.

Multiple of a number. A number of which it is a factor.

Per Cent. Number of hundredths; units out of a hundred.

Prime Number. A number with no other factor than itself and 1.

Product. The result of multiplication.

Power. The product of two or more equal numbers as factors.

Quotient. The result of division.

Reduction. Changing the unit of a number without changing its value.

Remainder. What is left when part of a number is taken away.

Root. One of the equal factors forming a power.

Square (Number). The second power or the product of a number multiplied by itself.

Square Root. One of the two equal factors of a square, or second power.

Subtraction. The process of taking part of a number out of it to find the remainder; finding the difference between two numbers.

Subtrahend. A number to be subtracted from another.

Signs and Abbreviations.

() as in $(3 + 4) \times 5 = 35$ } *Curves* or
as in $\overline{3 + 4} \times 5 = 35$ } *Vinculum*

shows that the numbers enclosed or beneath are to be treated as one number.

$\sqrt{\quad}$, the square root of.

$\sqrt[3]{\quad}$, the cube root of.

6% , $\frac{6}{100}$, .06, or 6 per cent.

2 , 3 , as in $5^2 =$ the square of 5, or 25; $4^3 =$ the cube of 4, or 64.

Dr., debtor.

Cr., creditor.

G.C.D., greatest common divisor.

L.C.M., least common multiple.

92. — Numbers

$$1. \ 24 = 12 \times 2.$$

$$2 = \frac{1}{12} \text{ of } 24.$$

Compared.

What is the difference between these two ways of comparing 24 and 2?

Oral.

2. Compare, in two ways, 12 and 36; 15 and 60.

3. Compare 7 and 49.

6. Compare 3 in. and 2 ft.

4. Compare 8 and 72.

7. Compare 2 yd. and 6 in.

5. Compare 42 and 6.

8. Compare 2 lb. and 4 oz.

9. Why is it that sometimes an integer and sometimes a fraction is used in comparing two numbers?

$$10. \ 16 = \frac{1}{2} \text{ of } x \text{ and } \frac{1}{3} \text{ of } y.$$

$$11. \ 18 = \frac{1}{5} \text{ of } x \text{ and } \frac{1}{x} \text{ of } 54.$$

12. With 72 compare 36, 24, 9, 16, 144, 720.

93. — Ratio.

The ratio of 6 to 2 is 3. $6 : 2 = 3$.

The ratio of 2 to 6 is $\frac{1}{3}$. $2 : 6 = \frac{1}{3}$.

Oral.

1. Read these five ways of expressing ratio: —

$$12 \div 24 = 1 \div 2; \ 12 : 24 = 1 : 2$$

$$\frac{12}{24} = \frac{1}{2}; \ 12 \text{ is to } 24 \text{ as } 1 \text{ is to } 2$$

The ratio of 12 to 24 is $\frac{1}{2}$

Ratio is expressed as the quotient of one number divided by another.

2. Compare 12 with 60. The ratio of $12\frac{1}{2}$ to 100 = x .
3. Which term of the couplet is the dividend? The divisor?
4. Mention the *Antecedent*, or first term of the couplet. The *Consequent*, or second term.

5. Compare $0.12\frac{1}{2}$ with 0.25
6. Compare 0.75 with $0.12\frac{1}{2}$

$8\frac{1}{3}\%$ with $16\frac{2}{3}\%$

$\frac{3}{4}$ with $\frac{1}{8}$

24 with 18

a month with a year

Compare the end numbers with those between them.

$$7, 8. \quad 12\frac{1}{2} \left\{ \begin{array}{ccccc} 18\frac{3}{4} & 50 & 6\frac{1}{4} & 75 & 87\frac{1}{2} \\ 62\frac{1}{2} & 200 & 31\frac{1}{4} & 137\frac{1}{2} & 150 \end{array} \right\} 37\frac{1}{2}$$

$$9, 10. \quad 16\frac{2}{3} \left\{ \begin{array}{ccccc} 66\frac{2}{3} & 266\frac{2}{3} & 41\frac{1}{3} & 366\frac{2}{3} \\ 100 & 500 & 300 & 433\frac{1}{3} \end{array} \right\} 33\frac{1}{3}$$

- 94.—*Oral Review.* 1. Give two equal factors of 3600.

At sight.

$$\sqrt{3600} = x.$$

2. Find the cost of $2\frac{3}{4}$ lb. honey @ 20¢,
- 3 qt. oil @ \$0.12 a gal., 12 oz. cheese @ 16¢ a pound.

3. How much is subtracted by dropping the figure 5 from 205? Strike out the *factor* 5, and what is the quotient?

4. How many 7's from 910 leave 700?

5. $\frac{1}{3}$ of 210 = x . $\frac{1}{30}$ of 210 = x . $\frac{1}{300}$ of 2100 = x .

6. What is the effect of dropping one cipher from the right of an integer? Two ciphers? What is the easiest way of finding $\frac{1}{3000}$ of 21,000?

7. Find the sum: $3600 \div 120$; $\frac{1}{30}$ of 2400; $\frac{1}{900}$ of 27,000.

8. Find the value of $\frac{7}{8}$ of $2400 - \frac{5}{6}$ of 1500.

9. Compare 3 and 15. If 15 oranges cost 40 cents, 3 cost $\frac{1}{x}$ as much, or y .
10. If 3 cost 7 cents, 15 cost x . 15 things will cost x times as much as 3 things, and \sim as much as 30 things.

11. 25% or $\frac{1}{4}$ of a thousand bricks are soft burnt. In another lot of $2\frac{1}{2}$ M. there are five hundred such. In which lot is the proportion of soft ones greater?

95. — For
Analysis.

1. 2 women receive \$ 2.10 for making button-holes. One makes 42; the other, 28. How shall they divide the money?
2. I pay 3 men \$ 36 for shingling a house. One works 6 d., the others, 3 d. each. How do they divide the money?
3. At the rate of 3 for 7 cents, a dozen eggs cost x .
4. If 6 tons cost \$ 24, $2\frac{1}{3}$ tons cost x dollars.
5. What will twice as much cost at half as much per ton?
6. With how few coins can 90 cents be paid?
7. A's money is $\frac{1}{2}$ of B's and $\frac{1}{3}$ of C's. C has \$ 45; B has x .
8. Divide $3 \times 19 \times 12$ by 19. Striking out a factor does what?
9. 25 is $\frac{5}{8}$ of x and $\frac{5}{8}$ of y .
10. Divide $8 \times 37 \times 6$ by 37. By 48. By 24.

96. — For Rapid
Figuring.
[p. 8.]

During the year ending June, 1892, the coinage of the United States was as follows:—

<i>Gold Coins.</i>		<i>Silver Coins.</i>	
Double Eagles,	1,086,280	Dollars,	8,329,467
Eagles,	892,153	Half-Dollars,	1,942,033
Half-Eagles,	968,191	Quarter-Dollars,	12,093,324
Quarter-Eagles,	7,561	Dimes,	26,654,641

1. Find the number of coins for the year.
2. The value of all the gold pieces coined.
3. Of all the silver coinage.
4. $\frac{1}{2}$ m. = x ft.
 $\frac{1}{4}$ m. = x ft.
 $\frac{1}{8}$ m. = x ft.
 $\frac{7}{8}$ m. = x ft.
5. 27 acres = x sq. rd.
 $32\frac{1}{2}$ A. = x sq. rd.
 $13\frac{1}{2}$ cu. ft. = x cu. in.
3782 in. = x ft. y in.
7. Begin at the hundreds' figure to multiply 846 by 327. Is the first partial product 2538 or 253,800, and why?
8. Multiply 478 by 837. Write in words the third partial product.

9. 50,880 lb. of iron are worth a pound of gold, 16 lb. of silver, 71 lb. of nickel, or 6360 lb. of lead. How many pounds of iron can be bartered for one pound of each of the other metals? Omit fractions in the quotient.

10. In the United States silver is produced at the rate of about 45 cubic inches every minute; how many cubic feet are produced on an average in a day?

11. A young man with an income of \$ 600 yearly, smokes 3 cigars a day, which he buys at the rate of 3 for a quarter. What is the annual cost?

97.—Oral Reviews. 1. When was a man born who, on August
For dictation. 12, 1797, was just threescore and ten?

2. Begin at 13 and give every 13th number to 130.

3. How many inches in $2\frac{1}{3}$ yd.? In $\frac{1}{2}$ of $3\frac{1}{3}$ yd.?

4. A and B get the same wages. A works 3 days, and B 4. They earn \$ 21. Find the rate per day. What does each get?

5. Count by 16's to 144. By 18's. By 24's.

6. How can you tell the number of desks in your school without counting each one?

7. What is the difference between rent and interest? What is the annual rent of a \$ 3000 house when 10 % is charged?

8. If I pay 9 cents for the use of a dollar 1 year, what should I pay for the use of it for 4 months, or $\frac{1}{x}$ of a year?

9. What is the interest of \$ 300 for a year at 6 %?

10. Count to 100, $6\frac{1}{4}$ at a time. Mention 5 different aliquot parts of a thousand.

98.—At Sight.

1. Give quickly. 6 times 15, 20, 25, 12, 24.

2. A rubber stamp printing the words "The Property of the City of Brooklyn" costs $\frac{1}{3}$ ¢ a letter. For the whole, the cost is x .

3. Fred Jones hires a bicycle @ 25¢ an hour. What does he pay for the use of it from 9 A.M. to 3.10 P.M.?

4. A owns a third of a farm, B one-half of it. One has 50 A. more than the other. How much has each?

Give results, or value of x.

5. $25 \div 10?$ 6. $250 \div 10?$ 7. $2.5 \div 10?$ 8. 18 is $\frac{9}{10}$ of
 $2.5 \times 10?$ $25 \times 10?$ $0.25 \times 10?$ $(20 + x)$
 $25 \div 2.5?$ $250 \div 25?$ $2.5 \div 0.25?$ 9. $20 - x = 25 - 10$

10. Explain the effect of moving the decimal point.

99.—Denominate 1. A girl is 5, her mother 30. In 25 years
 Numbers. the daughter's age will be what part of her
 Written. mother's?

2. $\frac{1}{8}$ of a barrel of flour weighs x . 44 barrels at \$ $5\frac{1}{4}$ are sold in $\frac{1}{8}$ -bl. bags at 70¢. What is the profit on 1 barrel?

3. \$100 = how many pounds sterling? How many francs? How many marks?

4. Give the total value in U.S. money of an English shilling, a mark, and a franc.

5. When lessons are \$30 a quarter, the average cost per week is x . How many weeks and days in a leap year?

6. A barrel holding 49 gal. contains x cu. in. How does it compare in size with a barrel that holds 3 bushels, or y cu. in.?

7. Find the profit on an 8-peck barrel of cranberries costing \$8 and retailing a 2 qts. for a quarter.

8. A ton of wheat contains x bushels. At $2\frac{1}{2}$ marks a bushel it is worth \$ y .

9. At 60 lb. to the bushel, 3 bushels to the barrel, 9 tons of beans will fill x barrels.

10. How many barrels will be required for 8 T. 8 cwt. sugar, the contents of the barrels being 268, 254, 278 lb., and an equal number of each size being used?

- 100.—Cancellation. 1. What is the effect of cancelling or striking out the same factor in both dividend and divisor?

Oral.

ing out the same factor in both dividend and divisor?

Illustrate the principle, using—

2. $100 \div 20$. 3. $72 \div 24$. 4. Any larger numbers.

5. Explain each of the following ways of dividing 48×26 by 3×13 .

First Method.

$$\begin{array}{r} 13 \\ \times 3 \\ \hline 39 \end{array} \qquad \begin{array}{r} 48 \\ \times 26 \\ \hline 288 \\ 96 \\ \hline 1248 \end{array} \qquad \begin{array}{r} 32 \\ 39 \overline{)1248} \\ \underline{117} \\ 78 \\ \underline{78} \\ 0 \end{array}$$

Second Method.

$$\begin{array}{r} 16 \quad 2 \quad 16 \\ 48 \times 26 \\ 3 \times 13 = 32, \text{ or } 3 \overline{)48} \\ \underline{32} \quad 2 \\ 13 \overline{)26} \\ \underline{26} \\ 0 \end{array}$$

6. Which way do you prefer, and why?

Choose a method of reckoning mentally:—

7. $(64 \times 120) \div (8 \times 2 \times 10)$ 9. $(111 \times 120) \div (40 \times 37)$
 8. $(156 \times 75) \div (13 \times 15)$ 10. $(128 \times 85) \div (17 \times 16)$
 11. State the principle of cancellation.
 12. Divide $320 \times 725 \times 960 \times 570$ by $250 \times 380 \times 144 \times 16$.

101.—Written.

Solve, using both methods:—

1. $(18 \times 19 \times 240) \div (57 \times 36 \times 16)$
 2. $(220 \times 231 \times 84) \div (21 \times 11 \times 44)$ 3. $63360 \div 5280$

Formulate the following problems; that is, make an equation showing the process of solving each problem.

Shorten the work by cancelling common factors where possible, and find the result.

4. A steamer runs 528 m. in 24 h. In 96 h. \sim ?
 5. How many periods of 28 h. in 7 wk.?
 6. 192 T. coal cost \$1152; 1536 T. cost how much?
 7. Give distance in miles between two stations on a double-tracked railroad laid with 1056 rails 30 ft. long.

8. A bag of sage weighs 100 lb. Grinding and sifting costs a dollar and wastes 30 lb. of stems. Estimate the value of 10 bags after sifting, if sold at the retail price of 10¢ a quarter pound.

9. Two letter-carriers serving a length of 16 blocks, 10 houses to a block, on both sides of two streets, deliver in one day 2440 letters, 760 newspapers, etc.; what is the average number of pieces left at each house in one week?

10. In a hundredth of the first 5 calendar months how many 8-hour watches?

**102.—Review
Problems.
Written.**

1. In 1894 the deposits in the Minnesota savings banks amounted to \$8,954,575; the average to a depositor was \$232.63. How many depositors were there?

2. U. S. post office statistics for 1894:

No. of post offices, 69,805	Receipts, \$75,080,479
Miles of postal routes, 454,746	Expenditures, \$84,324,414

Find average receipts to an office, and the average expenditures per mile of postal routes.

3. Yale's time in 1894 was 23 min. 47 sec.; course, 4 miles. Feet rowed per minute?

4. The lowest price reached by wheat in the Chicago market in 1894 was in September, when it touched 50. The highest was in April, 65½. I bought 8000 bushels at the highest rate and sold at the mean price. What did I lose?

5. Compare the mean diameter of the sun, 866,400 miles, with that of the earth, 7918 miles.

6. In the total eclipse of the moon, September, 1895, the moon entered the shadow on the 3d at 10:51.7 p.m., and left it at 2:45.7 a.m. on the 4th. Duration of the eclipse in minutes?

7. Ocean steamers sometimes use 300 T. of coal every 24 hours. This is x pounds per minute.

8. There are 9 miles of perspiratory ducts in the human body. How many ducts are there if each is $\frac{1}{4}$ inch long?

9. The velocity of light is 186,337 miles per second. Light from the sun reaches us in 8.3 minutes. What is the sun's mean distance from us?

10. The equatorial circumference of the earth is about $6\frac{2}{7} \times 3963.296$ miles. What is the length of a degree of longitude at the equator?

Fractions.

103. — Fractional Forms 1. What is a fraction? 2. Is $\frac{5}{3}$ a fraction or an integer? 3. In what sense may it be **Indicate Division.** improper to call $\frac{5}{3}$ or $\frac{3}{4}$ fractions? Are they mixed numbers? 4. What are they in form? In value? 5. Give other expressions that may be called *improper* fractions. 6. What can you say of the number of units in a proper fraction?

7. Show how the expression $\$ \frac{5}{4}$ may mean either 5 coins or 2 coins.

8. Compare the expressions $\$ \frac{9}{4}$, $\frac{1}{4}$ of \$9, and $\$ 9 \div 4$.

9. Read the following, first as collections of fractional units and then as merely fractional forms of indicating division: $\frac{16}{4}$, $\frac{27}{9}$, $\frac{84}{12}$, $\frac{100}{25}$, $\frac{500}{50}$. 10. Give their values in integers. 11. Which numbers have you used as dividends? 12. Which as divisors? 13. Are denominators or numerators dividends? 14. Which are divisors?

104. — Improper Fractions. 1. Give the quotients: $\frac{21}{3}$, $\frac{43}{8}$, $\frac{77}{16}$, $\frac{5}{6}$, $\frac{14}{15}$, $\frac{20}{3}$. 2. Give the value of these improper fractions: $\frac{100}{33}$, $\frac{60}{9}$. 3. How do the terms of improper fractions compare in size? 4. What kinds of units are mixed in $17\frac{1}{3}$? In \$18.45?

5. How is an improper fraction changed to an integer or to a mixed number? 6. $\frac{23}{3} = x$; $1\frac{2}{3} \cdot 2 = x$.

7. Can 2 be divided by 3? Explain. 8. What 3 equal numbers added together make 2? 9. Does $\frac{2}{3}$ mean 2 thirds of 1 thing, or $\frac{1}{3}$ of 2 things? Explain, using objects.

10. What is the quotient when the divisor is larger than the dividend? 11. What part of 3 is 2?

12. What part of 8 is contained in 5?

Give the value of x in—

$$13. \frac{28}{5} = x$$

$$14. \frac{35}{x} = 7$$

$$15. \frac{x}{3} = 16$$

$$16. 11 \div 12 = x$$

105.—Mixed Num- 1. How many 7ths equal 1? How many
bers changed to Im- 7ths in 5? How many in $5\frac{3}{4}$? 2. How many
proper Fractions. 12ths in 1? In $\frac{1}{2}$ of 1? In 8? In $\frac{7}{12}$? In

Oral.

$8\frac{7}{12}$? 3. Change $8\frac{2}{3}$ to an improper fraction.

4. In changing a mixed number to an improper fraction (a) what two numbers are multiplied?

(b) What two are added?

(c) How may your result be proved correct?

Change to mixed numbers or improper fractions. Complete each line in 20 seconds.

$$5. 2\frac{5}{8}, 3\frac{1}{9}, 4\frac{7}{10}, 5\frac{11}{12}, 6\frac{4}{15}, 100\frac{10}{100}, 2\frac{5}{36}, 66\frac{2}{3}, 45\frac{9}{10}.$$

$$6. \frac{11}{7}, \frac{40}{13}, \frac{26}{25}, \frac{50}{11}, \frac{129}{10}, \frac{19}{17}, \frac{51}{24}, \frac{80}{35}, \frac{3517}{100}.$$

$$7. \text{Compare } \frac{175}{111} \text{ and } 15\frac{1}{11}. \quad 8. \frac{900}{50} = 2 \text{ times } x.$$

$$9. \text{What mixed numbers equal } \frac{100}{6}, \frac{100}{8}, \frac{100}{3}, \frac{100}{12}, \frac{100}{16}, \frac{200}{3}?$$

106.—For Written *Change from the fractional to the mixed form,*
Work. *and vice versa:—*

$$1. 39\frac{4}{9} \quad 3. 497\frac{11}{12} \quad 5. 511\frac{19}{20} \quad 7. 401\frac{13}{101} \quad 9. 465.27$$

$$2. \frac{600}{16} \quad 4. \frac{1128}{25} \quad 6. \frac{5000}{12} \quad 8. 968\% \quad 10. \frac{1809}{9}$$

$$11. \frac{20000}{231} \text{ gal.} = \text{how many gallons?}$$

$$12. \text{How many eggs in } 196\frac{1}{12} \text{ dozen?}$$

$$13. \text{Add } \frac{1728}{144}, \frac{1936}{4}, \text{ and } \frac{23276}{18}.$$

$$14. \text{Find the difference between } \frac{\$600}{25} \text{ and } \frac{\$3755}{4}.$$

$$15. \text{Which is larger, } \frac{1317}{13} \text{ or } \frac{1429}{14}?$$

$$16. \text{Add } \frac{79}{100}, \frac{120}{100}, 67\%, 0.97, \text{ and } 37\%.$$

Change to mixed numbers after cancelling:—

$$17. \frac{6408}{16}$$

$$18. \frac{1728}{120}$$

$$19. \frac{15000}{400}$$

$$20. \frac{7585}{125}$$

107.—Changes in the Number of Fractional Units to correspond with Changes in their Size.

1. 3 weeks; 21 days. Compare the size of the units; the number of the units; the value of the two expressions.
2. Why is the larger number of no greater value than the smaller?
3. Give other examples of equal values expressed in units of different size.
4. Compare the fractions $\frac{1}{4}$ and $\frac{3}{12}$ as to size of units, number of units, and value. Illustrate by drawing diagrams, or folding paper.
5. In the same way compare $\frac{3}{4}$ and $\frac{9}{12}$.

6. How does an increase in the numerator of a fraction affect its value? A decrease?

7. An increase in the denominator does what? A decrease? Illustrate with $\frac{1}{6}$.

Both terms of a fraction may be multiplied or divided by the same number without changing the value.

8. If each term of $\frac{5}{12}$ be made $\frac{1}{2}$ as large, how is the value affected? Why is this?

9. If each term of $\frac{2}{3}$ be made 3 times as large, how is the value affected? Explain.

10. What is done to the fractional unit when the denominator is doubled? When it is halved?

11. What is done to a fraction when its numerator is doubled? When it is halved? 12. How do you tell whether a fraction is large or small?

13. Show the effect produced in a quotient by multiplying or dividing dividend and divisor by 5.

108.—The Terms of a Fraction made Larger or Smaller.

1. Applying the principle stated above, how may a fraction be expressed in larger terms? 2. How in smaller? 3. How do you change 3ds to 12ths? 12ths to 3ds?
4. How many 12ths in $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$?

5. In changing $\frac{2}{3}$ to 20ths why do you multiply both terms by 4 instead of by 5? 6. In changing $\frac{3}{4}$ to 21ths do you divide both terms by 6 or by 7, and why?

7. Change to 40ths: $\frac{8}{8}, \frac{7}{10}, \frac{5}{4}, \frac{9}{5}, \frac{40}{80}, \frac{24}{160}, \frac{50}{200}, \frac{3}{20}, \frac{160}{240}$.
8. $\frac{3}{7} = \frac{x}{21}, \frac{x}{49}, \frac{x}{28}, \frac{x}{42}, \frac{x}{84}, \frac{x}{91}$ 9. $\frac{x}{60} = \frac{1}{20}, \frac{4}{15}, \frac{7}{12}, \frac{11}{30}, \frac{10}{5}$.
10. $\frac{5}{9} = \frac{x}{36}, \frac{x}{45}, \frac{35}{x}, \frac{10}{x}, \frac{x}{54}, \frac{60}{x}, \frac{x}{90}, \frac{80}{x}, \frac{100}{x}$.

11. Without changing the value, make the denominators of these fractions 156: $\frac{1}{4}, \frac{1}{52}, \frac{8}{13}, \frac{11}{52}, \frac{102}{312}, \frac{3}{168}, \frac{1}{12}, \frac{5}{78}$.

12. In changing a fraction to lower terms, how do you affect the size of the fractional units? Their number?

13. Why does multiplying both terms of $\frac{5}{6}$ by 8 leave the value unchanged?

109. — Rapid
Changes.
At sight.

Give an equivalent fraction in larger or
smallest terms: —

- | | |
|--|---|
| 1. $\frac{3}{4} = \frac{x}{56}$ or $\frac{75}{y}$ or $z\%$ | 6. $\frac{21}{35}, \frac{64}{72}, \frac{27}{96}, \frac{36}{108}, \frac{73}{95}$ |
| 2. $\frac{7}{8} = \frac{x}{120}$ or $\frac{63}{y}$ or $\frac{31}{z}$ | 7. 75%, 0.50, 80%, 0.90 |
| 3. $\frac{4}{5} = \frac{x}{45}; \frac{5}{9} = \frac{y}{45}$ | 8. $\frac{30}{105} = x; \frac{75}{500} = \frac{15}{y} = \frac{z}{20}$ |
| 4. $\frac{3}{16}, \frac{5}{24}, \frac{12}{96}$ to 48ths | 9. $\frac{7}{8} = \frac{x}{400} = \frac{105}{y} = \frac{z}{720} = \frac{875}{v}$ |
| 5. $\frac{5}{12}, \frac{48}{120}, \frac{3}{4}$ to 60ths | 10. $\frac{96}{144} = \frac{16}{x} = \frac{12}{y} = \frac{z}{24} = \frac{v}{288} = \frac{288}{u}$ |

110. — Greatest
Common Divisor of
Two or More Num-
bers.

1. Why do we call 72 a *composite* number and 73 a *prime* number? 2. What common divisors have 36 and 108? 60 and 90? 3. What is their *greatest* common divisor (g.c.d.)?

4. Numbers without a common divisor are prime to each other. Explain which of these are prime to each other:—27 and 35; 16 and 45; 27 and 45.

5. What advantage is there in using the g.c.d. in changing fractions to smallest terms? 6. Change to smallest terms, using the g.c.d.: $\frac{25}{125}$, $\frac{108}{144}$, $\frac{69}{92}$, 80%, 0.125, $\frac{96}{120}$.

**III. — Finding
G. C. D. when not
readily seen.**

For small divisors. 1–8. Which of these numbers are multiples of 2? of 3? 4? 5? 6? 8? 9? 10?

360	3123	8397	2160
1728	6984	6624	3240

NOTE. — Any number is a multiple

Of 2, if it ends in 2, 4, 6, 8, or 0; i.e. when it is even;

Of 3, if 3 divides the sum of its digits;

Of 4, if the last two figures are zeros or express a multiple of 4;

Of 5, if it ends in 0 or 5;

Of 6, if it is even and divisible by 3;

Of 8, if the last three figures are zeros or express a multiple of 8;

Of 9, if 9 divides the sum of its digits;

Of 10, if it ends in 0.

For large divisors. 9. Using 8 and 12, or any other two numbers having a common divisor, show that —

Any divisor of two numbers is a divisor —

I. *Of their sum.*

II. *Of their difference.*

III. *Of any multiple of either.*

To change $\frac{247}{533}$ to smallest terms, applying the principles just stated.

247)533(2

494

39)247(6

234

g.c.d. = 13)39(3

39

10. *Dividing the greater number by the less and the last divisor by the remainder, we find 13 to be the g.c.d. Show —*

a. That any divisor of 247 and 533 must divide every 247 in 533 (III.), and also the remainder 39 (II.);

b. That, therefore, the largest common divisor cannot be more than 39;

c. That any divisor of 247 and 39 must divide every 39 in 247 (III.), and the remainder 13 (II.);

d. That 13 divides both, but no larger number would. Hence 13 is the g.c.d. of 247 and 533. $\frac{13}{13} \mid \frac{247}{533} = \frac{19}{39}$.

112.—Fractions	1. $\frac{209}{297}$	4. $\frac{69}{171}$	7. $\frac{217}{465}$	10. $\frac{477}{1325}$
to Smallest Terms.	2. $\frac{136}{187}$	5. $\frac{261}{290}$	8. $\frac{333}{481}$	11. $\frac{1261}{1856}$
<i>Written.</i>	3. $\frac{273}{325}$	6. $\frac{209}{285}$	9. $\frac{123}{328}$	12. $\frac{1656}{2232}$

113.—Like Fractions to Add and Subtract.
Oral.

- What are like numbers? Give examples.
- What is an integer? A fraction? Is a fraction a number?
- Show the difference between integral and fractional units.

4. With regard to each of the following numbers mention (a) the integral unit, (b) the size and kind of the fractional unit, (c) the number of fractional units:—
 $\frac{5}{8}$ pk.; $\frac{7}{12}$ yr.; $\frac{1}{6}$ S; $\frac{3}{4}$ m.; S 0.15; 6%

Like fractions have units of the same size and kind.

5. Which of the following fractions have units (a) of the same size? (b) Of the same kind? (c) Of the same size and kind? Which are like fractions? $\frac{5}{6}$ yr.; $\frac{1}{6}$ yr.; $\frac{1}{4}$ yd.; $\frac{7}{8}$ day; S $\frac{2}{3}$; $\frac{5}{8}$ yd.

6. Why not add $\frac{7}{8}$ lb. and S $\frac{1}{8}$? $\frac{2}{7}$ wk. and $\frac{1}{2}$ wk.?

7. $\frac{7}{12} + \frac{3}{12} + \frac{10}{12} = \frac{x}{12}$ 9. $\frac{x}{36} - \frac{14}{36} = \frac{21}{36}$ 11. $72\% - \frac{x}{100} = 0.48$

8. $\frac{37}{64} + \frac{9}{64} - \frac{x}{64} = \frac{10}{64}$ 10. $0.55 - x = 0.30$ 12. $16\frac{2}{3} \div 18\frac{2}{3} = x$

13. How are like fractions added? How subtracted?

114.—Unlike Fractions to Add and Subtract.
Oral.

- 14 d. \div 3 wk. = x wk. or y days.
- Mention several unlike fractions. Why do you call them unlike?
- $\frac{1}{8}$, $\frac{3}{8}$, $\frac{4}{8}$, $\frac{5}{8}$. Which of these fractions have a common numerator? A common denominator?

Which are like fractions? Which unlike?

4. $\frac{9}{16} + \frac{3}{4} + \frac{5}{8} + \frac{7}{16} + \frac{1}{4} + \frac{3}{8} = x$. Add the like fractions first. 5. $\frac{1}{6} + \frac{1}{12} = x$. Why change 18ths to 6ths? 6. Add $\frac{1}{24}$ to $\frac{3}{8}$. Why change 8ths to 24ths?

7. Which term is common to like fractions? Change $\frac{2}{3}$ and $\frac{8}{10}$ to a common denominator. To the least common denominator. To 20ths.

8. In adding $\frac{3}{4}$ and $\frac{1}{8}$ shall we use 4ths or 60ths as the common fractional unit? 4 or 60 as the common denominator? 9. Why is it easier to use the largest common units — or the smallest common denominator — in adding? 10. Give three steps in adding $\frac{7}{8}$ and $\frac{6}{24}$.

Before finding their sum or difference, fractions must be changed to like fractions.

115. — Oral or Written.

- | | | |
|--------------------------------|----------------------------------|----------------------------------|
| 1. Change to a common unit. | 4. $\frac{8}{10} + \frac{4}{5}$ | 5. $\frac{2}{3} + \frac{3}{4}$ |
| 2. Add or subtract numerators. | 2. $\frac{9}{16} + \frac{2}{32}$ | 6. $\frac{7}{8} - \frac{5}{6}$ |
| 3. Simplify the result. | 3. $\frac{7}{10} - \frac{3}{4}$ | 7. $\frac{3}{18} - \frac{7}{24}$ |
| 4. Find short methods. | 4. $\frac{3}{7} + \frac{2}{35}$ | 8. $25\% + \frac{7}{8}$ |

9. $\frac{1}{6} - \frac{8}{48} + \frac{10}{12} + \frac{1}{24}$.

- | | |
|--|---|
| 10. $\frac{1}{8} + \frac{1}{9} + \frac{1}{12} + \frac{1}{24} + \frac{5}{72}$ | 15. $\frac{2}{7} + \frac{5}{12} + \frac{5}{7} - \frac{1}{3} + \frac{1}{4}$ |
| 11. $20\% + \frac{1}{3} + 0.20 + \frac{4}{5}$ | 16. $\frac{5}{6} + \frac{1}{18} + \frac{2}{30} + 2\frac{1}{2}$ |
| 12. $\frac{2}{40} + \frac{1}{4} + \frac{7}{8} + \frac{1}{16} + 25\%$ | 17. $\frac{3}{8} - \frac{2}{3} + \frac{8}{9} + \frac{7}{8}$ |
| 13. $\frac{4}{5} + \frac{7}{12} - \frac{2}{3} + \frac{2}{60}$ | 18. $36\% - \frac{1}{4} + 0.14 - \frac{1}{5}$ |
| 14. $13\% + 0.12 + \frac{3}{4} + \frac{2}{3}$ | 19. $\frac{1}{4} + \frac{3}{5} + \frac{1}{2} - \frac{5}{10} + \frac{7}{10}$ |
| 20. $\frac{1}{7}$ wk. + $\frac{2}{365}$ yr. + $\frac{1}{8}$ d. - $\frac{1}{15}$ mo. + 180 min. = x | |

116. — Multiples:
Common Multiples;
Least Common Multiples.

NOTE. — The term "dividend" may be substituted for "multiple."

1. Show that 36, 60, 72, and 120 are multiples of 12.
2. Show that 50, 60, 80, and 120 are multiples of 10.
3. Show that 60 and 120 are common multiples of 10 and 12.
4. Show that 60 is the least common multiple (l.c.m.) of 10 and 12.

5. 2, 6, 3, 10, 15, 5 are factors of 30. Which of them are the *prime* factors of 30? What is the product of these prime factors?

6. 2, 6, 3, 14, 7, 21 are factors of 42. Select from them the prime factors of 42 and give their product.

7. The product of the prime factors of a number is always ∞ .

8. Show that any multiple of 30 contains all its prime factors.

9. Show that any multiple of 42 contains all its prime factors.

10. Show that any common multiple of 30 and 42 contains all the prime factors of each.

117.—Finding the
L. C. M.

Oral and written.

To find the *least com-* $30 = 2 \times 3 \times 5$
mon multiple of 30 and $42 = 2 \times 3 \times 7$
42; *i.e.* the multiple $42 \times 5 = 210$, the l.c.m.
containing only such prime factors as are

needed to produce each number separately.

1. Will a number whose factors are 2, 3, 5, 7 be a multiple of $2 \times 3 \times 5$ or 30? Of $2 \times 3 \times 7$ or 42? 2. Why need not the 2 and the 3 be used twice in this multiple?

3. What factor not found in 42 is needed in the common multiple?

4. Find the l.c.m. of 60 and 84.

5. Find the least number exactly divisible by 60, 72, and 108.

$$60 = 2 \times 2 \times 3 \times 5$$

$$72 = 2 \times 2 \times 3 \times 3 \times 2$$

$$108 = 2 \times 2 \times 3 \times 3 \times 3$$

$$5 \times 2 \times 108 = x$$

6. What prime factor of 60 is not found among the prime factors of 108? Of 72?

7. What is meant by the least common multiple of several numbers?

8. What is the l.c.m. of 60, 84, and 132?

9. Find the l.c.m. of 45, 90, 100, and 200.

$$45$$

$$90 = 2 \times 5 \times 3 \times 3$$

$$100$$

$$200 = 2 \times 5 \times 2 \times 2 \times 5$$

$$200 \times 3 \times 3 = x.$$

10. Is a multiple of 90 a multiple of 45? Compare 200 and 100 in this respect. How then may the process be shortened?

NOTE 1.—The prime factors of large numbers may often be got by finding composite factors first, and then the prime factors of these. Thus:—

$$120 = 10 \times 12 = (2 \times 5) \times (2 \times 2 \times 3); \quad 180 = 10 \times 18 = (2 \times 5) \times (2 \times 3 \times 3).$$

NOTE 2.—For other methods of finding the l.c.m. see the Appendix, p. 8.

Find the least common multiple of—

- | | | |
|--------------------|---------------------|--------------------|
| 11. 15, 21, 45 | 13. 16, 25, 80, 100 | 15. 34, 85, 51, 68 |
| 12. 16, 18, 27, 72 | 14. 12, 18, 96, 144 | 16. 480, 600, 1000 |

Practice in Changing Fractional Forms.

118. — Oral.

- Read as mixed numbers:
 $\frac{39}{8}, \frac{47}{9}, \frac{289}{14}, \frac{365}{30}, \frac{300}{29}$.
- Put into fractional form:
 $4\frac{5}{6}, 5\frac{6}{7}, 10\frac{3}{4}, 20\frac{3}{4}, 6\frac{7}{8}, 13\frac{1}{2}$.
- Change to smallest terms:
 $\frac{28}{44}, \frac{34}{51}, \frac{63}{72}, \frac{32}{72}, \frac{64}{96}, \frac{60}{84}, \frac{108}{120}$.
- Simplify the form:
 $\frac{20}{32}, \frac{100}{1}, \frac{18}{42}, \frac{132}{140}, \frac{1111}{1440}, 0.160$.
- Read as 144ths: 1, 2, $\frac{1}{12}$,
 $\frac{1}{16}, \frac{3}{16}, \frac{19}{72}, \frac{178}{288}, \frac{1000}{1440}, \frac{1000}{72}$.
- Change to smallest terms,
using g.c.d.: $\frac{25}{75}, \frac{27}{81}, \frac{52}{156}, \frac{48}{80}$,
 $\frac{320}{480}, \frac{72}{108}, \frac{72}{288}, \frac{47}{141}$.
- Find l.c.m. of 5, 7, 35, 70;
8, 16, 128.
- $6\frac{1}{4}$ = how many 8ths?
- $\frac{3}{7} = \frac{x}{56}$; $\frac{21}{x}$; $\frac{x}{63}$.
- $\frac{x}{72} = \frac{5}{6}$; $\frac{5}{12}$; $\frac{5}{8}$; $\frac{5}{9}$.
- What parts of 144 are 1, 2,
3, 4, 6, 8, 9, 12, 72?
- Compare in value:
 $\frac{125}{99}, \frac{35}{175}, \frac{91}{99}, \frac{77}{99}, \frac{40}{96}, \frac{36}{99}$.
- Read in order of size:
 $\frac{1}{10}, \frac{1}{3}, \frac{2}{7}, \frac{9}{15}, \frac{3}{11}, \frac{14}{15}, \frac{1}{21}, \frac{12}{12}$.

119. — Written.

- Write two large fractions,
using four figures.
- Express the same values
with 12 figures.
- Change to mixed num-
bers: $\frac{8243}{91}$; $\frac{1000}{33}$; $\frac{700}{8}$; $\frac{6251}{1000}$.
- Simplify the form of $\$1\frac{30}{20}$;
 $\frac{486}{12}$ doz.; $\frac{3527}{128}$ cd.
- Change $\frac{1500}{83760}$ to smallest
terms.
- Express $\frac{1}{2}$ as 625ths.
- $\frac{99}{9008}$ in smallest terms = ?
- Change $\frac{27}{81}$ and $\frac{16}{729}$ to like
fractions.
- Find the least common
multiple of 16, 48, 96, 108.
- Change to smallest terms,
 $\frac{440}{3080}$.
- Express $\frac{247}{333}$ with four
figures.
- Condense this expression:
 $140\frac{21000}{140000}$.
- Compare $196\frac{1}{3}$ and $295\frac{4}{5}$.
- What part of 1728 is 576?
- $\frac{63 \times 2000 \times 3\frac{1}{2}}{125 \times 1260 \times 70}$ = what?

120. — Fractions
Added or Subtracted.

Process.

$15 = 3 \times 5$
 $18 = 2 \times 3 \times 3$
 $18 \times 5 = 90$, l.c.m.
or l.c.d.

$$\frac{11}{15} \times \frac{6}{6} = \frac{66}{90}$$

$$\frac{7}{18} \times \frac{5}{5} = \frac{35}{90}$$

$$\text{Sum} = \frac{101}{90} = 1\frac{11}{90}$$

$$\text{Difference} = \frac{31}{90}$$

1. What is the first step to be taken in adding or subtracting $\frac{11}{15}$ and $\frac{7}{18}$? 2. Is the common fractional unit readily seen?

3. What is the l.c.m. of 15 and 18? 4. What then will be the smallest common fractional unit, or the least common denominator?

5. How many 90ths = $\frac{1}{15}$? $\frac{1}{18}$? 6. $\frac{1}{18}$ = how many 90ths? $\frac{7}{18}$?

7. How are the size and the number of fractional units changed by multiplying both terms of $\frac{1}{18}$ by 6?

8. What remains to be done after the fractions have been changed to like fractions?

121. — Written.

1. $\frac{7}{15} + \frac{4}{21} = x$

2. $\frac{31}{33} + \frac{17}{18} = x$

3. $\frac{7}{24} + x = \frac{11}{21}$

4. $\frac{29}{48} + \frac{29}{32} = x$

5. $\frac{8}{10} + \frac{625}{1000} = x$

6. $0.8 - 0.625 = x$

7. $80\% - \frac{5}{8} = x$

8. $\frac{3}{7} + \frac{5}{12} + \frac{23}{42} = x$

9. $51\frac{5}{2} + 21\frac{49}{65} = x$

10. $x + \frac{13}{48} = 7\frac{29}{60}$

11. Add $\frac{7}{40} + \frac{17}{30}$ to the sum of $\frac{61}{80}$ and $\frac{41}{60}$.

12. What shall be added to $\frac{5}{36}$ to make $\frac{13}{32}$?

13. Which is greater, $\frac{9}{25}$ or $\frac{7}{16}$? How much?

14. Add $\frac{35}{80}$ to $\frac{49}{112}$. What should be the first step here; and why?

15. $24\frac{1}{4} + 23\frac{1}{8} + 72\frac{3}{4} + 16\frac{5}{16} + 18\frac{8}{32}$. Add the fractions mentally.

16. Add $\frac{13}{40}$, 16%, 0.027, and $\frac{18}{1000}$.

17. $32\frac{5}{9} + 17\frac{4}{5} = x$

122. — Subtracting
Mixed Numbers.

Oral.

Process.

$$75\frac{7}{8} = 75\frac{28}{32} = 74\frac{100}{32}$$

$$57\frac{11}{4} = 57\frac{33}{12} = 57\frac{88}{32}$$

$$\text{Difference} = 17\frac{67}{32}$$

1. In subtraction of integers, what is done when a digit in the subtrahend is larger than the one above it in the minuend?

2. How is the $\frac{100}{32}$ obtained in the process at the left? The 74 ?

3. Tell how to take $7\frac{3}{8}$ from $15\frac{1}{4}$.

4. $12\frac{1}{4} - 7\frac{3}{4} = x$ 6. $10 - 3\frac{7}{8} - 2\frac{5}{8}$ 8. $9\frac{3}{4} - 5\frac{1}{2}$
 5. $100 - x = 8\frac{5}{8}$ 7. $5\frac{7}{8} - 3\frac{3}{8} - \frac{3}{4}$ 9. $33\frac{1}{3} - 12\frac{1}{3}$
 10. $20 - \frac{7}{8} - \frac{7}{8} - \frac{7}{8} - \frac{7}{8}$ 12. $15 - 2\frac{6}{7} - 2\frac{6}{7} - 2\frac{6}{7}$
 11. $9\frac{3}{5} - \frac{4}{5} - 5\frac{3}{10}$ 13. $20\frac{1}{2} - \frac{3}{4} - 75\% - 0.75$

123. — Written.

1. $63\frac{1}{2} - 41\frac{7}{8}$ 3. $72\frac{7}{8} - 15\frac{1}{4}$
 2. $94\frac{1}{6} - 18\frac{5}{4}$ 4. $19\frac{1}{6} - 5\frac{1}{8}$
 5. $14\frac{9}{5} - 6.51$ 6. $0.875 - 0.625 + \frac{3}{10} + \frac{17.5}{1000}$
 7. $114\frac{1}{30} - 16\frac{1}{2} + 84\frac{1}{30} - 16\frac{2}{3}$
 8. What is the difference between $87\frac{1}{2}\%$ and $66\frac{2}{3}\%$?
 9. What added to $17\frac{9}{10}$ gives $29\frac{9}{35}$?
 10. From $18\frac{1}{5}$ take 9.935

**124. — Problems
with Fractions.***Oral or written.*

1. $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{1}{7}$ of a number = 110. Is the unknown number larger or smaller?
 2. A barrel is $\frac{3}{4}$ full. Draw off $\frac{1}{3}$ of a barrel and $\frac{2}{5}$ of a barrel. What part remains?
 3. A stone wall cost \$1 a rod. What costs 3 days' work, or $6\frac{2}{3}$ rds., $5\frac{3}{4}$ rds., and $7\frac{5}{6}$ rds.?
 4. What % of an income is collected when 0.125 of it, 25%, 0.35, and 0.025 are paid in?
 5. A chimney contains 132 courses of brick. $\frac{1}{3}$ are under ground, $\frac{2}{5}$ roofed in; how many courses are exposed?
 6. How many cords of wood in $\frac{3}{2}$ cd. sawed by hand, $\frac{2}{128}$ cd. by machine, and $\frac{7}{16}$ cd. chopped?
 7. Two pumps contribute $\frac{1}{3}$ and $\frac{1}{4}$ toward filling a reservoir, springs contribute $\frac{1}{5}$ and $\frac{1}{6}$, surface water the rest. How much more do the pumps yield than other sources?
 8. In a 10 acre marsh lot three men cut $\frac{1}{4}$, $\frac{5}{12}$, and $\frac{2}{9}$ of the whole. What part remains for a fourth man to cut?
 9. If you invest $\frac{2}{13}$ of what you have in one way and $\frac{3}{5}$ in another, what remains?
 10. At \$3 a day what is due a man for working half a day, $\frac{5}{8}$ d., $\frac{3}{4}$ d., and $2\frac{1}{2}$ d.?

125.—To Multiply a Fraction by increasing the Number of Parts.

1. 9×7 units = x units. Does it matter whether these units are integral or fractional?
 2. Then it follows that 8×3 fifths = x fifths, and $9 \times \frac{7}{8} = \frac{x}{8}$.

3. $7 \times \frac{4}{9} = \frac{x}{9}$, or 3 and $\frac{y}{9}$

7. The product of $\frac{20}{11}$ by $7 = x$

4. $6 \times \frac{7}{10} = x$, or y and $\frac{1}{5}$

8. $24 \times \$\frac{2}{3} = x$

5. $\frac{8}{9} + \frac{8}{9} + \frac{8}{9} + \frac{8}{9} + \frac{8}{9} = x$

9. $100 \times \$\frac{2}{3} = x$

6. $\frac{7}{12}$ multiplied by 11 = x

10. $50 \times \$\frac{5}{6} = x$

11. $25 \times \$\frac{3}{8} = x$

12. Multiplicand $\frac{7}{15}$, multiplier 9, product?

13. In the preceding exercises, have we changed the *number* or the *size* of the fractional units?

126.—To Multiply a Fraction by Increasing the Size of Parts.

1. James has $\$ \frac{3}{4}$, Henry has twice as much. Has Henry 6 quarters or 3 halves?
 $2 \times \frac{3}{4} = \frac{3}{2}$ or $\frac{6}{4}$?

2. If I have ten $\frac{1}{4}$'s of a dollar and change them for as many coins of double the size, what fractional parts do I get, and what increase in value?

3. Compare $\frac{1}{16}$ and $\frac{1}{4}$. $4 \times \frac{7}{16} \text{ lb.} = \frac{7}{4} \text{ lb.}$ What change is made

in value when instead of 16ths, we take as many 4ths?

Find the product by increasing the size of the parts:—

4. $6 \times \frac{7}{12}$

7. $10 \times \frac{7}{10}$

10. $15 \times \frac{7}{10}$

5. $8 \times \frac{3}{4}$

8. $12 \times \frac{10}{36} \text{ yd.}$

11. $25 \times \frac{18}{100}$

6. $3 \times \frac{11}{24}$

9. $18 \times \frac{3}{54} \text{ m.}$

12. $36 \times \frac{15}{144}$

127.—Multiplier, a Fraction; Multiplicand, an Integer.

1. Which is larger, product or multiplicand?
 2. Is it proper to say either $\frac{1}{3}$ times \$6, or \$6 multiplied by $\frac{1}{3}$?
 3. What about the product here? Is anything really multiplied?

4. In finding $\frac{1}{4}$ of \$8 to be \$2, do we multiply or divide? 5. $\frac{2}{3}$ of 6 yr. (or $2 \times \frac{1}{3}$ of 6 yr.) is 4 years. What part of the multiplicand has been taken? What does multiplying mean?

6. Show that multiplying by a fraction is finding one or more of the equal parts of a number. 7. Is this more like multiplication or division?

8. $\frac{3}{8}$ of 24 h. 13. $\frac{5}{7}$ of 30 = $5 \times 4\frac{2}{7} = 20\frac{10}{7} = 21\frac{3}{7}$. Explain.
 9. $\frac{7}{9}$ of 36 in. 14. $\frac{5}{7}$ of 30 = $\frac{5 \times 30}{7} = \frac{150}{7} = 21\frac{3}{7}$. Explain.
 10. $1\frac{1}{2}$ of 84 mo. 15. $\frac{8}{9}$ of 20 18. $\frac{3}{16}$ of 100
 11. $\frac{9}{15}$ of 60 d. 16. $\frac{7}{12}$ of 50 19. 0.23 of 200
 12. $\frac{19}{25}$ of \$100 17. $\frac{5}{6}$ of 100 20. 24% of 500

128.—The Product of Fractions and Integers. 1. Show that $\frac{7}{10}$ of \$20 = $20 \times \$\frac{7}{10}$.
 2. What is the principle of cancellation?

Written.

3. Compare $\frac{18}{35}$ of $105 = x$ with $105 \times \frac{18}{35} = x$.
 4. $\frac{17}{60}$ of $144 = \frac{204}{5} = 40\frac{4}{5}$. Explain.

5. $1\frac{3}{8}$ of 105. 7. 18% of 250. 9. $1\frac{7}{8}$ of 225 tons.
 6. $65 \times \frac{7}{13}$. 8. $\frac{8}{68}$ of 85. 10. $840 \times 1\frac{7}{8}$ yd.

129.—One Factor a Mixed Number. 1. At \$3.50 a yard what will $\frac{3}{8}$ yard cost?
 2. If one revolution of a wheel requires

Written.

- $\frac{5}{6}$ of a second, how long will it take it to revolve 1000 times?
 3. $\frac{2}{3}$ of 400 = x ; $16 \times 400 = y$; $16\frac{2}{3} \times 400 = x + y$.
 4. $17 \times \frac{3}{8} = x$; $17 \times 300 = y$; $17 \times 300\frac{3}{8} = x + y$.
 5. At \$0.95 a pound what will $74\frac{7}{8}$ pounds of tea cost?

Process.

$$\begin{array}{r} \$0.95 \\ 74\frac{7}{9} \\ 9) \$6.65 \text{ or } a \times \$0.95 \\ 0.73\frac{8}{9} = \frac{7}{9} \text{ of } b \\ 3.80 = c \times \$0.95 \\ 66.50 = d \times \$0.95 \\ \$71.03\frac{8}{9} \text{ or} \\ \$71.04 \end{array}$$

Copy the accompanying process, supplying values for the letters.

NOTE. — When the cost includes a fraction of $\frac{1}{2}$ cent or more, it is customary to count the fraction as another cent. Why is this done? Answers given in this book conform to this custom.

6. Price \$8.75; quantity $18\frac{3}{8}$ cords; total cost?
7. Weight of one bag of coffee, $35\frac{7}{8}$ lbs.; 19 bags weigh what?

In the process at the left give the values of a , b , c , and d .

$$\begin{array}{r} 35\frac{7}{8} \text{ lb.} \\ 19 \\ 8) 133 \text{ or } 19 \times a \\ 16\frac{5}{8} \text{ lb. } 19 \times b \\ 315 \text{ " } 9 \times c \\ 35 \text{ " } d \times 35 \\ 681\frac{5}{8} \text{ " } \end{array}$$

8. 15 boxes; one weighs $18\frac{3}{4}$ lb.; all?
9. \$6.25 each; $27\frac{5}{8}$ yards; cost?
10. $15\frac{7}{12}$ miles an hour; 25 hours?
11. Rent, \$28; time, $7\frac{3}{5}$ months.
12. Time, $29\frac{1}{2}$ d.; sailing rate, 185 miles.
13. $\frac{5}{8}$ of \$785 was lost in speculation; what remained?

130.—Problems.

For written work.

1. What remains of a 49-yd. piece of cloth after selling $\frac{5}{7}$ of it, $\frac{2}{7}$ of the rest, and 4 yd.? What is the remnant worth at $8\frac{5}{8}$ ¢ per yard?
2. Another piece of 47 yards is damaged. One half sold at 7¢. Of the other half $2\frac{3}{4}$ yards were unsalable, but the rest went at 5¢. Give the total receipts.
3. At the rate of \$1 $\frac{5}{8}$ a day, figure a board bill in dollars and cents for 3 months from August 1.
4. If a glacier moves uniformly a hundred feet a year, how far does it go in 181 days?
5. A sawyer works up wood at the rate of $\frac{9}{16}$ cd. a day. What can he do in 26 weeks if he takes a half-holiday each Saturday?
6. A wind storm passes over $\frac{1}{3}\frac{1}{2}$ m. in $3\frac{3}{16}$ sec. In what time would it go a mile? 7. If it travels $\frac{3}{20}$ m. each second, how far would it go in an hour?

8. Find the cost of seven 50-gal. barrels of oil at three for \$16.71.
 9. Supposing an empty barrel to be worth \$1.25, what is the oil worth per gallon?
 10. An unfailing spring flows $37\frac{1}{2}$ barrels daily. How much would it yield in October?

131. — To Find One or More Equal Parts of a Fraction. 1. $\frac{2}{3}$ of 6 things (apples, dollars, fourths) = x .
 2. $\frac{2}{5}$ of 10 twelfths = x ; $\frac{2}{5}$ of $\frac{10}{12} = \frac{x}{12}$.

$$3. \frac{5}{6} \text{ of } \frac{12}{13} = \frac{x}{13}$$

$$5. \frac{8}{9} \text{ of } \frac{27}{35} = x$$

$$7. \frac{5}{13} \text{ of } \frac{39}{40} = x$$

$$4. \frac{7}{8} \text{ of } \frac{16}{25} = x$$

$$6. \frac{7}{10} \text{ of } \frac{40}{19} = x$$

$$8. \frac{11}{12} \text{ of } \frac{72}{5} = x$$

To separate $\frac{3}{4}$ into 5 equal parts and give the value of 3 such parts; that is, to find $\frac{3}{5}$ of $\frac{3}{4}$, or to multiply $\frac{3}{4}$ by $\frac{3}{5}$.

Process.

9. How does increasing the denominator affect the size of the fractional units? 10. If we make the denominator 5 times as large, how is the value of the fraction changed? $\frac{1}{5}$ of $\frac{3}{4} = x$. 11. If $\frac{1}{5}$ of a fraction = $\frac{3}{20}$, $\frac{3}{5}$ of it will be how many times $\frac{3}{20}$?

12. Which part of the process gives the product of the numerators? Of the denominators? 13. Make a rule for finding the product of two fractions (i.e. for finding one or more of the equal parts of a fraction).

132. — Fractions

Multiplied.

Cancellation.

1. Find $\frac{10}{12}$ of $\frac{24}{35}$, or multiply $\frac{24}{35}$ by $\frac{10}{12}$.

2. Of what use is cancellation? 3. On what principle is it based? 4. Which

Process.

$$\frac{10}{12} \times \frac{24}{35} = \frac{240}{420} = \frac{4}{7}$$

$$\frac{10}{12} \times \frac{24}{35} = \frac{4}{7}$$

is easier in the process at the right, to change the product to lowest terms or to cancel first?

$$5. \frac{9}{10} \text{ of } \frac{20}{27}$$

$$7. \frac{13}{21} \times \frac{35}{39}$$

$$9. 85\% \text{ of } \frac{10}{17}$$

$$6. \frac{4}{17} \text{ of } \frac{51}{60}$$

$$8. \frac{25}{36} \times \frac{24}{70}$$

$$10. 96\% \text{ of } \frac{25}{32}$$

133.—Multiplying Fractional Numbers.

Written.

1. Find the product of $\frac{1}{12}$ and $\frac{2}{7}$.
2. What is $\frac{2}{3}$ of $7\frac{1}{2}$ ($\frac{2}{3}$ of $\frac{1}{2}$)?
3. $8\frac{3}{4} \times 6\frac{3}{7} = x$ (First step?)
4. How may the product of small mixed numbers be found?
5. $2\frac{2}{3} \times 7\frac{5}{16}$
6. $4\frac{4}{5} \times 15\frac{5}{8}$
7. $\frac{1}{7}$ of $\frac{3}{5}$ of $3\frac{1}{2}$
8. $2\frac{2}{3} \times 4\frac{1}{5} \times 7\frac{1}{2}$
9. 16% of $5\frac{1}{3}$
10. $9\frac{1}{7} \times 12\frac{1}{10}$
11. $6\frac{5}{8} \times 246 = \text{what?}$ [\$ 68.]
12. $6\frac{3}{4} \times 137\frac{5}{8} = \text{what?}$

Explain the process.

$$\begin{array}{r} 246 \\ 6\frac{5}{8} \\ \hline 8)1230 \\ 153\frac{3}{4} \\ \hline 1476 \\ \hline 1629\frac{3}{4} \end{array} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} = (5 \times 246) \div 8$$

$$1476 = 6 \times 246$$

13. $8\frac{1}{3} \times 126$

14. $149 \times 7\frac{2}{3}$

15. $8\frac{5}{8} \times 209\frac{1}{2}$

16. $6\frac{5}{6} \times 194\frac{3}{5}$

17. $26\frac{1}{2} \times 795\frac{2}{3}$

18. $84\frac{7}{8} \times 641\frac{5}{12}$

19. $96\frac{7}{8} \times 109\frac{3}{10}$

Explain the process.

$$\begin{array}{r} 137\frac{5}{8} \\ 6\frac{3}{4} \\ \hline 6 \times 137 = 822 \\ \frac{3}{4} \times 137 = 102\frac{3}{4} \\ \hline 6 \times \frac{5}{8} = 3\frac{3}{4} \\ \frac{3}{4} \times \frac{5}{8} = \frac{15}{32} \\ \hline 928\frac{31}{32} \end{array}$$

134.—Business

Problems.

For written work.

Make out bills in full for —

1.

2.

17 doz. ... @ \$ 1.62 $\frac{1}{2}$

15 $\frac{2}{3}$ doz. ... @ \$ 1.00

17 $\frac{5}{8}$ yd. ... @ 10¢

1 $\frac{1}{4}$ yd. ... @ 62 $\frac{1}{2}$ ¢

3. Find the cost of $9\frac{3}{5}$ tons of coal at \$ 7.41.
4. Twenty pounds of sugar bought @ $4\frac{7}{16}$ are sold for \$ 1.25. At this rate what is gained on a barrel of 200 lbs.?
5. Oil is bought at \$ 3.50 for a 42-gal. barrel and retailed at $12\frac{1}{2}$ ¢. The gain is what part of the cost?
6. Oranges bought at 3 for 5¢ are sold at 4 for 9¢. What is gained on a box of 9 doz., 1 in 12 of which are worthless?
7. I can buy blank books of one dealer at the rate of \$ 1.25 a hundred; of another at \$ 1.60 a gross. How much is one offer better than the other?

135.—Products at

Sight.

Oral.

Multiply each fraction in the table by the number at the end of its line or column. Change any fraction in the product to a lower denomination when possible. Thus:—

$$4 \times \frac{5}{9} \text{ yd.} = \frac{20}{9} \text{ yd.} = 2\frac{2}{9} \text{ yd.} = 2 \text{ yd. } 8 \text{ in.}$$

	12	4	6	10	15	
2	$\frac{3}{4}$ qt.	$\frac{5}{9}$ yd.	$\frac{4}{15}$ h.	$\frac{7}{9}$ sq. yd.	$12\frac{1}{2}\%$	12
3	$\frac{5}{8}$ yd.	$\frac{7}{8}$ gal.	$\frac{1}{4}$ d.	$\frac{8}{15}$ min.	$16\frac{2}{3}\%$	11
4	$\frac{5}{8}$ pk.	$\frac{3}{16}$ lb.	$\frac{7}{12}$ ft.	$\frac{1}{12}$ sq. ft.	$33\frac{1}{3}\%$	10
5	$\frac{3}{4}$ ft.	$\frac{5}{12}$ yr.	$\frac{1}{10}$ T.	$\frac{9}{20}$ sec.	$62\frac{1}{2}\%$	9
6	$\frac{7}{8}$ wk.	$\frac{9}{10}$ in.	$\frac{8}{9}$ ed.	$\frac{7}{10}$ h.	$37\frac{1}{2}\%$	8
	7	8	9	12	7	

Division of Fractions.

136.—Fractions to be Divided. $\frac{3}{4} \div 5$. To divide $\frac{3}{4}$ into 5 equal parts.

1. Each part will be $\frac{1}{5}$ of $\frac{3}{4}$ or $\frac{3}{20}$.

[See §§ 107, 131.]

2. What is meant by $\frac{3}{4} \div 4$?

3. $\frac{3}{10} \div 5 = x$; $5 \times x = \frac{3}{10}$.

4. Divide $\frac{3}{4}$ ft. into 6 equal lengths. 5. $\frac{7}{12} \div x = \frac{7}{24}$.

6. $\frac{4}{11} \div 4 = \frac{x}{17}$. 7. $\frac{4}{11} \div 4 = \frac{15}{x}$. 8. Using the two preceding

examples as illustrations, give two ways of dividing a fraction into equal parts. 9. When is the second method used? 10. Which is shorter?

11. $\frac{1}{2}$ of $\frac{3}{10}\%$

14. $\frac{1}{2}$ of $\frac{4}{5}$

17. $0.15 \div 6$

12. $\frac{3}{8} \div 8$

15. $\frac{5}{8} \div 12$

18. $48\% \div 16$

13. $\frac{3}{8} \div 8$

16. $\frac{2}{3} \div 20$

19. $\frac{8}{100} \div 17$

137.—The Divisor $5 \div \frac{3}{4}$. *To find how many times $\frac{3}{4}$ is contained in 5.*
a Fraction; the Dividend an Integer. 1. How many times is $\frac{1}{4}$ contained in 1, or how many 4ths in 1?

2. How many 4ths in 5? $5 = \frac{x}{4}$ 3. 3's in 20? $\frac{20}{4} \div \frac{3}{4} = x$.
 4. 2 ft. \div 3 in. = x ; or, since dividend and divisor must represent like units, 24 in. \div 3 in. = x .
 5. $\frac{7}{9} \div \frac{4}{9} = x$. 7. $7 \div \frac{2}{5} = x$. 9. $\frac{8}{11}$ in 8, x times.
 6. $8 \div \frac{4}{3} = x$. 8. $\frac{5}{6}$ in 7, x times. 10. Divide 17 by $\frac{3}{4}$.

138.—The Dividend $\frac{3}{4} \div \frac{2}{3}$. *To find how many times $\frac{2}{3}$ is contained in $\frac{3}{4}$.*
and Divisor both Fractional. 1. Which is the larger, $\frac{3}{4}$ or $\frac{2}{3}$? Are the

- units of the same size? 2. $\frac{3}{4} = \frac{x}{12}$; $\frac{2}{3} = \frac{y}{12}$; $x \div y = 1\frac{1}{2}$.
 3. $\frac{1}{3}$ or $\frac{5}{12}$ is contained how many times in $\frac{4}{3}$ or $1\frac{2}{3}$? $12 \div 5 = x$.
 4. $\frac{5}{6} \div \frac{2}{3}$ 6. $\frac{8}{9} \div \frac{3}{4}$ 8. $\frac{5}{12}$ in $\frac{7}{5}$ 10. Divide $\frac{37}{12}$ by $\frac{5}{16}$
 5. $\frac{3}{4}$ in $1\frac{1}{2}$ 7. $\frac{7}{10} \div \frac{2}{3}$ 9. $\frac{4}{5} \div \frac{4}{7}$ 11. Divide 0.9 by 30%
 12. What is the first step in dividing days by hours? In dividing 4ths by 5ths? One fraction by another?

139.—Denominate Numbers. 1. $\frac{2}{3}$ of a case of slates at \$4 $\frac{1}{2}$ a case cost \$ x . 2. A ten-pound box of marbles contains $\frac{1}{2}$ yellow, $\frac{2}{3}$ blue, and 48 red. The box contains how many? 3. The box cost 30¢, and the marbles sell at ten for a cent. The profit is x .

4. A girl who earned $\frac{2}{3}$ of a dollar gave $\frac{2}{3}$ of it to one who had nothing. With the rest she bought three things that may have cost x , y , and z cents.

5. A dealer bought \$10 worth of oranges. After selling a fifth of them for \$3, he sells the rest for what the whole had cost. What was the profit?

6. $\frac{1}{4}$ of a bushel of berries are picked; $\frac{1}{2}$ of them are sold to one man, $\frac{1}{4}$ of the remainder to another. How many quarts are unsold?

7. A certain sand-glass runs ten minutes. It runs out twice during practice time at noon; $\frac{1}{30}$ of an hour are spent in the morning, and $\frac{7}{12}$ at night. Give the sum in hours. In 6 days it would be x .

8. At $5\frac{1}{3}$ ¢ each 64¢ buys x , and 80¢ buys y .

9. \$10,000 in postage stamps are divided among 4 offices. If $\frac{9}{10}$ of them are twos and the rest ones, how many ones does each office get? 10. How many twos are distributed?

140.—The Divisor

Inverted.

Oral and Written.

(a) $\frac{7}{8} \div \frac{3}{5}$. To find how many times $\frac{3}{5}$ is contained in $\frac{7}{8}$.

First Process.

$$\frac{7}{8} \div \frac{3}{5} = \frac{35}{40} \div \frac{24}{40} = \frac{35}{24} = 1\frac{11}{24}$$

1. To what common unit are the fractions changed?

2. How many times are 24 units contained in 35 units?

Second Process.

Analysis of Process. $\frac{1}{5}$ is contained

$\frac{7}{8} \div \frac{3}{5} = \frac{7}{8} \times \frac{5}{3} = \frac{35}{24} = 1\frac{11}{24}$ in 1 five times; $\frac{3}{5}$ is contained in 1, $\frac{1}{3}$ of 5 times or $\frac{5}{3}$ times. Since $\frac{3}{5}$ is contained in 1 $\frac{5}{3}$ times, in $\frac{7}{8}$ of 1 it will be contained $\frac{7}{8}$ of $\frac{5}{3}$ times or $\frac{35}{24}$ times = $1\frac{11}{24}$ times.

But $\frac{5}{3}$ is the divisor $\frac{3}{5}$ inverted. Hence to divide one fraction by another, we may

Multiply the dividend by the divisor inverted.

3. What advantage has the second process over the first?

4. What disadvantage may it possibly have?

Apply the shorter process to the following and explain it:—

5. $\frac{5}{7} \div \frac{3}{8}$

7. $\frac{6}{7}$ in $1\frac{1}{2}$

9. What part of $\frac{5}{6}$ in $\frac{3}{4}$?

6. $\frac{6}{11} \div \frac{4}{9}$

8. $\frac{4}{5}$ in $1\frac{7}{2}$

10. What part of $\frac{9}{10}$ in $\frac{3}{8}$?

NOTE.—Cancellation will often shorten the process in division as in multiplication of fractions.

11. $\frac{25}{27} \div \frac{5}{18}$

12. $\frac{5}{44} \div \frac{2}{11}$

13. $\frac{128}{250} \div \frac{16}{75}$

14. $\frac{210}{625} \div \frac{7}{25}$

141.—Division of Mixed Numbers; 1. Divide $12\frac{3}{4}$ by $3\frac{2}{3}$. (Change mixed numbers to improper fractions.)

Complex Fractions. 2. $15\frac{5}{6} \div 9\frac{3}{5}$ 5. $82\frac{2}{7} \div 32\frac{2}{3}$

Written. 3. $73\frac{5}{9} \div 57\frac{1}{6}$ 6. $1000 \div 66\frac{2}{3}$

4. $72\frac{9}{10} \div 42\frac{3}{4}$ 7. $100 \div 62\frac{1}{2}$

8. $\frac{2\frac{1}{2}}{7\frac{1}{3}}$ is a way of indicating the division of $2\frac{1}{2}$ by $7\frac{1}{3}$. $\frac{5}{2} = \frac{5}{2}$.

Such expressions are called *complex* fractions. To change them to simple fractions, multiply each term by a common denominator of the fractions. Thus:—

$$\frac{2\frac{1}{2} \times 6}{7\frac{1}{3} \times 6} = \frac{15}{44}$$

9. $\frac{16\frac{2}{3}}{100}$

10. $\frac{18\frac{3}{4}}{100}$

11. $\frac{5\frac{1}{2}}{30\frac{1}{4}}$

12. $\frac{\frac{3}{4}}{\frac{5}{16}}$

142.—Mixed Numbers Divided. 1. What are the two steps in dividing $1645\frac{5}{7}$ by 9?

Written.

2. $3476\frac{2}{6} \div 8 = x$

3. Divide $7329\frac{1}{5}$ by 12

4. $9213\frac{7}{8} \div 9 = x$

5. How many times is 11 contained in $876\frac{1}{4}$?

6. Divide $289\frac{3}{4}$ by 26. To what common fractional unit are both dividend and divisor changed?

Process.

$$\begin{array}{r} 26 \overline{) 289\frac{3}{4}} \\ 4 \quad 4 \end{array}$$

$$\begin{array}{r} 104 \overline{) 1159(11\frac{5}{16}} \\ 104 \\ \hline 119 \\ 104 \\ \hline 15 \end{array}$$

$\frac{1}{16}$ of 15 = $\frac{15}{16}$

Process.

$$\begin{array}{r} 9 \overline{) 1645\frac{5}{7}} \\ 182\frac{5}{7} \end{array}$$

a. $\frac{1}{9}$ of $1645\frac{5}{7} = 182$, and $7\frac{5}{7}$ remaining.

b. $\frac{1}{9}$ of $7\frac{5}{7} = \frac{1}{9}$ of $5\frac{1}{4} = \frac{5}{36}$.

7. $367\frac{2}{3} \div 24$

9. $4723\frac{2}{3} \div 105$

8. $846\frac{3}{5} \div 39$

10. $6948\frac{1}{6} \div 216$

11. If 75 boxes weigh $847\frac{7}{16}$ lb., what will one weigh?

12. 89 rd. = $1468\frac{1}{2}$ ft.; 1 rd. = x ft.

**143.—Dividing by
Mixed Numbers.**

Process.

$$\begin{array}{r} \$24\frac{2}{3}) \$1428 \\ \underline{8} \quad \underline{8} \\ 195 \quad) \quad 11424(58 \\ \underline{975} \\ 1674 \\ \underline{1560} \\ \$114 \\ \underline{8} = \$14\frac{1}{4} \end{array}$$

Oral.—1. If one chair costs $\$24\frac{2}{3}$, how many can be bought for $\$1428$, and how much will remain?

2. Why do we change dividend and divisor to 8ths? 3. The remainder is always a part of what? 4. What is the unit of the first dividend? Of the second? Of the remainder?

Written.—5. How many times is $\$42\frac{6}{7}$ contained in $\$2500$?

6. 1 sq. rd. = $272\frac{1}{4}$ sq. ft.; 1728 sq. ft. = x sq. rds.

7. Find the quotient and remainder; $2000 \text{ cu. ft.} \div 18\frac{3}{4} \text{ cu. ft.}$

8. At $\$0.87\frac{3}{4}$ each, how many spoons can be bought for $\$75$, and how much remains?

9. Of $\$525$ I spend as much as I can for bicycles at $\$125\frac{5}{8}$ each. With the remaining money, how far can I travel at 2 cents a mile?

144.—Fundamental Processes applied to Fractions.

PRACTICE TABLE.

<i>a.</i>	<i>b.</i>	<i>c.</i>	<i>d.</i>	<i>e.</i>	<i>f.</i>	<i>g.</i>	<i>h.</i>	<i>i.</i>
1.	$\frac{2}{3}$	$\frac{5}{9}$	$\frac{11}{12}$	$2\frac{3}{8}$	$6\frac{3}{8}$	$15\frac{5}{8}$	$316\frac{1}{2}$	$\$100$
2.	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{3}{8}$	$7\frac{3}{8}$	$18\frac{1}{8}$	$28\frac{1}{8}$	$493\frac{3}{8}$	250
3.	$\frac{5}{6}$	$\frac{5}{8}$	$\frac{4}{9}$	$5\frac{3}{10}$	$9\frac{1}{12}$	$21\frac{3}{8}$	$641\frac{3}{4}$	500
4.	$\frac{7}{8}$	$\frac{2}{15}$	$\frac{7}{24}$	$6\frac{5}{6}$	$10\frac{3}{10}$	$64\frac{2}{5}$	$827\frac{4}{5}$	576
5.	$\frac{4}{5}$	$\frac{6}{35}$	$\frac{3}{14}$	$9\frac{6}{7}$	$15\frac{2}{3}$	$85\frac{3}{4}$	$936\frac{7}{8}$	600
6.	$\frac{8}{9}$	$\frac{3}{10}$	$\frac{11}{30}$	$8\frac{7}{8}$	$21\frac{5}{8}$	$90\frac{9}{10}$	$1464\frac{1}{6}$	640
7.	$\frac{7}{10}$	$\frac{2}{35}$	$\frac{13}{40}$	$41\frac{1}{5}$	$9\frac{7}{8}$	$16\frac{1}{2}$	$2525\frac{1}{10}$	720
8.	$\frac{11}{12}$	$\frac{3}{16}$	$\frac{5}{18}$	$9\frac{3}{4}$	$12\frac{4}{15}$	$72\frac{1}{4}$	$4769\frac{3}{8}$	800
9.	$\frac{5}{7}$	$\frac{3}{9}$	$\frac{7}{105}$	$10\frac{4}{21}$	$18\frac{3}{4}$	$36\frac{5}{7}$	$8461\frac{3}{7}$	960
10.	$\frac{13}{15}$	$\frac{19}{50}$	$\frac{23}{60}$	$31\frac{9}{10}$	$20\frac{1}{15}$	$25\frac{4}{5}$	$7550\frac{7}{10}$	1000

TO THE TEACHER. — Each of the following 45 combinations may be applied to each number in the designated column so as to furnish ten examples, which may be assigned consecutively from I to 10, making 450 in all. How much to use depends on the degree of accuracy and facility attained or desired.

<i>Addition.</i>	<i>Subtraction.</i>	<i>Multiplication.</i>	<i>Division.</i>
1. $b + \frac{2}{3}$	11. $20 - e$	21. $5 \times b$	31. $b \div a$
2. $c + \frac{3}{4}$	12. $f - 5\frac{1}{2}$	22. $12 \times c$	32. $a \div c$
3. $d + \frac{7}{8}$	13. $b - c$	23. $a \times g$	33. $b \div c$
4. $b + c$	14. $d - c$ or $c - d$	24. b of d	34. $e \div d$
5. $c + d$	15. $f - e$	25. $c \times d$	35. $f \div e$
6. $b + c + d$	16. $g - f$	26. $b \times c \times d$	36. $g \div f$
7. $e + f$	17. $94\frac{7}{8} - g$	27. $c \times d \times e$	37. $h \div 9$
8. $f + g$	18. $h - g$	28. $f \times g$	38. $h \div 75$
9. $e + f + g$	19. $g - d$	29. $67 \times h$	39. $i \div e$
10. $e + f + g + h$	20. $g - e - f$	30. $g \times i$	40. $i \div g$
<i>Mixed.</i> — 41. $b + c - \frac{1}{2}d$			
	42. $g - (e + f)$	43. c of $f \div e$	
	44. $g \div e \times f$	45. $(b \text{ of } e) \div (e \text{ of } f)$	

145.—To be

Formulated.

Written.

1. An heir gets $\frac{1}{3}$ of an estate, then loses $\frac{2}{3}$ of his share. What part of the estate does he keep?

2. I buy at 20% discount. What is the total cost to me of goods sold regularly for \$1.42, \$3.98, \$57, \$0.16 $\frac{2}{3}$, and 9 pieces at \$0.31 $\frac{1}{4}$?

3. If $8\frac{1}{2}$ tons of coal cost \$48 $\frac{7}{8}$, what is the cost of 68 tons?

4. Property which cost \$5000 is rented for \$43 $\frac{1}{3}$ a month; what is the annual income to the owner after paying a tax of \$15 on a thousand?

5. Three cheeses, weighing respectively 34 $\frac{1}{2}$, 42 $\frac{3}{4}$, and 47 $\frac{5}{8}$ lb., were sold for \$20.60; what was the price per pound?

6. J. F. Sampson bought 72 $\frac{1}{2}$ bu. potatoes at 62 $\frac{1}{2}$ ϵ a bushel, and sold $\frac{2}{3}$ at 64 $\frac{1}{2}$ ϵ , the remainder at 75 ϵ ; what did he gain?

7. An electric launch was sold for \$285, or $\frac{1}{2}\frac{2}{3}$ of the cost. Find $\frac{2}{19}$ of the cost.

8. $\frac{2}{5}$ of a ton of hay @ \$20 pays for $1\frac{1}{3}$ tons of coal at how much a ton?

9. $16\frac{1}{2}$ ft. of 2-in. pipe @ $6\frac{1}{2}$ ¢, and 1020 ft. of 1-in. pipe @ $4\frac{1}{4}$ ¢, are exchanged for 120 lb. tubing at $11\frac{1}{8}$ ¢, and 134 ft. @ 9¢. What is the difference in value?

10. Two trains start together in the same direction. How far apart will they be in an hour if one goes a mile in $1\frac{3}{10}$ min. and the other in 85 sec.?

146.—Fractional Parts Compare with 100:—

of 100.

1. 50, 25, 75, 20, 40, 10, 30, 70.

At sight.

2. 5, 15, 4, 12, 16, 2, 6, 8.

Repeat rapidly, until thoroughly learned, the values of the following parts of 100:—

- | | | | | | | | | | | | | | |
|------------------|----------------|----------------|----------------|----------------|----------------|----------------|--------------------|----------------|----------------|----------------|----------------|-----------------|-----------------|
| 3. $\frac{1}{2}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{1}{4}$ | $\frac{3}{4}$ | $\frac{1}{5}$ | $\frac{2}{5}$ | 7. $\frac{2}{10}$ | $\frac{2}{12}$ | $\frac{2}{16}$ | $\frac{2}{20}$ | $\frac{2}{25}$ | $\frac{2}{30}$ | $\frac{2}{100}$ |
| 4. $\frac{3}{5}$ | $\frac{4}{5}$ | $\frac{1}{6}$ | $\frac{5}{6}$ | $\frac{1}{8}$ | $\frac{3}{8}$ | $\frac{5}{8}$ | 8. $\frac{3}{10}$ | $\frac{4}{12}$ | $\frac{5}{16}$ | $\frac{6}{20}$ | $\frac{7}{25}$ | $\frac{8}{30}$ | $\frac{1}{7}$ |
| 5. $\frac{7}{8}$ | $\frac{1}{10}$ | $\frac{1}{12}$ | $\frac{1}{16}$ | $\frac{1}{20}$ | $\frac{1}{25}$ | $\frac{1}{30}$ | 9. $\frac{3}{12}$ | $\frac{3}{16}$ | $\frac{1}{40}$ | $\frac{1}{30}$ | $\frac{5}{16}$ | $\frac{1}{16}$ | |
| 6. $\frac{2}{4}$ | $\frac{2}{6}$ | $\frac{3}{6}$ | $\frac{4}{6}$ | $\frac{2}{8}$ | $\frac{4}{8}$ | $\frac{6}{8}$ | 10. $\frac{7}{12}$ | $\frac{7}{16}$ | $\frac{9}{12}$ | $\frac{3}{40}$ | $\frac{9}{16}$ | $\frac{11}{12}$ | |

11. A nurseryman sells 2500 strawberry plants @ \$6 a hundred. They cost him $\frac{5}{8}$ as much to raise, and he gives an agent $\frac{1}{3}$ of the profit. How much does he gain?

147.—To Find
what Part or what
per cent One Num-
ber is of Another.

Compare one number with the other in each of the following columns. Thus:—

- A. 8 is $\frac{1}{3}$ or $33\frac{1}{3}\%$ of 24.
 B. 24 is 3 times 8 or 300% of 8.
 C. The ratio of 8 to 24 is $\frac{1}{3}$.
 D. The ratio of 24 to 8 is 3.

I.	II.	III.	IV.
1. 8, 24	25, $6\frac{1}{4}$	90, 18	1 d., 1 h.
2. 60, 12	$3\frac{1}{3}$, 10	1 oz., 1 lb.	1 wk., 1 d.
3. 24, 60	$\frac{5}{8}$, $\frac{1}{4}$	2 gal., 3 qt.	5 min., 25 sec.
4. 48, 72	72, 60	30, 50	144, 48
5. $16\frac{2}{3}$, $66\frac{2}{3}$	$37\frac{1}{2}$, $6\frac{1}{4}$	$\frac{4}{5}$, 0.16	$\frac{80}{100}$, $\frac{16}{100}$
6. $\frac{3}{8}$, $\frac{3}{4}$	$\frac{3}{8}$, $\frac{6}{8}$	0.68, 0.51	$4 \times \frac{17}{100}$, $3 \times \frac{17}{100}$
7. 375, 25	$\frac{5}{6}$, $\frac{2}{3}$	$10\frac{1}{2}$, $5\frac{1}{4}$	0.93, 0.31
8. 1, 100	10 %, 20 %	$\frac{10}{100}$, $\frac{50}{100}$	1, 200
9. 2.75, 5.50	\$ 0.75, \$ 1.25	\$ 1, 100¢	\$ 1, \$ 1.50
10. 1, 1000	$37\frac{1}{2}$, $87\frac{1}{2}$	$33\frac{1}{3}$, 100	$1.83\frac{1}{3}$, 1.00

148.—Finding
what %.

What number compares with 100 as—

- | | | | |
|--------------|------------|---------------------|------------------|
| 1. 4 with 28 | 15 with 40 | 6. 35 with 105 | 3 d. with 1 wk. |
| 2. 16 " 80 | 6 " 42 | 7. 4 in. with 1 yd. | 2 mo. with 1 yr. |
| 3. 25 " 150 | 24 " 42 | 8. 1 oz. " 1 lb. | 1 h. " 12 min. |
| 4. 45 " 15 | 48 " 36 | 9. 1 lb. " 1 oz. | 100 lb. " 1 T. |
| 5. 18 " 54 | 18 " 24 | 10. 2 qt. " 4 gal. | 1¢ " \$1 |

149.—A Part Given;
the Whole Required.

Oral.

- | | |
|--|--|
| 1. 12 is $\frac{1}{2}$ of x | 5. 28 is $\frac{2}{3}$ of x |
| 2. 16 is $\frac{1}{5}$ of x | 6. 36 is $\frac{3}{4}$ of x |
| 3. 24 is $\frac{1}{6}$ of x | 7. 72 is $\frac{4}{5}$ of x |
| 4. 19 is $\frac{1}{5}$ of x | 8. 100 is $\frac{10}{17}$ of x |
| 9. What is $\frac{7}{8}$ of 960 ? | 11. 450 is $\frac{15}{16}$ of what ? |
| 10. What part of $2\frac{3}{4}$ is $\frac{3}{4}$? | 12. 175 is $\frac{25}{31}$ of what ? |
| 13. $\frac{17}{100}$ or 85% of x tons of coal were sold. | |
| 14. What part of 1200 is found in 480 ? | |
| 15. $3\frac{2}{3}$ is $\frac{1}{7}$ of what ? | 18. $\frac{7}{8}$ is $\frac{4}{5}$ of what ? |
| 16. $12\frac{1}{2}$ is $\frac{5}{8}$ of what ? | 19. What part of $\frac{2}{3}$ is $\frac{5}{12}$? |
| 17. $14\frac{2}{7}$ is $\frac{2}{3}$ of what ? | 20. $18\frac{3}{4}$ is what part $62\frac{1}{2}$? |

150. — To find the Whole when a Part or Per Cent of it is known.

1. 16 is $\frac{1}{2}$ or 50% of x
2. 24 is $\frac{3}{4}$ or 75% of x
3. 32 is $\frac{2}{3}$ or 66 $\frac{2}{3}$ % of x
4. 40 is $\frac{5}{6}$ or 83 $\frac{1}{3}$ % of x
5. 56 is $\frac{7}{8}$ or 87 $\frac{1}{2}$ % of x
6. 20 is 25% of x
7. 15 is 12 $\frac{1}{2}$ % of x
8. 30 is 37 $\frac{1}{2}$ % of x
9. 2 $\frac{1}{2}$ is 10% of x
10. 3 $\frac{1}{2}$ is 33 $\frac{1}{3}$ % of x
11. $\frac{7}{8}$ is 20% of x
12. $\sqrt{25}$ is 5% of x
13. 36 is 18% of x
14. 9² is 27% of x

151. — For Frequent

Practice.

Oral.

1. $\frac{7}{10}$ of 10 $10 \times \frac{7}{10}$ $10 \div \frac{7}{10}$
10 is $\frac{7}{10}$ of what number?
2. Change to fractions of a dollar in lowest terms: \$0.12 $\frac{1}{2}$, \$0.375, \$ $\frac{2}{10}$ ⁵, \$0.66 $\frac{2}{3}$, 83 $\frac{1}{3}$ %,
3. Express in cents these parts of a dollar: —
 $\frac{2}{4}$ $\frac{3}{8}$ $\frac{1}{10}$ $\frac{2}{3}$ $\frac{2}{6}$ $\frac{5}{8}$ $\frac{5}{6}$
4. Of 1000 find —
 $\frac{1}{2}$ $\frac{1}{4}$ $\frac{3}{8}$ $\frac{5}{8}$ $\frac{7}{8}$
5. Use each of the following numbers as divisor of the one at the right or left of it: —
3 11 17 51 65 26 $\frac{2}{3}$ $\frac{4}{9}$ $\frac{10}{18}$ $\frac{4}{9}$ $\frac{15}{24}$
 $\frac{5}{6}$ 10 62 $\frac{1}{2}$ 1000 125 2000 $\frac{2}{3}$ $\frac{1}{4}$ $\frac{3}{10}$ $\frac{1}{3}$ $\frac{6}{10}$
6. What is 1000 times —
 $\frac{3}{4}$ $\frac{2}{3}$ $\frac{1}{5}$ $\frac{1}{12}$ $\frac{1}{6}$
7. Give the fewest cents that will pay for 1 when the price of 12 is —
15 16 18 20 25
30 35 38 40 42
45 50 65 70 88
8. Add the following fractions: —
 $\frac{2}{3}$ $\frac{4}{9}$ $\frac{10}{18}$ $\frac{4}{9}$ $\frac{15}{24}$
 $\frac{2}{3}$ $\frac{1}{4}$ $\frac{3}{10}$ $\frac{1}{3}$ $\frac{6}{10}$

Give rapidly the following parts of 100: —

9. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, etc., to $\frac{1}{10}$.
10. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{6}{7}$, etc., to $\frac{1}{10}$.

152. — General

Questions.

Give numbers when needed to explain your answers.

1. What are the processes of combining fractions? Of separating them?
2. Of what use is it to change the form of a fraction and not its value? Explain the principle.

3. State the method of finding the sum of two fractions if their units are not alike.
4. What is meant by "higher terms" and "lower terms"?
5. Show a connection between $\%$ and fractions.
6. Why is the product of two fractions less than either? 7. How can a quotient be larger than its dividend?
8. Compare common and decimal fractions.
9. In working with fractions what is the need of finding a greatest common divisor? A least common multiple?
10. What is the difference between a fraction and a fractional unit?

153.—Problems for *Explain exactly how you get each result:—*

Analysis.

1. 34% of certain telegraph lines are under ground. What $\%$ are above ground?
2. A foundry uses 100 T. of Swedish iron to 50 T. from other sources. What part or $\%$ of each class is used?
3. \$160 was $\frac{4}{5}$, or 16% of the profits, which were x .
4. 23% of the stock was glassware, 69% was china. The rest was in brass goods which were $x\%$ of the whole.
5. The 5000000 sq. m. of the Arctic Ocean are what $\%$ of the area of the Pacific, which is 16 times as large?
6. I gain 100% on $\frac{1}{2}$ my goods and sell the rest at cost. How much do I gain on \$100 invested?
7. In a 36-column newspaper what part of the whole space would be filled by 20 columns of advertisements? What per cent?
8. I lose half that I have, and 25% of the rest. What I keep is what part of what I lose?
9. The board of a horse is \$20, shoeing \$1.25, harness repairs \$0.25, use of carriage \$3.00, new whip \$0.50. Each item is what part of the whole? Give per cents when you can.
10. $3^3 = x\%$ of 9^2 $2^3 = x\%$ of 8^2
 $\frac{3}{4}$ of $\frac{1}{4}\%$ = $\frac{1}{2}$ of $x\%$ 10% of \$10 less 1% of itself = x

154. — For Oral

Analysis.

1. $10 = \frac{10}{11}$ of what number? $10 = \frac{10}{6}$ of what number?2. Give $\frac{4}{6}$ and $\frac{4}{12}$ their least common denominator.3. $\frac{2}{7}$ of a hill is dug away. How many times as much remains?
 $3 \times \frac{2}{7} + \frac{1}{2}$ of $\frac{2}{7} = x$. $1 \div \frac{2}{7} = y$.4. $10 \div \frac{1}{12} = x$. $10 \div \frac{5}{7} = y$.5. A train goes a mile in $1\frac{2}{3}$ min. How far will it go in an hour? 6. At 1 m. in 90 sec., how much in an hour? 7. 50 m. an hour = how much a minute?8. Express in lowest terms $\frac{60}{48}$. 9. At the rate of 48 m. an hour, how long does it take to go 1 m.?10. A mile in $1\frac{2}{3}$ min. is the same as 60 m. in ~?**155. — At Sight.**1. Reduce to lowest terms $\frac{510}{370}$; $\frac{707}{1400}$.2. Compare results: $224.7 \text{ yards} \div 7$; $224\frac{7}{10} \text{ yd.} \div 7 \text{ yd.}$ 3. $\frac{1}{11}$ of 10 = x . $\frac{5}{6}$ of 10 = y . 4. $\frac{2}{11}$ league = what part of $\frac{3}{11}$ league?5. $\frac{3}{6}$ A. = what part of $\frac{1}{6}$ A.? Of $1\frac{7}{4}$ A.?6. $\frac{3}{9}$ is contained in $1\frac{2}{3}$ how many times?7. Cancel mentally: $\frac{6}{7}$ of $\frac{5}{6} \times \frac{1}{4}$ of $\frac{1}{5} = x$.8. By getting a discount of $\frac{2}{3}$ I pay only \$3.33. What is the regular price?9. $\frac{170}{462} \div \frac{17}{231} = x$.

10. What is the least that will pay for 1 article, when the price per dozen is —

\$1.05 \$1.10 \$1.15 \$1.25 \$1.30 \$1.35 \$1.40 \$1.50

\$1.75 \$2.00 \$2.25 \$2.50 \$2.75 \$3.00 \$5.00

11. $\frac{3}{4}$ rd. @ \$1.25.13. $3\frac{1}{3}$ in $25x$ times.12. $5\frac{7}{16}$ lb. @ \$1.28.14. $40 - 16\frac{2}{3} - 7\frac{2}{3} = x$.

156. — For Dictation. 1. Add $1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$, $4\frac{1}{2}$, $5\frac{3}{4}$, $83\frac{1}{2}$.

Results orally.

2. $10 \times \frac{1}{4}$. 3. $\$1\frac{3}{8} - \0.625 .

4. $412\frac{1}{2} + 87\frac{1}{2} + 62\frac{1}{2} + 37\frac{1}{2} + 112\frac{1}{2} + 12\frac{1}{2}$; $66\frac{2}{3} - 16\frac{2}{3} - 8\frac{1}{3}$.

5. In a Fahrenheit thermometer what is the temperature when the top of the mercury column is $\frac{5}{16}$ of the distance in degrees from zero to the freezing point.

6. After gaining $\frac{1}{10}$, or 10%, I have \$99. What had I at first?

7. Find $2\frac{1}{2} \times 90$. What is $\frac{1}{100}$ of it? 6% of it?

8. What part of 100 is $9\frac{1}{11}$? Give $\frac{2}{11}$ of 100; $\frac{3}{11}$; $\frac{6}{11}$; $\frac{9}{11}$.

9. How much for a dozen at 16 for a quarter? At 4 for 5¢? At 20 for a quarter? At 3 for 10¢? At 3 for 5¢?

10. A newspaper weighing 4 oz. may be mailed for 1¢; what will it cost to send —

1 lb. 10 oz. 5 oz. $4\frac{1}{8}$ oz. $\frac{3}{4}$ oz.

The rate for other printed matter and for seeds is 1¢ for every 2 oz. Give the cost as before.

157. — Fractional

1. Compare the cost of —

Measures.

$7\frac{1}{2}$ A. at 10¢ a foot and 560 A. at 25¢ a

Written.

square rod.

2. Corn is worth $53\frac{1}{4}$ ¢ a bushel. \$1000 buys x bushels, with y cents remaining. If sold at a profit of $\frac{1}{10}$, what will be the gain?

3. Copper sells for £39 5s. a ton. A pound is worth about x cents.

4. At $3\frac{1}{3}$ leagues an hour, how long will it take to go 500 m.? How far would a ship sail in a week?

5. Find the average of these prices for oats: $29\frac{3}{4}$, $29\frac{5}{8}$, $28\frac{5}{8}$, $29\frac{7}{8}$, $30\frac{1}{4}$, $30\frac{3}{8}$.

6. If pork costs $12\frac{35}{100}$ ¢ a pound, what is the profit on 4 bbl. retailed at 14¢ (200 lb. to a barrel)?

- 7 A barrel of 42 gal. will fill how many cans containing $1\frac{7}{8}$ pt. ?
8. When \$80 are earned in a month and $\frac{5}{12}$ of it spent, the savings of $2\frac{1}{12}$ years at that rate would be \$ x .
9. A chest contains $7\frac{1}{4}$ lb. tea @ 55¢, $13\frac{1}{2}$ lb. @ 35¢, and $9\frac{3}{8}$ lb. @ 27¢. The mixture is worth x ¢ a pound, and \$10 would buy y lb., with z ct. remaining.
10. $\frac{1}{4}$ of a cargo is the captain's share. It consists in part of 14 cwt. coffee @ 28¢, 230 lb. cloves at 18¢, 180 lb. ginger at $10\frac{3}{8}$ ¢, 15,000 gal. molasses @ $25\frac{1}{2}$ ¢, 2580 bags sugar @ \$3.12 $\frac{1}{2}$. What is the value of the captain's share of these goods ?

158. — Oral Problems. 1. Mention 3 fractions of a dollar which together equal \$ $\frac{2}{3}$; \$0.50.

2. What sum becomes \$2 after spending $\frac{4}{10}$, $\frac{1}{8}$, and $\frac{6}{10}$ of it ?
3. Which is more, $\frac{5}{10}$ of a share or $\frac{6}{13}$ of one ? When the difference is \$10, what is the whole share ?
4. Of the 8000 bills presented in the first session of the 53d Congress only 1500 were considered. What fraction was that ?
5. A ship was insured for 50% of its value. $\frac{3}{4}$ of the insurance was \$4500. The ship was worth x .
6. I pay for a purchase with a \$5 bill, and receive in change one of each of the current coins of the United States amounting to x .
7. The back of a bench is $\frac{11}{8}$ as high as the seat, which is 2 ft. Find the difference in inches.
8. One plant urn holds $1\frac{2}{11}$ pk. How many such can be filled from 3 bu. of earth ? Out of $3\frac{1}{2}$ bu. what part could not be used ?
9. How much velvet at \$4.37 $\frac{1}{2}$ a yard can be bought for the price of $4\frac{3}{8}$ yd. satin at \$2.66 $\frac{2}{3}$ a yard ?
10. $\frac{7}{9}$ of a telephone pole is the part above ground. $\frac{8}{10}$ of this part is painted brown, and the remaining 7 ft. are painted white. What fraction of the whole is white ? How long is the pole ?

159.—For Rapid Figuring. 1. Paid at different times $\frac{1}{3}$, $\frac{1}{6}$, and $\frac{7}{12}$ of a debt. The balance was \$1170.61. What was the whole debt?

2. $\frac{1}{3}$ of an estate is divided equally among 14 persons, another $\frac{1}{3}$ among 9 persons. One of each of these shares is to be given to the heir of the remaining third. What part of the whole does he receive?

3. From a life-saving station to the end of the northern beat is $2\frac{1}{10}$ m. How many full steps will a surfman take in going and returning, if his steps average $2\frac{2}{12}$ ft. each?

4. The Minot's Ledge light revolves twice a minute. It is lighted from sunset to sunrise. How many revolutions does it make between 5.52 P.M. and 5.53 A.M.?

5. Four equal farms, all adjoining, are offered for house lots. Parts of each are sold as follows: $\frac{8}{9}$, $\frac{6}{10}$, $\frac{9}{10}$, $\frac{9}{12}$. Add the fractions, and tell what the sum shows.

6. What part of a mile is covered by 22 revolutions of a wheel 18 ft. round?

7. Divide 231 cu. in. into 3 equal integral parts; into 33. How else can it be exactly divided?

8. Just when does $\frac{1}{3}$ of a common year end? $\frac{1}{3}$ of a leap year?

9. A bushel of potatoes is commonly 60 lb. A thousand 56-lb. bushels are what part by weight of 1000 60-lb. bushels?

10. What other fractional units besides $\frac{1}{2}$ and $\frac{1}{3}$ express integral parts of 43,560 sq. ft.?

160.—Business Transactions.

Find the amount due on the following purchases:—

Written.

[See pp. 8-9.]

1. $3\frac{1}{2}$ yd. silk @ \$1.37 $\frac{1}{2}$; $8\frac{1}{3}$ doz. buttons @ 15¢; $7\frac{1}{2}$ sticks braid @ 75¢; 23 yd. ribbon @ 16 $\frac{2}{3}$ ¢.

2. 6470 ft. fencing @ 9 $\frac{5}{12}$ ¢; 3 lots land, 10280, 7595, 8122 sq. ft., at 5 $\frac{1}{4}$ ¢; 3400 bricks @ \$8.15 per M.

3. Find the total weight in pounds:—

1 T. $3\frac{3}{8}$ cwt. 2560 oz. $\frac{1}{4}$ T. 5 lb. 8 oz. 5 cwt. 9 lb.

4. How many pounds altogether will fill 600 4-oz. bottles, 360 2-oz. bottles, and 36 doz. boxes, each containing 20 2-gr. pills? What fraction of this number would represent the same weight in avoirdupois pounds?

5-7. A. & H., B. C. & Co., F. S. T. are the initials of three firms. Make out three bills from these entries on the day-book of James Graeme, who sold the goods. Use the present date. (See p. 40.)

B. C. & Co., 23 $\frac{1}{2}$ M. ft. pine @ \$55

F. S. T., 371 locust posts @ 62 $\frac{1}{2}$ ¢

1090 split rails @ 18 $\frac{3}{4}$ ¢

A. & H., 4 $\frac{3}{8}$ M. brick @ \$8

1830 Welsh slate @ 10 $\frac{1}{4}$ ¢

A. & H., 86 $\frac{1}{2}$ ft. 2-in. plank @ 3¢

3 cd. 5 ft. slabs @ \$3.25

F. S. T., 500 ft. sheathing @ \$42.50

1 $\frac{1}{8}$ M. ft. flooring @ \$20

B. C. & Co., 22 bbl. lime @ 67 $\frac{3}{4}$ ¢

13 T. coal @ \$4.75

teaming 13 \times 37 $\frac{1}{2}$ ¢

B. C. & Co., 7 $\frac{3}{8}$ M. ft. spruce @ \$31.

7 bbl. Portland cement @ \$2.75

F. S. T., 350 ft. drain pipe @ 62 $\frac{1}{2}$ ¢

A. & H., $\frac{3}{4}$ cd. hard wood @ \$8.75

8. Rule paper for an account that you keep with John Holmes. It will show that he is debtor for all that is sold to ~ by ~, and creditor for all that is paid to ~ by ~, as below.

JOHN HOLMES		Dr.	Cr.
189- May 1	To 3 shares Mill stock	312	00
	By $\frac{1}{2}$ River pasture (3 A.)		45 00

Make out the account as above from the following memoranda, supplying different dates:—

$3\frac{1}{2}$ days repairing Holmes's fence @ \$2.50. Credit him for use of his oxen same time @ \$2. Sold him 23 bbl. apples at \$1.40. Bought 3 hogs (732 lb.) at $11\frac{1}{2}\phi$, and $1\frac{1}{2}$ loads hay \$15 (both of Holmes). Sold him 15 young maples at $\$1.06\frac{1}{4}$, and 3 hoops containing 35 ft. strap iron at $2\frac{3}{4}\phi$. Holmes paid cash \$100.

9. Draw off in proper form (p. 18) a cash account for July containing the following items (cash must be credited with all that you take from it to spend): Left over, saved, or "on hand" from last month, \$2.47, and had a present of five dollars vacation money on the Fourth, besides the 75ϕ I earned at the fair the day after. My ticket to Newburn was \$1.38, the luncheon 30ϕ , and the express 25ϕ : this was on the 6th. In just a week I had to spend all the rest of my money except 44ϕ ; three dollars being for a hat and new ribbon, and all but the 44ϕ for a ticket to Groveton. There I found waiting a money order for \$15.

10. Rule paper as on p. 18, and make up a balanced cash account. ("Cash" is debtor for all that you entrust to it.) (Aug. 1st) Bought 9 shares A. and F. stock @ $\$109\frac{2}{3}$. Sold 8 shares @ $\$109\frac{1}{2}$. (2d) Paid note of E. F. James, \$128, with 3 yr. interest ($3 \times \$7.68$). (5th) Sold 8 bbl. lime @ $\$0.83\frac{1}{2}$. (9th) Sawing wood, $18\frac{1}{2}$ cd. @ \$1.20, paid to Tom. (12th) From Edwards, \$36 for $5\frac{1}{4}$ squares roofing @ *v.* (20th) Taxes on \$14,000 @ \$14.25, paid. (27th) Received cash for $50\frac{1}{2}$ gal. oil @ $8\frac{1}{4}\phi$. (29th) Sold 2 cases eggs, 48 doz., @ $16\frac{2}{3}\phi$. July 31 I had on hand \$2000 cash.

161.—Fractional

Figure the exact value of—

Measures.

1. 290 ft. hemlock boards @ $1\frac{3}{8}\phi$.

Written.

2. $\frac{1}{2}\frac{9}{4}$ ream @ $\frac{3}{4}\phi$ a sheet.

[See pp. 8–9.]

3. 3.25 bu. @ 2 qt. for $\$ \frac{1}{4}$.

4. Pay for $\frac{1}{2}$ yr. (even weeks) @ $\$2.37\frac{1}{2}$, only Sundays excepted.

5. $\frac{7}{32}$ g. gr. hooks and eyes at 5ϕ a doz.

6. $\frac{1}{4}\frac{9}{7}$ of a 235-lb. barrel of sugar @ $4\frac{7}{12}\phi$.

7. $11\frac{3}{7}$ long T. @ \$5 a short T. 8. $2\frac{1}{3}$ oz. opals @ \$3 a pwt.
9. $6\frac{1}{7}$ lb. avoirdupois @ 10 gr. for \$0.01.
10. A pound bar of silver, less 760 gr., @ \$7.80 a pound troy.
11. A bottle contains 20.79 cu. in. of varnish. What is it worth @ $16\frac{2}{3}\phi$ a pint?
12. If the duty were $\frac{1}{6}$ the cost of certain fabrics, the duty on enough to cover $\frac{1}{2}$ a square rod @ \$2 a yard would be x dollars.
13. Find the cost of $16\frac{1}{3}$ sq. yd. @ \$0.375. The value of 1 sq. ft. = x .
14. If roofing that would sell for $2\frac{5}{8}\phi$ by the sq. in. can be had for $\frac{4}{7}$ as much when bought by the sq. yd., what would be the cost of 1 square (100 sq. ft.) at the yard rate?
15. How many 30 ft. rails will reach a mile? $\frac{3}{8}$ of such a rail is how many yards long?
16. 90° of latitude = x miles. The distance round the world on the same circle would be y miles.
17. When $1^\circ = 69\frac{1}{6}$ m., $1'' = x$ feet.
18. $\frac{3}{16}$ A. = x sq. rd. = y sq. yd. How many squares 6 in. on a side would cover 1 sq. ft.? 1 sq. yd.? $\frac{3}{16}$ A.?
19. If a square top desk is $22\frac{3}{8}$ in. on a side, what is its perimeter? $\frac{1}{8}$ of this distance from one corner would reach how far beyond the second corner?
20. A picture moulding is $1\frac{1}{2}$ in. wide. How much farther is it round the outside than round the inside of the frame?

**162. — Denominate
Numbers.**

Written.

1. $2\frac{1}{4}$ lb. troy sold by the 3 at $30\phi = x$.
2. $1\frac{1}{16}$ tons cream of tartar fills how many 4-oz. boxes?
3. $32\frac{5}{8}$ yd. lace billed at \$87 costs how much a yard?
4. £10 4s. is the amount of an invoice of a case of cutlery containing 12 boxes @ x shillings a box.

5. 17s. is what part of a pound? of \$4.866? $\frac{17}{20}$ of \$4.866 = how many shillings? How many pence?

6. Show in figures whether a nautical mile is more or less than $1\frac{1}{6}$ times a common mile.

7. One piece of cloth is twice as long as another, which is reduced to $1\frac{5}{12}$ yd. by cutting off $9\frac{3}{4}$, $3\frac{1}{8}$, and $2\frac{5}{8}$ yd. How long is the first piece in yards and inches?

8. With hay at \$13 $\frac{3}{4}$ a ton, what fraction of a ton is worth \$13? How many pounds?

9. When 115 votes are in favor of a project and 46 are against it, what part of the whole are opposed?

10. Find the profit on 10 bales of wool bought at $16\frac{1}{4}$ ¢ and sold at $16\frac{3}{8}$ ¢, total weight $22\frac{1}{2}$ cwt.

163. — 1st, Written 1. At an auction, one buyer bids $\frac{7}{15}$ of the Results; cost; another, $\frac{4}{5}$. The difference is \$75.

2d, Oral What is the lower bid?

Analysis. 2. From a grove of 40 trees $13\frac{3}{4}$ bu. chestnuts are saved for market. Supposing squirrels to have taken 3 qt. from each tree, what fraction of a bushel did each tree bear? (Make statement.)

3. 573 ft. of fence are to be built. The posts are 1 ft. through, and the space between posts is $3\frac{1}{3}$ yd. How many posts will be required?

4. How long will it take to perform a task of which only $\frac{2}{15}$ can be done in $\frac{5}{9}$ d.?

5. Mary can do the washing in 3 h.; Sarah, in 4 h. What part can each do in an hour? If they work together, how long should it take to do it?

6. John hoes a corn-field in 4 d. Last year his brother could work $\frac{1}{2}$ as fast; this year $\frac{2}{3}$ as fast as John. How much can each do in a day? How long will it take them together to hoe the field?

7. Four men are partners in business. Three furnish \$ 21,000 capital; the other gives only his services, and has $\frac{5}{26}$ of the profits. Two furnish $\frac{2}{7}$ and $\frac{3}{9}$ of the capital; how much capital does the other furnish?

8. If a glass jar contains a hundred thousand fish-eggs, how many jars will hold 8,000,000? If $\frac{13}{16}$ are hatched, how many on an average are lost from each jar?

9. The catch of shad for a certain period is valued at \$ 145,000. What part of this is \$ 4000, the cost of hatching the eggs and stocking the waters?

10. When oysters yield $1\frac{1}{4}$ gal. to the bushel, a 25-gal. barrel can be filled from x bushels in the shell.

164.—To Illustrate 1. A square floor contains 64 sq. yd. Draw by Diagrams. it, and mark it off in squares. Show the dimensions of a square carpet that covers $\frac{9}{16}$ of it.

2. Represent a street having a branch $\frac{3}{7}$ as long.

3. Show how many sq. ft. in $\frac{1}{9}$ sq. yd. + $\frac{2}{9}$ sq. yd. + $\frac{12}{9}$ sq. yd.

4. 1 rod = $16\frac{1}{2}$ ft. How far is it round 3 sides of a rod square?

5. Count the square inches in a square foot. Draw a heavy line round $\frac{6}{14}$ of $\frac{7}{12}$ of them.

6. $1\frac{5}{12}$ sq. ft. contains how much more than 144 sq. in.?

7. The distance between two fence posts is 10 ft. How many $3\frac{1}{2}$ in. slats, 3 in. apart, will be needed?

8. Set the hands of a clock dial at $1\frac{0}{12}$ h. after $4\frac{1}{12}$ h.

9. A postage stamp covers $1\frac{5}{16}$ sq. in. How many times as large is an envelope of 21 sq. in. area?

10. The circumference of a wheel is $12\frac{57}{100}$ in., which is about $3\frac{1}{7}$ times the diameter. Draw the diameter.

165.—For Written

Work.

*From dictation
if possible.*

1. $37\frac{1}{2}$ yd. @ $\$3\frac{3}{5}$ a yard.
2. Clifford can mow the lawn in 1 hour. Clifford and Leonard can do it in 24 min. In 1 min. what part is done by Clifford? By Clifford and Leonard? By Leonard? How long would it take Leonard to do the whole?
3. Steaming from New York harbor at 10 A.M. Saturday, and reaching Holyhead in 54 min. over $5\frac{1}{2}$ d., what time is it in New York?
4. If morning comes $4\frac{2}{3}$ h. sooner in Holyhead than in New York, what time will it be by a Holyhead clock?
5. A piece of ground sells for $\$12,600$, which is $\frac{3}{5}$ of what it might bring if properly drained. How much would be added to its value by draining?
6. A tent contains 40 metres of canvas $\frac{3}{4}$ metre wide. If the stuff were $\frac{5}{6}$ metre wide, how much would be required?
7. $7\frac{2}{3}$ yd. silk $\frac{3}{4}$ yd. wide = $6\frac{9}{10}$ yd. of what width?
8. $\frac{2}{5}$ of a box of pens costs 25¢; 18 boxes cost what?
9. A man who received $\frac{7}{8}$ of his father's property gives to his own son $\frac{3}{4}$ of what he received. Who then has $\frac{7}{32}$ of the whole?
10. Two men require $8\frac{1}{2}$ days to take account of a stock of goods. Six men would need what time?

166.—For Written

Work.

Make statement and cancel when possible:—

1. How many tables can be made in 15 wk. at the rate of one in $5\frac{5}{8}$ d.?
2. $2\frac{3}{4} \div \frac{2}{1\frac{1}{2}} = x$.
3. $x = 1 \times \frac{7}{8}$ of $14\frac{2}{3}$ multiplied by $23\frac{3}{4}$.
4. What mixed number is the quotient of $927\frac{1}{3} \div 11\frac{7}{12}$?
5. What fraction is the quotient of $\frac{2}{3} \div \frac{2}{3}$?
6. A contractor is to receive in three equal payments $\frac{8}{9}$ of his expenses in digging a cellar. What may the first payment be after 6 men with horses have worked $19\frac{1}{2}$ days at $\$2.25$ a day?
7. The wall cost $\$372$ to build, and the grading $\$286.53$. On the same conditions each payment would be what?

8. In sowing a field, one kind of seed is used at the rate of $12\frac{1}{4}$ bu. to 5 acres. What will be required to sow $22\frac{2}{7}$ acres, using $\frac{5}{7}$ as much to the acre as before?

9. What will be the profit on 1200 rolls of wall paper costing $15\frac{1}{2}$ ¢ a roll and selling for $1\frac{1}{3}$ as much, — excepting $\frac{1}{5}$ of the lot which, being damaged, sells at cost?

10. A sold goods at a profit of $\frac{1}{10}$ to B, who made a profit of $\frac{1}{5}$. B sold them to C for \$738, which was $1\frac{1}{8}$ times what they cost him. What was the cost to A?

Decimal Fractions.

167.—The Orders; [Review §§ 4–15.] 1. By a decimal system the Point. we mean ~.

2. Compare in value any one digit with another written at the left of it.

3. Compare the values of the 3's in 33.; in 3.3; in 0.33; in 0.033.

4. Explain which numbers are integers; decimals; mixed decimals.

5. The value of a figure depends on what two things?

7654321.234567.

6. What figure stands for tenths? Hundredths? Thousandths?

7. Compare the position and the value of the 3's. Of the 5's.

8. Of what orders are the 6's? The 7's?

9. What is the use of the decimal point? 10. Why should the 2d figure at the left denote *tens*, but the 2d figure at the right, *hundredths*?

168.—The Denom- 1. The denominator of a decimal fraction inator. is ~. 2. Why need we not write it?

3. Are $\frac{8}{10}$, $\frac{8}{100}$, $\frac{8}{1000}$ common or decimal fractions?

4. How would you write them decimally?

5. Compare the number of places each takes up with the number of ciphers in its denominator.

6. Make a rule for pointing, so as to show what the denominator is.

169.—Reading
Decimals.

1. What part of 140.040 should be read first?
2. Where is "and" used in the reading?
3. Where does the numerator begin, and where end?
4. Which is the easier way of finding the denominator of 0.040,—
(a) By counting from the point—"tenths, hundredths, thousandths"; or (b) By imagining a 1 with *three* ciphers annexed?

Read:—

- | | | | | | |
|--------------|----------|-------|-----------------|----------|---------|
| 5. 0.307 | 0.0307 | 0.010 | 6. 0.710 | 64.0700 | 500.005 |
| 7. 330.03 | 3003.075 | | 8. 37290.760009 | 808.08 | |
| 9. 94.07508 | 6.06008 | | 10. 64.000109 | 0.009004 | |
| 11. 0.407501 | 900.09 | | 12. 6000.006 | 0.06006 | |

170.—Complex
Decimals.

Since 84 per cent, or 84%, means 84 hundredths or 0.84, in what two ways might we read—

1. 0.06 $33\frac{1}{3}\%$ $0.16\frac{2}{3}$
2. $0.87\frac{1}{2}$ $62\frac{1}{2}\%$ $0.03\frac{1}{3}$
3. $3\frac{1}{3}$ thousandths is written $0.003\frac{1}{3}$. To which order of units does the $\frac{1}{3}$ belong?
4. Give other *complex decimals*.

171.—Writing
Decimals.

Oral.—1. When there is no integral part, what may be written in its place before the decimal point? For what purpose?

2. If the denominator contains three ciphers, how far from the decimal point must the numerator end?
3. If that numerator contains but one figure, in what place must it be written?

Write from dictation or at sight:—

- | | |
|------------------------------|---------------------------------------|
| 4. 8 thousandths | 9. 804 hundred-thousandths |
| 5. 17 tenths | 10. 400 and 4 ten-thousandths |
| 6. Three and a fifth % | 11. Forty thousand forty millionths |
| 7. 3075 millionths | 12. Seven hundred six thousandths |
| 8. $\frac{1}{2}$ a hundredth | 13. Two million 71 and 404 millionths |

172. — Problems. *Try to show that —*

1. A millionth of a mile = about $\frac{1}{16}$ of an inch.
2. $\frac{1}{1,000,000}$ yr. = about $\frac{1}{2}$ a minute.
3. A millionth of an acre ($43,560 \times 144$ sq. in.) = a square about $2\frac{1}{2}$ in. on a side.
4. A hundred-millionth of the distance to the sun = ~.
5. What is the name given to 0.0000001 of the distance from the equator to the pole?

173. — Reducing to

Smaller Terms.	1. Change $\frac{1\frac{2}{3}}{1\frac{2}{3}}$ to smallest terms. What principle applies? 2. Prove that $\frac{800}{1000} = \frac{8}{10}$. 3. Compare the numerators, the denominators, and the value of 0.90 and 0.9.
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4. Dividing both terms of a fraction by 100 has what effect on its value?
5. Omit the zeros at the right in 0.360 and 0.400. How is the numerator affected? The denominator? The value?
6. State the principle.

Read as printed; then in smallest decimal terms, telling what g.c.d. you use: —

- | | | | | |
|------------|--------|---------|--------------|-----------|
| 7. 0.0400 | 6.6450 | 0.03070 | 9. 84.700 | 6.040000 |
| 8. 7.09000 | 0.060 | 0.5000 | 10. 921.7600 | 800.80000 |

174. — To Larger

Terms.	1. Explain any change of value made by annexing a cipher to 8; to 120; to 0.3.
--------	--

2. Explain why $\frac{3}{10} = \frac{30}{100}$ and $0.3 = 0.30$.
3. When both terms of a fraction have a zero annexed, how is the size of the unit changed? The number of units?
4. How may you know the number of zeros in the denominator of a decimal?
5. Is the value of a decimal affected by annexing zeros, as in 0.4 changed to 0.400? Explain the principle.

Read as printed; then with one, two, and three ciphers annexed:—

6. 0.5 1.23 0.07 0.101 7. 37.4 0.008 15.0 245.6

8. Read the preceding numbers, and announce each change of value when the point is moved one place to the right. 9. To the left.

175.—Decimals Simple.

to Common Fractions. $0.75 = \frac{75}{100} = \frac{3}{4}$

1. Explain the first change; then the second change.

2. Give directions for the two steps of this process.

Change to common fractions. Give steps orally.

3. 0.2 0.4 0.5 0.6 0.80

4. 0.25 0.12 0.50 0.24 0.70

5. 0.08 0.125 0.45 0.480

6. 0.875 0.00875 0.0064

(Use small factors in reducing to lowest terms.)

7. 0.000125 0.0625 0.375

8. 0.01728 0.0675 0.4375

176. Complex.

$0.18\frac{3}{4} = \frac{18\frac{3}{4}}{100}; \times 4 = \frac{75}{400} = \frac{3}{16}$

1. A complex decimal is ~.

2. $18\frac{3}{4}\%$ means ~.

3. Multiplying both terms of $\frac{18\frac{3}{4}}{100}$ by 4

has what effect on the value?

Why? 4. On the form? 5. State

the principle. 6. Explain the

whole process.

Change hundredths or per cents to common fractions:—

7. $33\frac{1}{3}\%$ $0.16\frac{2}{3}$ $66\frac{2}{3}\%$

8. $0.83\frac{1}{3}$ $0.08\frac{1}{3}$ $50\% + 8\frac{1}{3}\%$

9. $12\frac{1}{2}\%$ $0.37\frac{1}{2}$ $87\frac{1}{2}\%$

10. $0.03\frac{1}{3}$ $4\frac{1}{6}\%$ $0.06\frac{1}{4}$

11. $6\frac{2}{3}\%$ $25\% + 6\frac{1}{4}\%$ $0.31\frac{1}{4}$

12. Subtract each of the preceding from 100%

177.—Written or Change to common fractions:—

Oral. 1-10. $43\frac{3}{4}\%$ $0.56\frac{1}{4}$ $62\frac{1}{2}\%$ $0.68\frac{3}{4}$ $81\frac{1}{4}\%$
 $0.41\frac{2}{3}$ $0.93\frac{3}{4}$ $0.91\frac{2}{3}$ $22\frac{2}{3}\%$ $0.29\frac{1}{8}$

11-20. Subtract each of the preceding from 100%, and change the remainder to a common fraction.

178. — Common Fractions to Decimals.

Written.

$\frac{7}{8}$ = what decimal?

Method.

$$\begin{array}{r} 8 \overline{)7.000} \\ 0.875 \end{array}$$

1. Write decimally: — $\frac{70}{100}$ $\frac{700}{1000}$ $\frac{7000}{10000}$.
 2. What is the difference between 7., 7.0, 7.00, and 7.000?

3. $\frac{7}{8} = \frac{1}{8}$ of \sim . If $7 = 7.000$, $\frac{1}{8}$ of 7.000 or $\frac{1}{8}$ of 7000 thousandths = x thousandths.

Write as 100ths, 1000ths, or 10000ths, etc.

$$\begin{array}{llll} 4. \frac{7}{20} & 6. \frac{13}{20} & 8. \frac{3}{80} & 10. \frac{15}{16} \\ 5. \frac{13}{40} & 7. \frac{27}{80} & 9. \frac{7}{16} & \end{array}$$

179. — Reducing to Complex Decimals.

Oral and Written.

Method.

$$\begin{array}{r} a. \\ 12 \overline{)5.00} \\ 0.41\frac{2}{3} \end{array}$$

$$\begin{array}{r} b. \\ 12 \overline{)5.000} \\ 0.416\frac{2}{3} \end{array}$$

$$\begin{array}{r} c. \\ 12 \overline{)5.0000} \\ 0.4166 + \end{array}$$

5. Compare the value of the quotients in a and b . Are both exact?

6. Is the quotient in c exact?

The + stands for what?

Change to complex decimals of three places: —

$$\begin{array}{ll} 7-11. & \frac{7}{12} \quad \frac{5}{6} \quad \frac{9}{16} \quad \frac{4}{15} \quad \frac{3}{4} \\ 12-17. & \frac{5}{9} \quad \frac{7}{30} \quad \frac{7}{9} \quad \frac{7}{15} \quad \frac{17}{16} \end{array}$$

Change to incomplete decimals of four places: —

$$\begin{array}{ll} 18-22. & \frac{5}{6} \quad \frac{11}{12} \quad \frac{5}{9} \quad \frac{6}{11} \quad \frac{11}{16} \\ 23-27. & \frac{7}{15} \quad \frac{1}{600} \quad \frac{7}{18} \quad \frac{3}{13} \quad \frac{12}{13} \end{array}$$

180. — Rapid Reduction.

Oral.

Change to hundredths or per cents: —

$$\begin{array}{ll} 1. & \frac{1}{2} \quad \frac{1}{6} \quad \frac{1}{4} \quad \frac{1}{3} \quad \frac{1}{5} \\ 2. & \frac{1}{10} \quad \frac{1}{8} \quad \frac{2}{5} \quad \frac{1}{16} \quad \frac{1}{20} \end{array}$$

$$\begin{array}{ll} 3. & \frac{1}{25} \quad \frac{3}{8} \quad \frac{3}{10} \quad \frac{1}{12} \quad \frac{3}{5} \\ 4. & \frac{9}{10} \quad \frac{5}{8} \quad \frac{5}{6} \quad \frac{11}{20} \quad \frac{4}{5} \\ 5. & \frac{7}{10} \quad \frac{7}{20} \quad \frac{7}{8} \quad \frac{9}{16} \quad \frac{7}{12} \\ 6. & \frac{5}{12} \quad \frac{7}{25} \quad \frac{13}{20} \quad \frac{3}{16} \quad \frac{24}{25} \\ 7. & \frac{3}{20} \quad \frac{5}{16} \quad \frac{11}{12} \quad \frac{11}{25} \quad \frac{19}{20} \end{array}$$

Change to the other form:—

8. $\frac{15}{16}$ $0.81\frac{1}{4}$ $\frac{17}{20}$ $91\frac{2}{3}\%$ $\frac{11}{12}$ 9. $0.93\frac{3}{4}$ $\frac{13}{16}$ $43\frac{3}{4}\%$ $\frac{7}{16}$ $41\frac{2}{3}$
 10. $\frac{11}{16}$ $56\frac{1}{4}\%$ $\frac{17}{25}$ $0.31\frac{1}{4}$ $58\frac{1}{3}\%$
 11. Give the difference between the last fifteen numbers and 100%.

DECIMALS ADDED AND SUBTRACTED.

181.—For Addition. *Without copying, write the sum of each Written. column, and of each line.*

- | 1. | 2. | 3. | 4. | 5. |
|-------------|----------|------------|-------------|----------|
| 6. 96.475 + | 186.32 | + 0.4875 + | 0.64985 + | 396.47 |
| 7. 83.8 + | 62.379 + | 2.93 | + 3.207 | + 82.379 |
| 8. 5.42 + | 4.8 | + 8.479 | + 0.0439 | + 6.4 |
| 9. 16.783 + | 9.54 | + 6.53 | + 9.642 | + 180.09 |
| 10. 4.09 + | 72.683 + | 2.946 | + 8.78314 + | 34.769 |

182.—Written. *Rewrite in columns, as integers and decimals; then add.*

- | | | | | | |
|-----------------------|-------------------|------------------|--------------------|------------------|----------------------|
| 1. 16.372 | $8\frac{5}{8}$ | 79.42 | $14\frac{1}{4}$ | $86\frac{3}{4}$ | $21.05\frac{1}{2}$ |
| 2. $216\frac{1}{2}$ | $34\frac{7}{8}$ | 19.3794 | $12\frac{1}{20}$ | $97\frac{3}{10}$ | $16\frac{145}{1000}$ |
| 3. $25\frac{25}{100}$ | 0.847 | $96\frac{4}{16}$ | $29\frac{13}{25}$ | $\frac{3}{10}$ | $62\frac{1}{2}\%$ |
| 4. $\frac{3}{8}$ | $87\frac{1}{2}\%$ | 0.758 | $93\frac{1}{2}\%$ | $\frac{1}{8}$ | $12\frac{1}{2}\%$ |
| 5. 0.3 | 2.03 | $\frac{75}{100}$ | $\frac{605}{1000}$ | 116% | 6.4837 |

183.—Oral. 1. How may common and decimal fractions be added?

2. Compare the denominators of the amounts in the last two exercises with the denominators of the addends.

3. What steps are to be taken in finding the sum or the difference of complex decimals?

4. Define the two ways of combining numbers.

5. When part of a number is given (0.57 is part of 0.96), how is the other part found?

184.— For
Subtraction.

*Find the difference, — first explaining whether
the denominators must be alike :—*

Written.

- | | | | | | |
|-------------------|----|-------------------|----|----------------------|----|
| 1. | 2. | 3. | 4. | 5. | 6. |
| 7. $3.64 - 1.873$ | | 9. $41. - 13.074$ | | 11. $6.39 - 0.497$ | |
| 8. $1.9 - 0.3694$ | | 10. $9.87 - 4.3$ | | 12. $2.641 - 0.0994$ | |

13. Take seventeen hundred eight ten thousandths from twenty-four and six thousandths.

14. From eighty-six tenths take forty-three thousandths.

185.— Written.

Find the difference. —

186.— Written.

Carry results to three places :—

- | | | |
|-----------------------------|-----------------------------------|--|
| 1. $17.38 - 200$ | 6. $9\frac{4}{5} - 7968$ | 1. $(6\frac{1}{4} + 2.35) - 5\frac{5}{8}$ |
| 2. $4\frac{1}{8} - 7.011$ | 7. $\frac{7}{8} - 64\%$ | 2. $(0.9 + \frac{17}{100}) - (\frac{63}{1000} + 0.7\frac{1}{2})$ |
| 3. $2.0875 - \frac{5}{16}$ | 8. $\frac{85}{10} - 2\frac{1}{2}$ | 3. $(100 - 37\frac{1}{2}\%) + (0.625 - \frac{1}{2})$ |
| 4. $1. - 0.83\frac{1}{3}$ | 9. $10. - 8.3\frac{1}{8}$ | 4. $[1.00 - (0.08\frac{1}{3} + \frac{1}{12})] - \frac{1}{3}$ |
| 5. $0.1 - 0.083\frac{1}{3}$ | 10. $6.4 - \frac{5}{6}$ | 5. $[(\frac{1}{12} - 0.91\frac{2}{3}) + 2\frac{3}{4}] - 0.16\frac{2}{3}$ |

DECIMALS MULTIPLIED.

187.— At Sight.

1. Compare the product and the multiplicand when the multiplier is 1. When it is more than 1. When less than 1.

2. Multiplication is \sim .

3. Moving the decimal point one place to the left has what effect in 18.4? In 0.15? 4. What is the shortest way of dividing 247. by 100? Of finding 0.01 of it?

5. Read quotients after dividing by 10. 6. By 100.

72.46 18. 0.875 625. 3.7 0.033 $\frac{1}{3}$ 9.

- | | | |
|----------------------------|------------------------------|---------------------|
| 7. $\frac{1}{10}$ of 2.46? | 8. $\frac{1}{1000}$ of 37.6? | 9. 0.0001 of 3500.? |
| $\frac{1}{100}$ of 32? | $\frac{1}{1000}$ of 0.9? | 0.001 of 25.? |

188.—The Process 1. Having found $\frac{1}{10}$ and 0.001 by moving Analyzed. the point, how would you find $\frac{3}{10}$ or 0.015?

2. To find 0.7 of 50

$$0.1 \text{ of } 50 = \sim$$

$$0.7 \text{ of } 50 = 7 \times \sim = x$$

or

$$7 \times 50 = \sim$$

$$0.1 \text{ of } 7 \times 50 = 0.1 \text{ of } 350 = x$$

3. To find 0.15 of \$ 300

$$\frac{1}{10} \text{ of } \$ 300 = \sim$$

$$\frac{1.5}{100} \text{ of } \$ 300 = \sim \times 15 = \$ x$$

or

$$\$ 300 \times 15 = \sim$$

$$\$ 300 \times \frac{1.5}{100} = \frac{1}{10} \text{ of } \sim = \$ x$$

4. To find 0.9 of 0.03

$$\frac{9}{10} \text{ of } \frac{3}{100} = \frac{27}{1000} = 0.027$$

Compare the denominators and account for the number of ciphers in each.

5. Count the decimal places in the factors. Why are there three in the product?

6. How many decimal places in the product of 0.46×0.38 ? of 0.372×0.471 ? Explain with common fractions.

7. A product has 7 decimal places; one factor 2; the other x .

The product contains as many decimal places as its factors.

189.—For Rapid *Show what process you use, and see that your*
Analysis. *result is a reasonable one.*

1. 0.06 of 200 5% of 500

5. 2.5×0.6 2.5×0.30

2. $\$ 90 \times 0.9$ 12% of 1000

6. 80% of \$ 400 16% of 40

3. 0.6 of 0.6 0.12 of 0.12

7. $33\frac{1}{3}\%$ of 360 $0.2 \times 0.2 \times 0.2$

4. 0.08×0.5 20% of 60 yr.

8. 1.2×1.2 200×0.003

9. Had you first to multiply or to divide in these examples?

10. A man of 50, spending 30% of his life abroad, is at home x yr.

11. 2% of \$ 5000 being counterfeit, \$ x is good.

12. $23 \times 375 = 8625$.

13. $0.23 \times 37.5 = y$

$2.3 \times 375 = x$

$2.3 \times 3.75 = z$

$w \times 3.75 = 0.8625$

$230. \times u = 8625$.

14. How would you write the product of 0.02×0.004 ?

190.—For Written
Work.

- | | | |
|------------------------|------------------------------------|---------------------------------|
| 1. 8.47×9.432 | 3. 3.468×2.008 | 7. $0.36 \times 3.6 \times 36$ |
| 2. 0.84 of \$9.60 | 4. $0.8\frac{1}{3} \times 5.076$ | 8. \$800 \times 2.4, less 75% |
| | 5. $0.037\frac{1}{2} \times 12.50$ | 9. 8% of 0.08×8000 |
| | 6. $1.8\frac{3}{4} \times 0.360$ | 10. $1\frac{1}{2}$ m. + 0.64 m. |

191.—Problems to
Dictate.

Written results.

- | |
|---|
| 1. 218 lb. sugar @ \$0.06 $\frac{1}{4}$. Deduct 1%. |
| 2. A gross of blank books at \$0.11 $\frac{3}{4}$ each. |
| 3. A man's salary is \$1200. If reduced 12 $\frac{1}{2}$ %, it becomes x . |
| 4. Some coal is 8 $\frac{1}{2}$ % slate. The pure coal in 1500 T. is x tons. |
| 5. 120 yd. cotton @ \$0.07125. 6. 306 million \times 17 ten-millionths. |
| 7. 0.375 T. hay weighs x lb. 8. The square of 0.75 is what? |
| 9. An \$8000 house rents for 12%. The monthly rent is x . |
| 10. The owner has this house insured for $\frac{3}{4}$ of its value, or x , paying 2% or y to the people who insure it. |

DECIMALS DIVIDED.

192.—The Process
Analyzed.

- [Review pp. 70–71.]
- One of the 8 equal parts of \$0.48 is x .
 - 0.06 is contained in 0.48 x times.
 - What decimal part of 6 is found in 4?
 - What is division? Division of fractions?

5. Explain division of decimals by moving the point in 17.28 to right or to left.

6. With what divisors can you find the quotient in that way?

Multiplying or dividing both dividend and divisor by the same number makes no change in the quotient.

To divide 0.144 by 0.09

Process.

$$\begin{array}{r} 0.09 \overline{)0.144 = 9.} \\ \underline{1.6} \end{array}$$

To divide 63.44 by 25.6

Process.

$$\begin{array}{r} 25.6 \overline{)63.44 = 256.} \\ \underline{512.} \\ 122.4 \\ \underline{102.4} \\ 20.00 \\ \underline{17.92} \\ 2.08 \end{array}$$

7. In the example at the left how is the divisor changed from 0.09 to 9? For what purpose?

8. How and why must the dividend also be changed? State the principle. 9. After dividing 14 by 9, how many tenths in all remain to be divided?

10. Explain the process shown in the second example. When there is a remainder, how do you continue the division?

11. How may you always have an integral divisor?

12. Give directions for division of decimals in five steps:—

I. Setting down. II. Pointing. III. Dividing. IV. Placing and pointing the quotient. V. Managing a remainder.

193.—Written

Exercise.

Notice whether the quotient will be larger or smaller than the dividend.

1. $21.6 \div 0.006$

6. $102.01 \div 1.01 + 12.5\%$ of 100

2. $0.4913 \div 1.7$

7. $\$8.281 \div \frac{91}{100} - 6.25\%$ of \$ 3.00

3. $2.1952 \div 0.028$

8. $4.096 \div 0.0064 + 0.82369 \div 28.7$

4. $1.521 \div 3.9$

9. $67.24 \times 82\% - 67.24\% \div 82$

5. $0.6345 \div 0.009$

10. $400 \div 0.66\frac{2}{3} + 876.16 \div 0.296$

194.—At Sight.

1. 38.025 is to be divided into 195 parts.

Will each part be more or less than $\frac{1}{2}$?

2. How many places would there be in the quotient if the division were exact?

3. How would you write decimally the quotient of 3 in 100?

4. The quotient of 6 in 0.1?

5. Add these parts of 100: $18\frac{3}{4}\%$, 6.25% , $0.62\frac{1}{2}\%$, $\frac{5.9}{100}$.

6. 1% of it is \dots $\frac{1}{10}$ of 1% of it is \dots $1\frac{1}{2}\%$ is \dots
 7. 1.5% of \$10 = x . 0.5% = \dots $\frac{1}{3}\%$ of \$12 = \dots
 8. $1 - 0.9 = x$. Find the difference between 0.009 and 0.01
 9. $0.1 - 0.01 = x$. 10. $0.01 - 0.002 = y$.

195.—For Dictation. 1. A camel goes 3.5 m. an hour. How far in 10 h.? 2. Express 18 h. in hundredths of a day. 3. Express 200 min. decimally in hours. 4. Is a week more or less than 0.02 yr.? 5. Reduce to lowest terms 0.00004
 6. In a month there are about $2\frac{1}{2}$ million seconds. How many in $\frac{1}{3}$ of a month?

7. The interest of \$1 for 1 d. is $\frac{1}{6}$ of a mill. How would you write it decimally? Did you choose a complex or an incomplete decimal? 8. Find the interest of \$1 for 1 mo. at the same rate. 9. How would you write decimally $\$ \frac{1}{2} + \frac{1}{2}\text{¢} + 5\text{ d.}$ 10. 1 mill is what fraction of 10 eagles?

196.—For Analysis.

[See tables, p. 8.]

1. $\text{£ } 0.1 + \text{£ } \frac{2}{3} = x$ shillings.
 2. $\sqrt{\frac{4}{9}} - 0.1\frac{2}{3} = x$.
 3. $\frac{1}{62\frac{1}{2}} = \frac{x}{1000}$.
 4. 12 pence ($d.$) make a shilling ($s.$). Which is more, 1 $d.$ or $\text{£ } 0.004$?
 5. 5 score is what % of 200?
 6. $8\frac{1}{3}\%$ of a gross = 3 doz. — x .
 7. $\frac{5}{8}$ of a ream of note-paper calls for what part of a thousand envelopes.
 8. 10% of a ream = x sheets.
 9. A quire = $x\%$ of a ream.
 10. What % of a pound = 1 oz.?
 11. A 12 oz. pound is how many thousandths of an avoirdupois pound?
 12. A long T. = $\frac{2\frac{1}{2}}{2000}$ of a short T. Express as a mixed decimal.

197.—Sight

Problems.

1. When coal is \$5 a ton, what part of a ton is worth 40¢? \$2.45? 2. At \$5 a ton, 2400 lb. cost x . 3. When a short ton is sold for the cost of a long ton, $\frac{2\frac{1}{2}}{2000}$ or what per cent is gained?

4. 9 is what part of 144? What %? 11% of 50 is x . 5. 1760 is x % of the number of feet in a mile.

6. A centimeter = $\frac{1}{100}$ of a meter. A coil of wire 100 meters long is notched at intervals of 5 centimeters. How many notches in the whole length of it?

7. Find the balance of this account by inspection. Which party is described as *Dr.* and which as *Cr.*?

NEW YORK, JAN. 30, 189-.

Mr. J. L. Adams

In acct. with B. C. D. & CO.

Dr.

Cr.

<i>Jan. 2</i>	<i>To Mdse. as by bill</i>	<i>\$ 114 81</i>	<i>Jan. 1</i>	<i>By bill for services</i>	<i>\$ 150 00</i>
<i>9</i>	<i>Freight prepaid</i>	<i>2 13</i>	<i>5</i>	<i>By goods returned</i>	<i>14 81</i>
	<i>Storage of barrels</i>	<i>50</i>		<i>By allowance for damages</i>	<i>2 00</i>
<i>16</i>	<i>Cash on acct.</i>	<i>50 00</i>			

8. Multiply the sum of 11.507 and 2.09 by the difference between 10.85 and $10\frac{3}{4}$.

9. How much land can be bought for \$1000.00 at 25¢ an A.? at \$6.25? Add $18\frac{3}{4}$ % to the total cost in each case.

10. Of what two equal numbers less than 1 is 0.25 the product? Find the square root of 0.0081.

198.—For *Read rapidly,*
Frequent and change to or
Practice. from the decimal
form:—

- $\frac{9001}{10000}$ $\frac{9}{3000}$ $\frac{35}{500}$ $\frac{1000}{125}$
- $0.31\frac{1}{4}$ $0.037\frac{1}{2}$ $3.7\frac{1}{2}$
- $6\frac{3}{8}$ % $\frac{1}{3}$ % $\frac{17}{4}$ $\frac{17}{40}$

- $\frac{10101}{10}$ $\frac{40800}{200}$ $\frac{935}{5000}$
- 60.004 0.048 6.666 +
- $\frac{30250}{250}$ 0.0006 $\frac{333}{6000}$
- 20.2020 202.020 2020.20
- 10.12 \$10.012 $\$ \frac{112}{10}$
- $\frac{17}{20}$ $\frac{3}{8}$ $\frac{5}{9}$ $\frac{3}{7}$
- $\frac{7}{2100}$ $\frac{48}{1920}$ $\frac{360}{1800}$

199.—Written Work; 1. $18\frac{3}{4}$ hundredths of \$476.

Oral Analysis.

2. 18.75% of \$476 $\times 100\%$.

3. A city contains 40,000 persons, 26% in the first ward, 32% in the second, 21% in the third, and x persons in the fourth. How many in each ward?

4. $4.5796 \text{ m.} \div 2.640$. Give result in feet. Make statement and cancel.

5. A ball containing 2.58 cu. in. weighs 4 oz. Give the number and combined weight of as many balls as contain 665.64 cu. in.

6. $30\frac{3}{8}$ is the product, $17\frac{1}{2}$ is one factor, $100 \times$ the other = x .

7. $0.2 + 0.04 + 0.0001$ will contain what part of $0.9 + 4$?

8. Reduce $\frac{2.5}{100}$ to thousandths. 9. $\frac{333\frac{1}{3}}{1000}$ = what 5-place decimal? 10. Reduce 0.1875 to a complex decimal of three places. To one of two places. To a common fraction.

200.—For Dictation; Results decimally:—

Written. $\frac{2005}{1000}$ $\frac{83\frac{1}{3}}{1000}$ $\frac{8\frac{1}{3}}{100}$

4-6. Write as decimal fractions in lowest terms: 10% 68% 7.3%.

7-9. Write as decimal and as common fractions: $\frac{1\frac{1}{4}}{100}$ $\frac{1\frac{1}{4}}{10}$ $\frac{2\frac{1}{2}}{1000}$

10. Add as decimals: $\frac{1}{12}$; $\frac{1}{16}$; $\frac{1}{15}$.

11-13. A pint is what decimal part of 1 qt.? 1 pk.? 1 bu.?

14-15. Find the value, in lowest denominations, of 0.125 gal.; of 0.375 qt.

16-17. Give products: $1\frac{1}{2}$ ft. by 1.5; $2.16\frac{2}{3}$ ft. by 6.

18-19. Give the area in sq. ft.: $0.22\frac{2}{3}$ sq. yd.; $66\frac{2}{3}\%$ of 3 sq. yd.

201.—Problems for Statement.

1. A double eagle weighs 516. gr.; find the weight of gold that pays for 10,120 bu. of wheat @ 80¢.

2. What is the rate per cwt. when the charges are \$86.25 on $1\frac{3}{8}$ T., $\frac{3}{4}$ T., 2.0625 T., and $1.5\frac{3}{8}$ T.?

3. How many pounds in each and all of four bins containing respectively 2.750 T., $2\frac{3}{8}$ T., 5.167 T., and $7\frac{1}{2}\frac{3}{8}$ T.?

4. 240 gal. of vinegar costing 20¢ a gallon are reduced by 15% of water and sold at the same rate. The profit is x .

5. 21% of a gang of 200 workmen receive \$48.30. The wages of the rest are 20% higher. What fraction over a dollar a day does a workman of each class receive?

6. Express as a decimal part of a year: 1 mo.; $1\frac{1}{2}$ mo.; 1 day.

7. What decimal part of a year has passed with Aug. 21?

8. What date is $\frac{2}{7}\frac{4}{3}$ of a year after June 1st?

9. 390 books cost \$54.60 to bind. What was the entire cost of each copy if the binding was 0.14 of it?

10. If 1 pound loaf of bread occupies 128 cu. in., how many equal 1 cu. ft.? The size of 1 loaf = what part of 1 cu. ft.

202. — Processes 1. Change with Decimals. to 128ths: —

$$\frac{21}{4} \quad \frac{15}{256} \quad \frac{39}{96}$$

2. Change to 4-place decimals and add: —

$$\frac{5}{18} \text{ and } \frac{17}{32}$$

3. Change to 100ths: —

$$\frac{9}{17}; \frac{3}{40}; \frac{4}{75}$$

$$4. \frac{3}{50} + \frac{23}{100} + 0.0871$$

5. Find the difference in 4-place decimals: —

$$\frac{1}{11} - \frac{1}{12}; \frac{1}{20} - \frac{1}{21}; \frac{1}{1} - \frac{1}{1000}$$

$$6. 1.5 = \frac{15}{x} \quad \frac{1}{10} \text{ of } 1.5 = y$$

$$1.5 + 0.15 = \frac{11}{10} \text{ of } x.$$

7. Add decimally: —

$$\frac{5}{8} \quad \frac{7}{9} \quad \frac{4}{11}$$

8. Reduce to lowest terms: —

$$7.08 \quad 0.0708 \quad 0.70080$$

9. Write as complex decimals: —

$$10.1833 + \quad 0.87625 \quad 0.1875$$

$$10. \sqrt{0.6400} = x.$$

203. — Problems.

In solving use simple methods and few figures.

1. The area of a floor is to the area of its supports as 10: $\frac{1}{16}$. What area of support is given to 20 sq. ft. of floor?

2. The number of feet in a mile is what per cent of the number of inches?

3. $16\frac{1}{2}$ ft. = 1 rd. One girl lives 370 rd. from school; another 142.35 rd. on the same road. Their houses are x ft. or y ft. apart.

4. A boy walks $\frac{1}{8}$ m. and 300 steps more, each measuring 2.2 ft. How far in all?

5. \$81271.08 is to be divided among 7 heirs. 5 of them share equally; the others receive each a double portion. What is the amount of a 2% tax on one of the 5 equal shares?

6. How much wheat at 73¢ a bushel pays for 30 sheep, in three lots, weighing 500, 600, and 700 lb., if taken at the rate of \$3.017 $\frac{1}{2}$ for each sheep on the foot?

7. After melting $\frac{5}{8}$ of a sheet of metal and later $\frac{1}{32}$, there was $\frac{1}{32}$ of a square foot left. How many square inches were in the part first melted?

8. If a workman saves \$62.40 in a year by taking 20¢ each day from his wages, how long would it take 4 men at the same rate to save \$124.80?

9. When the cost of transporting coal is 2¢ a ton for each mile, and the freight on 400 tons is \$200, what is the distance?

10. A lot of cord wood is $\frac{3}{16}$ beech, 0.21875 birch, 0.1875 maple, $\frac{3}{32}$ ash, 10 cd. oak, $\frac{1}{8}$ poplar, and $3\frac{1}{8}\%$ pine and fir; in all x cd.

204. — Mixed Examples.

1. At 1.25 cu. ft. to the bushel, compute the value at 57 $\frac{1}{2}$ ¢ a bushel of a bin containing 4000 cu. ft.

2. $72.012 \div 2.64$

3. $70.397 \div 0.9023$

4. Simplify $\frac{1}{272\frac{1}{4}}$

5. 1 sq. yd. = what part of 1 sq. rd.?

6. 160 sq. rd. = 1 A.

1 sq. rd. = x A.

1 sq. in. = x sq. ft.

7. From $1234\frac{7}{12}$ take $347.921\frac{2}{3}$.

8. Find the profit on
 274 bbl. at \$4.11 $\frac{1}{2}$
 128 " at 3.96 $\frac{3}{4}$
 if sold by the pound @ 3¢, 196 lb. to the bbl.
9. Find the value of £23,738 at \$4.8665.
10. Find the cost of expressing \$1,000,000 gold at $\frac{7}{160}$ of 1%.

- 205.—Denominate Numbers. 1. 1 s. = x ¢. £1 1s. or 1 guinea = how many pence? 2. $\frac{1}{4}$ d. or 1 farthing + $\frac{3}{4}$ d. = what part of a pound? Half a crown (1 crown = 5s.) is worth how many cents?
3. Find the difference between \$100 and the value of £10 10s. + 25 M. 50 pf. + 20 fr. 50 ct. In what (fewest) pieces of money could the difference be paid?
4. \$75 will buy how many pounds? The remainder will buy how many shillings and pence?
5. Add £4 6s. 9d., 21s. 4 $\frac{1}{2}$ d., 5s., £4 10s.
6. Reduce to grains: 3 sc. + 15 gr. + 4 dr. + 1 oz.
7. Find the difference between 1 lb. 5 oz. 3 dr. and 2 lb. 7 dr.
8. At \$7.00 a pound, 3 dr. of saffron cost what?
9. Find the value of 1000 gr. of a metal worth \$20 an ounce.
10. Three packages of opals weigh each 1 oz. 7 pwt. 5 gr. Find the total weight.

206.—1. Give the latitude of the north pole; the south pole; the equator; a point 691 $\frac{3}{4}$ m. south of the equator.

2. Two persons set out from the same point. One goes 42° north, the other 311 $\frac{1}{4}$ m. south. How many miles apart are they? How many degrees?

3. 1° of latitude = x m.; 1' = y m.; 1" = z ft.

4. The difference in latitude of two places is 15 minutes. How many miles apart are they, if one is due north of the other?

5. A steamship laying telegraph cable finds a depth of 120 fathoms after steaming 600 knots off shore. Give the depth in feet. The distance in common miles.

6. If a gold dollar weighs 25.8 gr., how many double eagles can be made from 3 lb. 3 oz. 372 gr.?

7. 1 gal. = x cu. in. 1 qt. = y cu. in. How many gallons altogether can be put into two tanks holding 10 cu. ft. and 8900 cu. in. respectively.

8. 1 peck = x cu. in. 1 qt. = y cu. in. How many 2-bushel bags can be emptied into a bin containing a number of cu. ft. equal to $6\frac{2}{3} \times 4\frac{1}{6} \times 3\frac{3}{4}$?

9. How many 6 gal. be reduced to pints? Change 2.5 gal. to gills. Express $\frac{5}{8}$ bu. in the largest possible units.

10. What multiplier will change tons to hundredweight? Change to lower denominations: 0.875 T.; 0.625 bu.; 0.55 qt.

207.—Interest.

a General Method.

1. I live in a hired house worth \$6000. For the use of the house for a year I pay $\frac{1}{10}$ (10%) of its value. The year's rent is x dollars. This is y dollars a month, and (counting 30 days to a month) z dollars a day.

2. If I had used the money which the house cost, \$6000, for a year at the same rate, 10%, the annual interest would have been x dollars. For 6 mos. it would have been $\frac{1}{2}$ of \$ x or \$ y .

3. Value of house used, \$3000; rate of rent, 5%; year's rent, \$ x ; 4 months' rent, \$ y ; 1 month's rent, \$ z .

4. Money used, \$3000; rate of interest, 5%; year's interest, \$ x ; 4 months' interest, \$ y ; 1 month's interest, \$ z .

5. What is the difference between *rent* and *interest*?

6. Interest is an allowance to the owner for the use of his money. The Principal is the money used.

The Amount is the sum of interest and principal.

The Rate of interest is the number of hundredths of the principal paid for a year's use of it.

The principal is \$200. The rate is 6%. Give the interest for 1 year; 2 yrs.; 3 yrs.; 4 yrs.; 5 yrs. The interest for 1 mo.? 2? 3? 4? 5? 6? 7? 8? 9? 10? 11?

7. What is a year's interest of \$300 at 2%; 3%; 4%; 5%?

Find the interest

What shall I pay for the use

- | | |
|---|--|
| 8. Of \$ 300 at 4% for 2 yrs. | 11. Of \$ 1000 for 2 yrs. at 10% |
| 9. Of \$ 500 at 6% for 3 yrs. | 12. Of \$ 600 for $\frac{5}{6}$ yr. at 10% |
| 10. Of \$ 800 at 7% for $\frac{1}{2}$ yr. | 13. Of \$ 800 for $\frac{3}{4}$ yr. at 4% |

208.—Interest: a 1. In most business transactions 30 days
 General Method. make a month. A month's interest is \$ 60; a
 Time in Months. day's interest is \$ x ; 10 days' interest is \$ y ;
 20 days' interest is \$ z ; \sim days' interest is
 $\sim \times \$ 2$.

2. The interest of \$ 300 at 10%
 for 1 year = 10% of \$ 300 or \$ x ;
 for 1 month = $\frac{1}{12}$ of \$ x or \$ y ;
 for 1 day = $\frac{1}{360}$ of \$ y or \$ z .

3. Reckoning 30 days to an interest month gives 360 days to an interest year. A day's interest is what part of a year's interest?

17 days' interest is $\frac{17}{360}$ of a year's interest.

4. To find the interest of \$ 240 for 2 yr. 5 mo. at 5%.

B.

A.
 \$ 240 = Principal
 .05 = Rate
 \$ 12.00 = Int. for 1 yr.
 $2\frac{5}{12}$ = Time in yrs.
 500
 2400
 \$ 29.00 = Int for $2\frac{5}{12}$ yrs.

$$29 \times \frac{1}{12} \times \frac{5}{100} \text{ of } \$ 240 = \$ 29$$

C.

	\$ 240 ²⁰
100	5
12	29
	\$ 29

In *B*, what represents a year's interest? A month's interest? 2 yr. 5 mo. or 29 mo. interest? How is the process shortened? In *C* the numbers are arranged in columns for cancellation, dividends at the right, and divisors at the left.

5. To find the interest of \$720 at 8% for 3 yr. 8 mo.

A.

	60
100	\$720
12	8
	44
	480
	3520
	176
	\$211.20

B.

$$\frac{44}{100} \text{ of } \frac{8}{100} \text{ of } \$720 = \$211.20$$

C.

$$3\frac{2}{3} \times 8\% \text{ of } \$720 = \$211.20$$

Explain each process.

What is the interest —

6. Of \$840 for 1 yr. 9 mo. at 10%?
7. Of \$360 for 4 yr. 10 mo. at 5%?
8. Of \$960 for 1 yr. 8 mo. at 4%?
9. Of \$1000 for $12\frac{1}{2}$ yr. at 8%?
10. Of \$400 for 2 yr. 5 mo. at 7%?

209.—Interest: a

General Method.

Time in Days.

	\$500
100	5
10 360	36
	\$2.500

1. To find the interest of \$500 for 1 mo. 6 d. at 5%.

a. 1 year's int. = $\frac{5}{100}$ of \$500, or \$2.

b. 1 day's int. = $\frac{1}{360}$ of \$2, or \$ $\frac{1}{180}$.

c. 36 days' int. = $36 \times \$\frac{1}{180}$, or \$2.

Explain the process at the left.

2. To find the interest of \$480 for 2 mo. 12 d. at 9%.

	12
	\$480
100	9
40 360	72
	\$8.64

a. $\frac{9}{100}$ of \$480 = int. for 1 mo.

b. $\frac{9}{360}$ of $\frac{9}{100}$ of \$480 = int. for 1 d.

c. $72 \times \frac{9}{360}$ of $\frac{9}{100}$ of \$480 = int. for 12 d.

What is the advantage of arranging dividends and divisors on either side of a vertical line?

3. The interest of \$600 at 4% for 60 days.

Find the interest of—

4. \$250 for 1 mo. 15 d. at 6%
5. \$120 for 80 days at 7%
6. \$372 for 36 days at 10%
7. \$336 for 8 mo. 10 d. at 4%
8. \$1728 for 2 mo. 17 d. at 9%
9. \$800 for 9 mo. at $7\frac{1}{2}\%$
10. \$1000 for 93 d. at $4\frac{1}{2}\%$

- 210.**—Interest: a General Method. Time in Years, Months, and Days.
1. To find the interest of \$840 for 3 yr. 7 mo. 11 d. at 4%. (360 d. to an interest year.)
 2. Explain the cancellation in the process at the left. Which numbers are then to be multiplied? 3. Need they be rewritten? Which is used as multiplier?
 4. Explain the two steps in dividing by 9000. What principle is applied?
 5. How is the amount found?

Process.

$$\begin{array}{r}
 \begin{array}{r}
 100 \\
 90 \ 360
 \end{array}
 \begin{array}{r}
 \$840 \\
 4 \\
 \hline
 1301 \\
 52040 \\
 \hline
 104080 \\
 9.000 \overline{) \$1092.840} \\
 \hline
 \$121.426+
 \end{array}
 \end{array}$$

- a. 3 yr. 7 mo. 11 d. = 1301 d.
- b. 1 yr.'s int. = $\frac{4}{100}$ of \$840, or \$x.
- c. 1 day's int. = $\frac{1}{360}$ of \$x or \$y.
- d. 1301 day's int. = 1301 \times \$y or \$z.

Compute the interest under the following conditions. Try to forecast the results approximately.

Principal.	Time.	Rate.	Principal.	Time.	Rate.
6. \$270.	1 yr. 7 mo. 20 d.	6%	14. \$363.42	1 yr. 5 mo. 12 d.	4%
7. 500.	2 yr. 8 mo. 7 d.	10%	15. 78.30	15 mo. 15 d.	8%
8. 810.	3 yr. 11 mo. 10 d.	4%	16. 1566.	4 yr. 7 d.	3%
9. 144.	1 yr. 1 mo. 1 d.	8%	17. 1800.	9 mo. 17 d.	$3\frac{1}{2}\%$
10. 696.	4 yr. 4 mo. 4 d.	3%	18. 427.50	117 d.	12%
11. 475.	1 yr. 10 mo. 27 d.	9%	19. 849.78	10 mo. 29 d.	4%
12. 84.50	2 yr. 6 mo. 15 d.	$4\frac{1}{2}\%$	20. 648.	1 yr. 1 mo. 21 d.	7%
13. 720.	5 yr. 5 mo. 7 d.	5%	21. 2100.	5 yr. 5 d.	5%

211. DEFINITIONS.

[FOR REFERENCE.]

Amount in computing interest. Interest and principal added.

Antecedent. The first term of a ratio; the dividend.

Bill. An itemized statement showing to whom and by whom goods have been sold, or services rendered, and giving dates, quantity, price, and amount.

Common Denominator of Two or More Fractions. One showing the size of some fractional unit in which all may be expressed.

Complex Fractions contain a fraction in the numerator, in the denominator, or in both.

Complex Decimals have a common fraction in the numerator, as $0.27\frac{1}{2}$.

Compound Number. Two or more denominate numbers used to express one quantity; a denominate number having two or more integral units of the same kind of measure, as $3^{\circ} 5'$.

Consequent. The second term of a ratio; the divisor.

Couplet. The two terms of a ratio.

Decimal Fractions, or Decimals. Any number of 10ths, 100ths, 1000ths, etc.; commonly expressed at the right of the decimal point without written denominator.

Denominate Number. One in which the unit is a measure, as 3 lb.

Improper Fraction. A number not less than 1 expressed in the form of a fraction.

Interest. An allowance to the owner for the use of his money.

Invoice. A bill of goods sold.

Least Common Denominator of Two or More Fractions. One showing the size of the largest fractional unit in which all can be expressed.

Like Fractions have fractional units of the same size and kind.

Mixed Decimal. A number consisting of an integer and a decimal fraction.

Principal. A sum upon which interest may be allowed.

Proper Fraction. A number less than 1; a true fraction.

Rate of interest. Per cent of the principal allowed for a year's use of it.

Ratio. The relative size of two numbers expressed by their quotient.

Reciprocal of a Number. $1 \div$ the number; the fractional unit expressed by that number as denominator, as $3, \frac{1}{3}$.

Reciprocal of a Fraction. $1 \div$ the fraction, or the fraction inverted.

Simple Fraction. One having only integral terms.

Terms of a Fraction. The numerator and denominator.

Terms of a Ratio. The antecedent and consequent.

Measurements.

[REVIEW THE TABLES ON pp. 8-9.]

212. — Of Lines.

1. Beginning with the shortest, name the five ordinary units used in measuring lines, or lengths, or distances.
2. Compare an inch with a foot. An inch with a yard.
3. Compare a foot with a yard; with a rod; with a mile.
4. What part of a mile is a rod? 5. What is $\frac{2}{11}$ of a rod?
6. $\frac{5}{16}$ m. = x rods.
7. 91 in. = x ft. + y in. 8. $3 \text{ ft.} \times 5\frac{1}{2} \times 320 = 1 \text{ m.} = x \text{ ft.}$
9. Learn the distance from home to school by measuring and counting your steps, or in some more exact way.
10. Estimate in rods and then in feet the dimensions of your schoolroom; schoolhouse; school lot; the width of the road or street. Test your estimates by measuring.

213. — Length

Measures.

At sight.

1. $\frac{5}{12}$ yd. = x ft. y in. 2. $62\frac{1}{2}\%$ of a mile = x rods.
3. At a cent a foot, 4 rods of picture cord will cost x cents.
4. At 12¢ a yard, picture moulding for a room 25 ft. long and 20 ft. wide will cost x .
5. $\frac{1}{2}$ rd. = x ft. 6. $\frac{1}{11}$ rd. = x ft. or y in. 7. 100 in. = x yd. y ft. z in. 8. 1000 rd. = x m. 9. 2 m. 40 rd. = x rd.
10. At the rate of 3 m. an hour, how many rods can you walk in 15 min.?

214. — Length

Measures.

Written.

1. An ocean steamship 660 feet long is what part of a mile in length?
2. Steaming 22 miles an hour is at the rate of x feet every second.
3. Crossing the Atlantic, a distance of 3100 miles, in 5 days $7\frac{1}{2}$ hours, the average rate is x miles per hour.

4. Mt. Everest is said to be 29,002 feet or x miles high.
5. The distance from the equator to the north pole is ten million meters. Calling a meter 39.37 inches, what is the distance in miles?
6. One wheelman rides 24 miles an hour. Another rides $\frac{1}{4}$ mile in 34 seconds. Compare the distance per minute each one rides.
7. A race-course was 30 knots. The time of the winning yacht was 3 hours 25 minutes. This was x feet per minute.
8. A horse trotted a mile in 2.04, or at the rate of x rods and y feet per second.
9. In May, 1893, the Empire State Express ran from Syracuse to East Buffalo, 145.6 miles, in 2 hours 21 minutes. Find the rate per hour.
10. The driving-wheel of the locomotive was $6\frac{1}{2}$ feet in diameter. Calling this $\frac{7}{2}$ of the circumference, and making no allowance for slipping, how many revolutions would it make?

215. — Surface**Measures.**

[Review Tables, p. 9.]

1. What is a plane surface?
2. What are the boundaries of surfaces called?
3. What kind of lines bound rectilinear surfaces (*rect-* meaning *right* or *straight*)?
4. What is the shape of most of the common units of surface measure?
5. Describe a square; an oblong, or rectangle.
6. Name the five square measures of surface, beginning with the smallest, and giving the length of each.
7. An acre is not a square measure. It contains x square rods.
8. $12^2 = x$; $(5\frac{1}{2})^2 = y$; $(16\frac{1}{2})^2 = z$.
9. Give the length of a square yard in yards; in feet; in inches.
10. The length of a square rod in yards; in feet; in inches.
11. What is the length of a square mile in rods? in yards? in feet?

216. — Of Surfaces**or Areas.**

1. Draw a diagram to show what a square inch is. Is it an inch square? Might it contain a square inch of surface and be of some other shape?

2. Draw a diagram as an example of a square foot. If your paper is too small, draw it on a scale of $\frac{1}{2}$ or $\frac{1}{3}$ or $\frac{1}{4}$; that is, make it $\frac{1}{2}$ or $\frac{1}{3}$ or $\frac{1}{4}$ of its actual length. 3. Divide your drawing to show the number of square inches in a square foot. How many are there?

4.

5.

6.

$$\frac{1}{2} \text{ sq. ft.} = x \text{ sq. in.} \quad \frac{1}{6} \text{ sq. ft.} = x \text{ sq. in.} \quad 36 \text{ sq. in.} = \frac{1}{x} \text{ sq. ft.}$$

$$\frac{1}{3} \text{ sq. ft.} = y \text{ sq. in.} \quad \frac{1}{8} \text{ sq. ft.} = z \text{ sq. in.} \quad 60 \text{ sq. in.} = \frac{5}{y} \text{ sq. ft.}$$

$$\frac{1}{4} \text{ sq. ft.} = y \text{ sq. in.} \quad \frac{1}{12} \text{ sq. ft.} = z \text{ sq. in.} \quad 108 \text{ sq. in.} = \frac{z}{4} \text{ sq. ft.}$$

7. $66\frac{2}{3}\%$ of a square foot = x sq. in. 8. $1\frac{1}{2}$ sq. ft. = x sq. in.

9. Represent a square yard in outline; scale, $\frac{1}{6}$. Separate it into square feet. 10. How many square inches in it?

217. — Of Land

Areas.

Written.

1. 1 rod = x yd. = y ft. = z in.

2. Draw a diagram on a scale of $\frac{1}{33}$ ($\frac{1}{33}$ of z inches long) to represent a square rod. From one corner mark the yards along two sides.

3. Separate it into square yards. You find that you have 25 squares, x half-squares, and y quarter-squares.

4. If convenient, outline a square rod on the schoolroom floor; imagine one on the ceiling, and show how far it would extend; or have one marked off in the school-yard.

5. How many square yards in an acre of land? 6. Draw a figure to represent an acre, 10 rods wide and 16 rods long; scale, $\frac{1}{4}$ in. to a rod. Divide your drawing to represent square rods.

7. What part of an acre does the school lot equal? 8. $\frac{7}{8}$ A. = x sq. rd.

9. How many acres in a square mile? In 10% of it? In $\frac{7}{8}$ of it?

10. A western township is 6 miles square. It contains x square miles or sections, and the distance around it is y miles.

11. Find some piece of ground which you can show to contain about 1 A.

218. — Surface**Measures.***Written.*

1. Change 20,000 sq. in. to square feet.

2. Change 12,371 sq. ft. to square rods.

3. Change 287 sq. rd. to square feet.

4. 1350 sq. m. = x A. 5. Change an acre to square feet.

6. Bought $\frac{7}{8}$ A. for \$400, and sold it at a dime per square foot.
How much did I gain or lose?

7. A farmer owns five fields or lots measuring as follows: 80 A., 200 sq. rd.; $\frac{3}{8}$ sq. m.; $87\frac{1}{2}\%$ of an acre; and 435,600 sq. ft. What is the acreage of this farm?

8. In 20,000 sq. ft. how many square rods?

9. Add:

17 sq. ft. 19 sq. in.

19 sq. ft. 75 sq. in.

42 sq. ft. 108 sq. in.

96 sq. ft. 121 sq. in.

10. What will be the cost of a school lot containing 32,670 sq. ft. at \$5000 per acre and \$12.50 per square rod for filling and levelling?

219. — Lines.

1. Draw horizontal, vertical, and inclined or oblique lines. 2. Name the three kinds that you have drawn and describe them according to their direction. Try to make your description exact and brief before consulting page 147.

3. With reference to each other, two lines may be parallel or perpendicular. How many pairs of parallel lines on pages 116–117?

4. What are parallel lines?

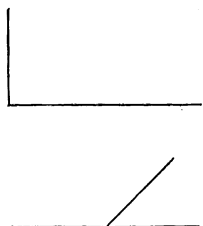
5. Describe straight lines and curved lines. 6. What is a line?

220. — Angles.

1. Draw an angle and show its sides and vertex, or their point of meeting.

2. Two lines having different directions and meeting at a point make an angle. To measure an angle is to measure this difference in the direction of the lines. Repeat the table for circular measure (p. 9).

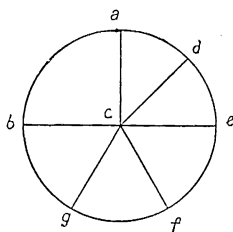
3. If you prolong the sides of an angle, do you increase its size?



4. Draw two intersecting lines so as to make four equal angles.
5. The lines thus drawn are perpendicular to each other and the angles are right angles. Define perpendicular lines.
6. What is a right angle?
7. Draw two inclined lines perpendicular to each other.
8. Compared with right angles what are acute angles? Obtuse angles?
9. Show how many of each of the three kinds are on pages 116 and 117.
10. Adjacent angles have one side in common. With two strokes of a pencil draw four angles; then draw four figures showing the four pairs of adjacent angles that you have made.
11. Unequal adjacent angles are *oblique* angles. "Oblique" means ~?

221.—Divisions of Circle; Degrees of Arcs and Angles.

Oral.



1. What is a circle?
2. What name is given to the curve that bounds it?
3. A diameter of a circle bisects it. How is the diameter indicated in the figure?
4. How many radii are drawn?
5. The surface enclosed by *bca* is a quadrant; by *bcb*, a sextant. These same names are applied to the arcs *ba* and *bg*, as the circumference is often called a circle. What part of a circle or circumference is *bca* or *ba*? *bcb* or *bg*?
6. For convenience in measuring arcs and angles, every circumference, whether large or small, is divided into 360 equal parts or degrees (360°). How many degrees is a semicircle? a quadrant? a sextant? a sign or 12th of a circle?
7. Each degree is divided into 60 minutes ($60'$), and each minute into 60 seconds ($60''$).

$$15^\circ = u'$$

$$300' = w^\circ$$

An arc of 30° contains y'

$$8. 10^\circ = v''$$

$$600'' = x'$$

$\frac{3}{4}$ of a circle contains z°

9. Over how many degrees does the long hand of a watch move in an hour? in 30 min.? in $\frac{1}{4}$ h.? 10. In 20 min.? in 25 min.? in 35 min.? in $\frac{1}{2}$ day?

11. Of the six angles in the figure which is the right angle? How many degrees in the arc between its sides? What arc measures a right angle?

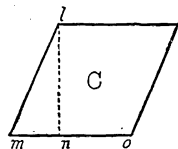
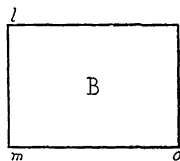
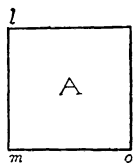
12. Which three angles in the figure are equal? What is the size of the arc that measures each? Each is an angle of x° .

13. How many degrees in half a right angle?

14. Draw angles of 90° ; 45° ; 60° ; 105° ; 120° . Which are obtuse? Which acute?

222. — The Six Quadrilaterals.

Oral.



1. Notice how many sides these figures have, and define a quadrilateral.

2. Which have their opposite sides parallel? What is a parallelogram.

3. Which of the quadrilaterals has only two parallel sides? What is a trapezoid?

4. Which is a quadrilateral without parallel sides? What is a trapezium?

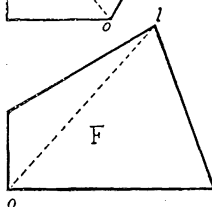
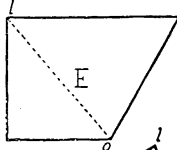
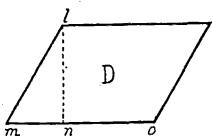
5. Which two parallelograms are equilateral? Equiangular? Rectangular?

6. What name is given to a quadrilateral with four right angles? 7. To an equilateral rectangle?

8. Which of the parallelograms have only oblique angles? What is a rhomboid?

9. Which rhomboid is equilateral? What is a rhombus?

10. Show the propriety of each of the following terms as applied to A: quadrilateral, parallelogram, rectangle, square, equiangular, equilateral.



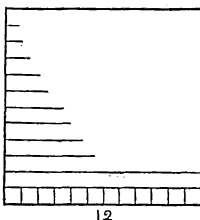
11. Of each parallelogram which line is the base? 12. Which shows the altitude or height of the parallelogram?

13. The altitude and base must always be perpendicular to each other. Try to tell why.

14. A straight line, like lo , that joins the vertices of opposite angles is a diagonal. Prove by cutting—

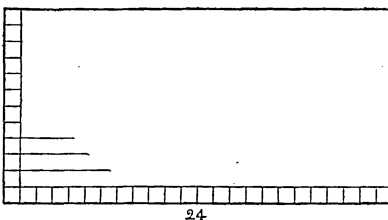
- (1) that a diagonal bisects a parallelogram.
- (2) that the opposite angles of a rhomboid are equal; and by measuring prove,
- (3) that the sum of the angles of a quadrilateral equals four right angles.

223.—Of Rectangles. 1. An inch-wide rectangle 12 in. long contains x sq. in.; a 12-inch-wide rectangle of the same length contains $12 \times x$ sq. in., or y sq. in. (See the figure.)



2. What is the area of a rectangle 15 in. square? 3. Of a rectangle 1 ft. 6 in. square?

4. A foot-wide rectangle 24 ft. long contains x sq. ft.; a 12-ft.-wide rectangle of the same length contains $12 \times x$ sq. ft. or y sq. ft.



5. Find the area of rectangles 15 ft. long, 7 ft. wide; 13 ft. \times 20 ft.; $16\frac{1}{2}$ ft. long, $16\frac{1}{2}$ ft. wide; $5\frac{1}{2}$ yd. \times $5\frac{1}{2}$ yd.

6. A piece of land measures 25 rods one way and 20 rods the other way. Find its area in square rods; in acres. Which of

the following statements or equations is right for the second answer?

$$\frac{25 \times 20 \text{ sq. rd.}}{160} = x; \quad \frac{25 \times 20 \text{ sq. rd.}}{160 \text{ sq. rd.}} = x.$$

7. A kindergarten table 4 ft. 3 in. long and 20 in. wide is marked off into inch squares; how many are there?

8. How many square yards in a web of cotton $40\frac{1}{2}$ yd. long and 3 ft. wide?

9. In a flag $10\frac{1}{2}$ ft. long and $\frac{2}{3}$ as wide, how many square yards of bunting, not allowing for seams?

10. A patchwork quilt measuring 3 yd. by 3 yd. is made of 4-inch silk squares; how many are there?

224. — Superficial

Contents of Rectangles.

Written.

NOTE. — Draw diagrams to illustrate; make statements, and cancel when possible.

1. What shall I pay Mr. Bates for concretizing a walk 60 ft. long, 4 ft. wide for half its length, and 3 ft. wide the rest of the way? His price is 75 cents per square yard.

2. Mr. Cross fenced his strawberry patch, which was 4 rods wide and 100 feet long, with three lines of barbed wire at $1\frac{1}{4}$ ¢ per foot. The posts cost \$7. How many quarts of berries at 5¢ a quart must he sell to pay for the fence?

3. When an acre of land is 40 rods long, what is its width?

4. Mrs. Fiske bought the equivalent of a square yard of 4-inch ribbon for \$4.50; what was the price per running yard?

5. How many square tiles 9 inches long will lay a floor 12 ft. wide and 27 ft. long?

6. A roll of oilcloth 72 in. wide is 30 ft. long; what is it worth at $62\frac{1}{2}$ ¢ per square yard?

7. How many square feet of glass in your schoolroom windows?

8. Of blackboard surface in the room?

9. Drawing paper measuring 24×36 is cut into 9×12 pieces. How many pieces will a ream furnish?

225.—Of Carpeting, Tiling, etc.

Oral and written.

1. Ingrain carpets are generally woven in strips a yard wide; other carpets, three-quarters of a yard. What two advantages come from running the strips lengthwise of the floor rather than across it?

2. On floors of the following widths which width of carpet could be used without either cutting or turning under any strip?

12 ft. 15 ft. $22\frac{1}{2}$ ft. 27 ft. $13\frac{1}{2}$ ft. 20 ft. 18 ft.

3. How many strips of ingrain carpet will be needed for a room 18 ft. square? How many running yards? How many yards must be bought if a quarter-yard is wasted in matching each two strips?

4. How many strips of brussels or tapestry carpet will be needed for a room 15 ft. wide and 21 ft. long? How many yards, if it matches without waste? Find the cost at \$1.25 a yard.

5. Find the cost of covering a floor 14 ft. by 20 ft. with yard-wide carpet at 75¢, no strips to be cut, nor allowance made for waste. 6. What will it cost using three-quarter carpet at \$1.50, on the same conditions? 7. Using 4-ft. oilcloth at \$1?

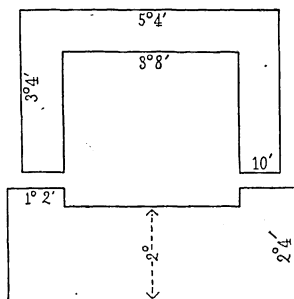
Find the cost of carpeting floors under the following conditions (strips that are cut cost as if whole):—

	Length of room.	Width of room.	Width of carpet.	Allowance for matching.	Cost per yd.
8.	18 ft.	14 ft.	1 yard	$1\frac{1}{2}$ yds.	\$ 0.90
9.	22 ft.	18 ft.	$\frac{3}{4}$ yard	$2\frac{1}{4}$ yds.	4.25
10.	$16\frac{1}{2}$ ft.	$13\frac{1}{4}$ ft.	1 yard	$\frac{7}{8}$ yd.	0.87 $\frac{1}{2}$
11.	$21\frac{1}{4}$ ft.	20 ft.	$\frac{3}{4}$ yard	$2\frac{3}{8}$ yds.	1.37 $\frac{1}{2}$

12. How many 8-in. marble tiles are required to cover a hearth 2 ft. by 4 ft. 8 in.? 13. To cover a floor 20 ft. by $46\frac{1}{2}$ ft.?

14. The areas to be tiled about a fireplace are: one of 5 ft. 3 in. by 1 ft. 9 in.; two of 2 ft. $10\frac{1}{2}$ in. by 1 ft. 9 in.; one of 1 ft. 9 in. by 6 in.; one of 5 ft. 3 in. by 2 ft. $4\frac{1}{2}$ in. Find the total area.

15. How many tiles $1\frac{1}{2}$ in. by 3 in. are required?



16. Find the area of the surfaces shown at the left. 17. How many 2-in. tiles are required to cover them? [$^{\circ}$, ', p. 148.]

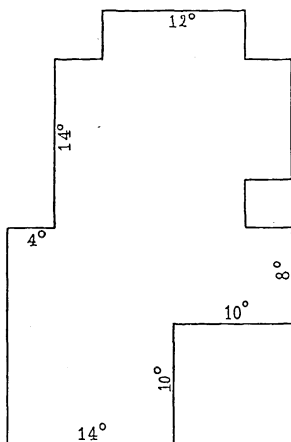
18. A room is $13\frac{1}{2}$ by 18, and 8 ft. high. How many rolls of wall paper are required, each being 8 yd. by 18 in., no allowance made for doors, windows, or baseboards?

226. — Of Roofs, Pavements, etc.

1. $9\frac{1}{2}$ squares of slating are required to cover a certain roof. This is equal to how many square yards? If the slates are 8×16 , and each course overlaps 10 inches of the one below it, find the number of slates used.

2. How many blocks 6×4 inches will be used in paving a four-rod square?

3. How many tin plates 13×19 must be used for 1 square of roofing, if they are lapped or folded $\frac{1}{2}$ in. on each side?



4. Three piazza roofs about a house measure in feet 30×8 , 24×7 , $7\frac{1}{2} \times 12\frac{3}{4}$. How much less than 5 squares do they contain?

5. A house lot contains $\frac{1}{4}$ A. How many sq. ft.? The house is $27\frac{1}{4} \times 40$. What would it cost to sod the remainder at \$1.50 a square?

6. Let this figure represent the outline of a cellar. Copy, and divide it into 5 rectangles. From the given dimensions find those of each rectangle.

7. How many square yards of cement would be required to cover the bottom of the cellar?

**227.—Areas and
Perimeters of
Rectangles.**

For oral analysis.

1. A chess-board contains 64 squares $1\frac{1}{2}$ in. long. What is its perimeter? If it has an inch-wide border, what?
2. In a 2-inch square how many $\frac{1}{4}$ -inch squares? How many $\frac{1}{2}$ -inch?
3. Compare the perimeter of a 4-foot square and an equal surface 8 feet long.
4. My sidewalk is 10 ft. wide besides the curb, and 100 ft. long. How many 4×8 bricks in it?
5. Compare a 4-inch square and a 12-inch square as to length and area.
6. A marble slab 4 ft. by $2\frac{1}{2}$ ft. was sold for \$4.50; price per square foot?
7. What is the area of a square that can be set off with 200 feet of rope?
8. How many boards 9 inches wide make a close fence 8 feet high around three sides of a square lot 180 feet long?
9. A hall measures 12 feet by 36 feet. How many breadths of yard-wide carpet would be needed? How many yards, allowing 3 yards wasted in matching the pattern?
10. A room 14 ft. by 18 ft. is to be covered with yard-wide carpet at \$1. Which is the cheaper way to run the strips? Why?

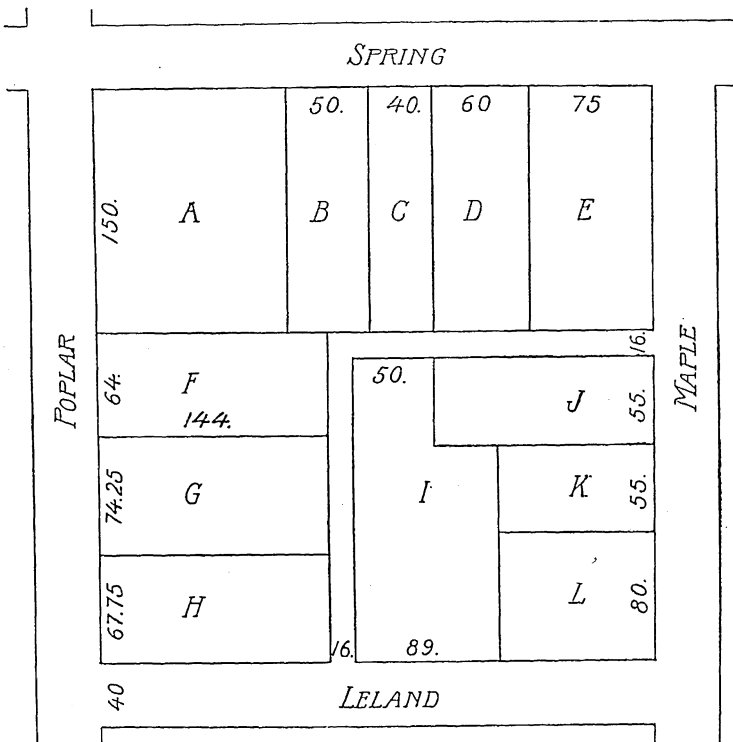
228.—Of City Land. 1. Mr. Sharp bought land bordering on

Written.

Spring Street between Poplar and Maple at [See next page.] 3 cents per square foot, which he cut up into building lots. He first laid out a 40-foot street in the rear, which he called Leland Street. What did he pay for the land?

2. He sold lot C to a civil engineer for his services in surveying and making plans, plus an additional 2 cents per square foot. What did the survey, etc., cost?

After reserving lot A for his own dwelling-house, he sold the remaining 10 lots by auction at the following prices.



Find the proceeds of the sale of each lot.

- | | | |
|--------------------------------------|--------------------------------------|---------------------------------------|
| 3. Lot B for $12\frac{1}{2}\text{¢}$ | 6. Lot F for $18\frac{3}{4}\text{¢}$ | 10. Lot J for $19\frac{1}{2}\text{¢}$ |
| 4. Lot D for 15¢ | 7. Lot G for 21¢ | 11. Lot K for $17\frac{3}{4}\text{¢}$ |
| 5. Lot E for $22\frac{3}{4}\text{¢}$ | 8. Lot H for $20\frac{1}{4}\text{¢}$ | 12. Lot L for $25\frac{1}{8}\text{¢}$ |
| | 9. Lot I for $17\frac{5}{8}\text{¢}$ | |

13. Before the sale, he opened and laid out a 16-ft. alley from Maple Street to Leland. What did the alley cost him, \$85 being paid for labor?

14. The grading of Leland Street cost him \$3.75 per square rod. What did the street cost, including land and labor?

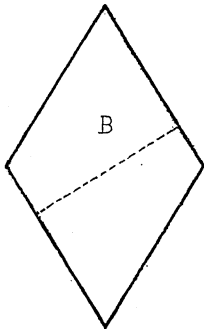
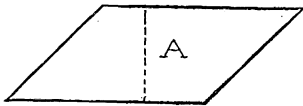
15. Mr. Sharp laid a sidewalk 8 ft. 8 in. wide on two sides of his own lot A. The 8-inch edge-stones cost him 80 cents per running foot. The brick cost \$12 per thousand, and the labor \$58.25. The bricks were $8 \times 4 \times 2$ and laid flat. What did his walk cost? (Make statement.)

16. The owner of lot I paid an average of \$3.50 per rod for fencing. It cost him \$ x if he paid for only half of the division fence.

17. The abutters combined, and concreted the alley at $56\frac{1}{4}$ cents per square yard. What was the total cost? 18. What part of the whole cost should the owner of J pay? 19. What is the assessment of the owner of I?

20. Leland Street is accepted by the city and paved at a cost of \$3 per square yard, the abutters agreeing to pay 25% of the cost of the part adjoining their property. What is the assessment on lot L?

229.—Of Rhomboids.



1. Cut a rhomboid from paper.
2. Divide it along any altitude line.
3. Adjust its parts so as to form a rectangle.

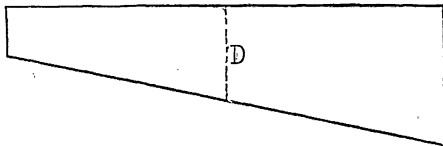
4. Compare the base and altitude of the rectangle with the base and altitude of the rhomboid.

5. How is the area of the rectangle found?

6. How, then, may the area of the equivalent rhomboid be found? Area = base \times altitude.

Find areas of rhomboids with—

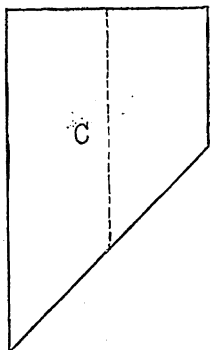
7. Base $12\frac{1}{2}$ ft., altitude $7\frac{1}{2}$ ft.
8. Base 16 rd., altitude 40 ft.
9. B = $2\frac{1}{2}$ yd., A = 18 in.
10. A = $8\frac{3}{4}$ ft., B $5\frac{1}{3}$ in.

230.—Of Trapezoids.*Oral.*

1. Cut out a trapezoid having two right angles.

2. Divide it along a middle line parallel to its parallel sides.

3. Adjust the parts so as to make a rectangle. Notice where the parallel sides of the trapezoid are to be found in the rectangle.



4. Compare the area of the rectangle with the area of the trapezoid.

5. Show that one dimension of the rectangle equals the sum of the parallel sides of the trapezoid.

6. Show that the other dimension of the rectangle equals one-half the altitude (length or width) of the trapezoid.

7. How is the area of the rectangle found?

8. The base and altitude of the rectangle correspond to what lines in the trapezoid?

9. How, then, may the area of the trapezoid be found?

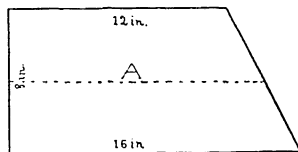
231.—Of Trapezoids.

At Sight.—1. In connection with this trapezoid explain this statement:

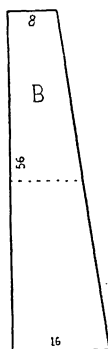
$$(12 + 16) \text{ sq. in.} \times \frac{8}{2} = 112 \text{ sq. in.}$$

2. Show that the average or mean or middle length of the trapezoid is $(12 + 16) \div 2$.

3. Is there any difference in value between $\frac{12+16}{2} \times 8$, $(12+16) \times \frac{8}{2}$, and $\frac{(12+16) \times 8}{2}$?



4. In what three ways may you state the process of finding the area of a trapezoid?



5. What is the altitude of trapezoid B? $\frac{1}{2}$ the sum of its parallel sides, or mean width? Explain

$$\frac{8 + 16}{2} \times 3 = x.$$

6. In trapezoids $\left\{ \begin{array}{l} \text{mean length} \times \text{width, or} \\ \text{mean width} \times \text{length} \end{array} \right\} = \dots$

Written. — Find the area of trapezoids measuring —

7. Parallel sides 16 ft. and 24 ft.; altitude 13 ft.
 8. Parallel sides 25 in. and 24 in.; altitude $4\frac{1}{2}$ in.
 9. Parallel ends 13 in. and 16 in.; length 14 ft.
 10. A trapezoidal board is $7\frac{1}{2}$ in. wide in the middle and $16\frac{1}{2}$ ft. long.

232. — Of Oblique-Angled Trapezoids.

1. Cut an oblique-angled trapezoid along its middle line and place its parts end to end to form a rhomboid.

2. What lines of the trapezoid form base and altitude of the rhomboid?

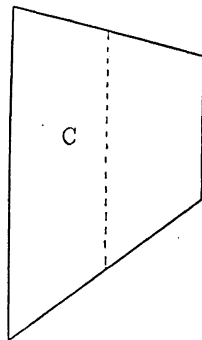
3. How may the area of the rhomboid be found?

4. Show how an oblique-angled trapezoid may be changed to an equivalent rectangle.

5. Find the area of a trapezoid measuring $22\frac{3}{4}$ ft. in altitude, $72\frac{5}{8}$ ft. and $85\frac{3}{4}$ ft. along its parallel sides.

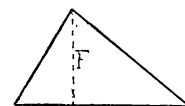
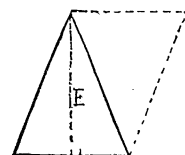
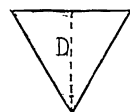
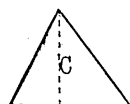
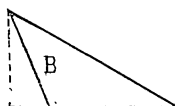
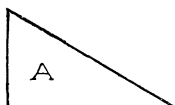
6. A 10-ft. wall is 128 ft. on the ground and 122 ft. along the top. What will it cost to paint both sides at 9¢ a square yard?

7. Draw a rectangle to represent the area painted. (Scale, $\frac{1}{1200}$)



233. — Of Triangles.

Written.



1. Show by measuring with a protractor, or by cutting and laying the angles together, that the sum of the angles of a triangle equals two right angles (180°).

2. How many right angles may a triangle contain? How many obtuse? How many of 60° ?

3. Find the size of the third angle when two angles of a triangle measure 90° , 30° ; 120° , 40° ; 65° , 35° ; $62\frac{1}{2}^\circ$, $87\frac{1}{2}^\circ$.

4. Triangles are named from their largest angles, — *right*, *obtuse*, and *acute*. How many of each kind are represented here?

5. Named from their sides, triangles are *equilateral* (three sides equal), *isosceles* (two sides equal), or *scalene* (three sides unequal). Give examples of each from the drawings.

6. Cut from paper any one of the four parallelograms (pp. 116, 117) and find its area.

7. Bisect it along one of its diagonals. What is the area of the resulting triangle?

8. Compare the base and altitude of the parallelogram with those of the triangle.

9. How is the area of the parallelogram found? The area of the triangle?

Find areas of triangles of the following dimensions: —

10. Base = 40, alt. = 18.

11. Base = 60, alt. = 25.

12. $B = 3$ ft., alt. = 9 in.

13. $B = 4$ rd., alt. = 7 ft.

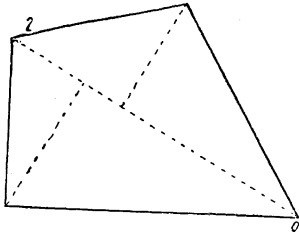
14. $19\frac{1}{2}$ ft., $2\frac{1}{2}$ yd.

15. 38 rd., $22\frac{1}{4}$ ft.

16. Show that $B \times \frac{1}{2}$ alt., $\frac{1}{2} B \times$ alt., or $\frac{1}{2}(B \times \text{alt.}) = \sim$

234. — Of Trapeziums.

Written.



1. Draw or cut out a trapezium.
2. Separate it into two triangles along one of its diagonals, as *lo*.

3. Find the dimensions of each triangle and its area.

4. What will the area of the trapezium be?

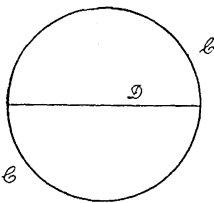
5. The diagonal of a trapezium is 24 inches; the altitudes perpendicular to it are 18 inches and 15 inches, respectively. What is its area?

6. The diagonals of a trapezium cross at right angles. The point of intersection is 50 feet from the upper end of each diagonal. One diagonal is 100 feet long, and the other 150 feet. Find the area.

235. — Of Circles:

**Diameters and
Circumferences.**

Oral and written.



1. Bring to school the results of very carefully measuring the diameter and circumference of several circular objects: plates, rings, covers, wheels, or coins.

2. In each case divide the circumference by the diameter, carrying the division to several decimal places, and compare the quotients.

3. If you have measured and divided accurately, the quotient in each case will be 3.1416-. What does this show?

4. In like manner divide your diameters by the corresponding circumferences. Your quotient should always be 0.31831. What does this show?

5. How many diameters make a circumference?

6. What part of a circumference equals a diameter?

7. The diameter of a circle is 10 ft.; the circumference = 3.1416×10 ft., or π ft.

8. The circumference of a circle is 10 ft.; the diameter = 0.31831 of 10 ft., or x ft.

9. Compare $10 \div 3.1416$ and 0.31831×10 . Which is easier, to divide by 3.1416 or to multiply by 0.31831?

10. 3.1416 = the ratio of the circumference to the diameter. It is represented by the Greek letter π (English p). D = diameter; C = circumference; R = radius. Interpret the following:—

$$C = D \times \pi; D = C \div \pi; \frac{1}{\pi} = 0.31831; D = C \times \frac{1}{\pi}.$$

Find the diameter or circumference or radius. Forecast the result.

11. $D = 20$ ft.; $C = x$.

14. $C = 4$ ft. 8 in.; $D = x$.

12. $C = 205$ ft.; $D = x$.

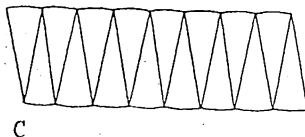
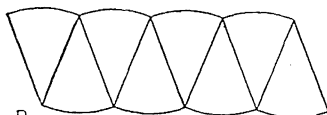
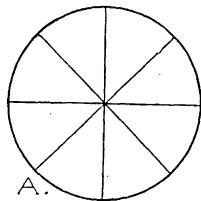
15. $D = 16\frac{1}{4}$ in.; $C = x$.

13. $R = 90$ ft.; $C = x$.

16. $C = 2$ yd. $1\frac{1}{2}$ ft.; $R = x$.

236.—Circles
changed to Equi-
valent Rectangles.

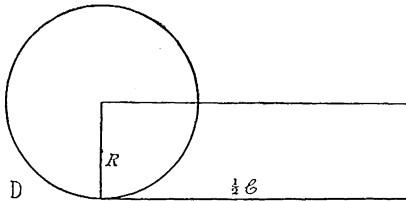
With Objects.—1. Cut a circle from paper. Bisect it and cut each half into fourths. 2. Arrange the eight sectors as in B, forming a figure somewhat like a rhomboid.



3. Cut another equal circle, divide it into sixteen equal sectors, and adjust them to form a figure still more like a rhomboid.

4. Of the two rhomboids, which is more nearly a rectangle? What part of the circle is its base? Its altitude?

5. Imagine a circle cut into a thousand equal sectors, arranged as before. Would the figure formed be a rhomboid or a rectangle? 6. Compare its base with a straight line. What can be done to make its base more nearly straight? 7. The circle would then be changed to a rectangle



having a base equal to $\frac{1}{2}C$ and an altitude equal to R . How would the area of this rectangle be found?

8. Explain the figure D.

9. Interpret the equation:

$$\text{Area of circle} = R \times \frac{C}{2}.$$

10. $R = 4$; $C = \text{about } 25$; $A = x$.

Find the area of circles:—

11. Radius, 6 ft.; Cir. x

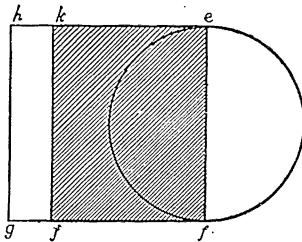
13. Cir. 100 ft.; Diameter, x

12. Diameter, 10 ft.; Cir. x

14. Cir. 50 ft.; Radius, x

237.— Diameter of Circle Given, to find the Area.

Oral.—1. $efgh$ is the square of the diameter of the circle. A circle is what part of the square of its diameter?



2. Explain the following (as shown in §§ 235-6):—

$$(a) A = D \times \frac{C}{4}; \text{ but } C = D \times 3.1416;$$

hence,

$$(b) A = D \times \frac{D \times 3.1416}{4}; \text{ cancelling,}$$

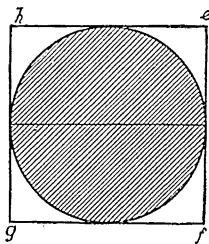
we have

$$(c) A = D \times D \times 0.7854; \text{ but } D \times D = D^2; \text{ hence}$$

$$(d) A = D^2 \times 0.7854; \text{ that is, a circle is } 0.7854 \text{ of the square of its diameter.}$$

3. In the figures at the left the shaded portion is the area of the circle. What decimal part of the square is it?

What decimal part of the square is not shaded?

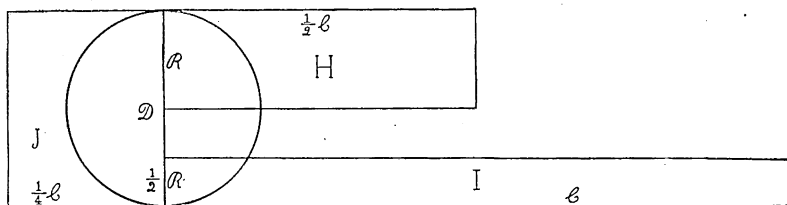


4. A circle is what part of its circumscribed square?
5. How is the area of a circle found from its diameter?

Written. — 6. What is the area of a 5-foot circle?

7. What part of 3.1416 is 0.7854?
8. Find the area of a 12-inch circle.
9. What is the cost of a circular piece of aluminum at 30 cents per square inch, the radius being 3 inches?
10. What is the area of a circular pond 200 feet in diameter?

238. — The Area of Circles. 1. In the following figure we have three rectangles each equivalent to the circle.



Which one is twice as long and half as wide as H? Which is half as long and twice as wide as H?

2. The length of each is what part of the circumference?
3. The width of each is what part of the diameter?
4. The diameter of a circle is 10 feet. What is its area? Explain each of these three methods of finding it.

$$[\text{Rect. H.}] \frac{10}{2} \times \left(\frac{10 \times \frac{0.7854}{3.1416}}{2} \right) = 78.54. \quad \text{Formula: } R \times \frac{C}{2} = A.$$

$$[\text{Rect. I.}] \frac{10}{4} \times (10 \times \frac{0.7854}{3.1416}) = 78.54. \quad \text{Formula: } \frac{R}{2} \times C = A.$$

$$[\text{Rect. J.}] 10 \times \left(\frac{10 \times \frac{0.7854}{3.1416}}{4} \right) = 78.54. \quad \text{Formula: } D \times \frac{C}{4} = A.$$

5. The circumference of a circle is 12 feet. Explain this method of finding its area:—

$$C = 12 \text{ feet.}$$

$$D = 0.31831 \text{ of } 12 \text{ feet.}$$

$$(0.31831 \times 12) \times \frac{12}{4} = 11.45916.$$

Written.—Find the area of circles when—

6. Diameter = 40

8. Radius = 24

10. $C = 200$

7. Circumference = 80

9. Diameter = 36

11. $D = 50$

239.—Oral Review. 1. What objects before you are nearest in length to a yard? to a foot? to a rod?

2. How many degrees measure a \angle ? Can you find as you look about any except right angles?

3. After going $\frac{1}{4}$ round the world, x° complete the circuit.

4. If the angles of a $\triangle = 2 \angle$'s, each angle in an equilateral \triangle measures x° .

5. How may the area of a triangular park be found?

6. How much of an 8-in. square is not covered by a 7-in. square? by a 6-in. one?

7. A rhombus containing 3 sq. yd. contains how many sq. ft.?

8. Which measures more, a rhombus or a square with the same perimeter?

9. The top of a round table has what part of the area of that of a square table of the same diameter?

10. A triangle = $\frac{1}{2}$ the area of ~.

240.—Drawing and Measuring of Figures. 1. Draw a 2-in. circle; then the largest possible square inside, and one of its diagonals.

2. Calling the diagonal the base of the triangles, what is the area of each?

3. The area of the largest square drawn in a 3-in. circle = x .

4. Draw a 1-in. square and a 1-in. rhombus. Are their bases the same? Their altitudes? Their areas?

5. Using 1-in. lines, make a rhombus with an altitude of $\frac{1}{2}$ in. Its area will be ~~~.

6. Using 1-in. lines, make a figure of as small area as you can. What is its shape?

7. As a ring is flattened does its capacity change? Does the length of its perimeter?

8. Draw a trapezoid; the horizontal and vertical lines may be 1, $1\frac{1}{2}$, and 2 inches. Divide it into a rectangle and a triangle. Find the areas of each, and add; then find the area of the trapezoid in the usual way.

9. Draw a rectangle and a second figure with the same length of line, but no L's. What is its shape? How do the two differ in area?

10. Explain what dimensions you need to know in order to find the area of a trapezium that you have drawn.

241. — Problems in 1. A button is 4.7124 in. round. How long
Measuring Circles. a button-hole is required?

Written.

2. Find the circumference of the base of a lamp chimney that is $2\frac{3}{8}$ in. across.

3. A circus ring is $414\frac{7}{10}$ ft. round. Find the distance to the centre in rods.

4. A hogshead is a little over $12\frac{1}{2}$ ft. round the middle. Will it go through a doorway that is 3 ft. 10 in.?

5. If a mountain is 10 m. round, what distance might be saved by tunnelling?

6. A pie is cut accurately into 6 equal pieces. Which is longer, the curved edge or the straight one?

7. The hubs of two wheels are alike, but the spokes of one are 3 in. longer. How much greater is its circumference?

8. If a barrel is 18 in. over the chine, how much strap iron will be required to make 100 end-hoops with 3-in. laps? Make a statement.

9. In a lawn 100 ft. square the circular basin of a fountain is 40 ft. from each side. Draw a figure, and find the area of the greensward.

10. When you know the area of a circle, how can you find the radius?

242. — Of Solids.

[See p. 9.]

1. Lines have one dimension; viz. ~~~.

2. Surfaces have two dimensions; viz. ~~~ and ~~~.

3. Solids occupy space and have three dimensions, viz. ~~~, ~~~, and ~~~.

4. Mention the three common solid measures in the order of their size. 5. Compare each one with the one next larger or smaller.

6. 10 cu. ft. = x cu. in.

8. 10 cords contain x cu. ft.

7. 20 cu. yd. = x cu. ft.

9. 720 cu. ft. = x cu. yd.

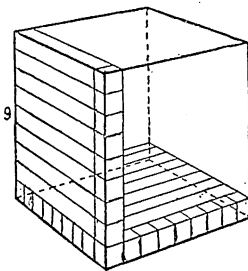
10. State the method of finding the number of cubic feet in 20,000 cubic inches.

243. — Of Cubes.

Oral. — 1. What is a rectangle? 2. A solid bounded by six rectangles is a *rectangular solid*.

3. A cube is a solid with six square faces. How many corners, edges, angles, has a cube?

4. Is a common brick a cube? Is it a rectangular solid? What is an equilateral rectangular solid?



5. Describe an inch-cube, or a cubic inch; a cubic foot; a cubic yard; a 2-foot cube.

6. What is a 9-inch cube? How many cubic inches in a 9-inch cube?

[See the figure.] Along one edge of a cube there is a row of x cu. in.; 9 such rows make a tier of $9 \times x$ cu. in. or y cu. in.; 9 such tiers contain $9 \times y$ cu. in. or 729 cu. in.

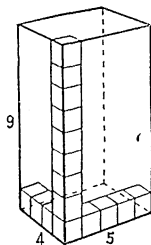
Statement: $9 \times 9 \times 9$ cu. in. = z cu. in.

Written. — In a similar or better way show the contents of —

7. A 6-in. cube 9. A 5-foot cube 11. A 10-yard cube
 8. An 8-in. cube 10. A 12-foot cube 12. A 20-in. cube
 13. 5^3 ? 7^3 ? 9^3 ? 12^3 ? 6^3 ? 10^3 ? 11^3 ? 20^3 ?
 14. $\sqrt[3]{64}$? $\sqrt[3]{216}$? $\sqrt[3]{1728}$? $\sqrt[3]{512}$? $\sqrt[3]{1331}$? $\sqrt[3]{729}$? $\sqrt[3]{27000}$?

244. — Rectangular Prisms.

1. Prisms are named from their bases, — square, rectangular, triangular, hexagonal, etc. Name some familiar objects that are rectangular prisms; that are square prisms. Which kind includes the other?



2. Find the cubical contents of a rectangular prism whose dimensions are 9 in., 4 in., and 5 in.

Notice in the figure the number of cubic inches in a row, the number of rows in a tier or layer, and the number of tiers in the prism; and then explain the statement: $9 \times 4 \times 5$ cu. in. = x cu. in.

3. How many inch cubes may be put into a box 10 in. long, 8 in. wide, and 5 in. deep?

4. A trunk measures 3 ft., 20 in., and 18 in. Find its cubical contents. Why not multiply 3 by 20 by 18 instead of 36 by 20 by 18?

Find the contents of rectangular prisms of these dimensions: —

Length.	Width.	Height.	Length.	Breadth.	Depth.
5. 16 ft.	10 ft.	8 ft.	8. $42\frac{1}{2}$ ft.	20 ft.	$13\frac{1}{3}$ ft.
6. 1 yd.	2 ft.	9 in.	9. $12\frac{1}{2}$ yd.	10 ft.	16 in.
7. 24 ft.	16 ft.	4 yd.	10. $20\frac{1}{2}$ ft.	$17\frac{1}{3}$ ft.	6 in.

245. — Of Cord-Wood.

Oral and written.

1. Wood for fuel, sold by the cord, is usually in sticks of what length? 2. In what form are they piled to make a cord? 3. Give the dimensions of a cord; a half-cord; $\frac{1}{8}$ of a cord or a cord foot.

4. What kind of solid does a half-cord resemble? 5. Explain:
 $8 \times 4 \times 4$ cu. ft. = x cu. ft. as applied to cord-wood.

6. A pile of 4-foot wood, 4 ft. high and 8 ft. long, contains a cord.
 If 16 ft. long? 24 ft.? 32 ft.? 96 ft.?

7. A pile of 4-ft. wood of the usual height must be how long to
 contain 10 cords? 12 cords? 25 cords?

8. How many cords in a pile of 4-ft. wood, 4 ft. high and 18 ft.
 long? Explain the following statement, and show what may be
 cancelled:—

$$\frac{4 \times 4 \times 18 \text{ cu. ft.}}{128 \text{ cu. ft.}} = x.$$

9. Bought a pile of 4-foot wood 30 ft. long and 8 ft. high at \$6
 per cord.

$$\frac{4 \times 8 \times 30}{128} \times \$6 = x.$$

In the statement what represents the number of cubic feet? The
 number of cords? The cost of all?

Find the value of piles of wood, as follows:—

Length.	Width.	Height.	Price.	Length.	Width.	Height.	Price.
10. 24 ft.	4 ft.	6 ft.	\$4.	13. 24 ft.	4 ft.	$7\frac{1}{2}$ ft.	\$3.50
11. 40 ft.	8 ft.	8 ft.	5.	14. 20 ft.	3 ft.	$12\frac{1}{2}$ ft.	5.00
12. 60 ft.	10 ft.	4 ft.	6.	15. $16\frac{1}{2}$ ft.	44 in.	$2\frac{2}{3}$ ft.	4.25

246.—Of Lumber.

Written.

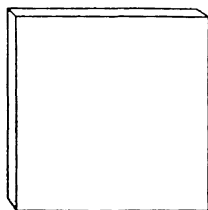
1. Timber sawed for building purposes is
 lumber. What forms can you mention besides
 boards, planks, joists, and beams?

2. In measuring lumber no attention is paid to the thickness
 unless it exceeds an inch. A board 12 ft. long and 12 in. wide
 and 1 inch or less in thickness contains 12 sq. ft.; if 10 in. wide,
 it contains $\frac{10}{12}$ or $\frac{5}{6}$ of 12 sq. ft. or = x sq. ft.

3. A board 15 ft. long, 8 in. wide, and $\frac{3}{4}$ in. thick contains how
 many square feet?

NOTE.—If 15 ft. long and one foot wide, it would contain x sq. ft.; being $\frac{3}{4}$ of a foot wide, it will
 contain, etc.

4. 10 16-ft. boards averaging 9 in. in width contain x square feet.
Explain: $10 \times \frac{3}{4}$ of 16 sq. ft.



5. A board 1 inch thick and a foot square is a *board foot*. x of them piled together would make a cubic foot.

6. A board 10 ft. long, 12 in. wide, and 1 inch thick contains 10 bd. ft. If 2 in. thick, it would contain $2 \times$ as many bd. ft. or x . If $1\frac{1}{2}$ in. thick? If $1\frac{1}{4}$ in. thick? If $2\frac{1}{2}$ in. thick? If a ft. thick?

7. Find the contents of a 3-in. plank 15 ft. long and 10 in. wide.
Explain: $3 \times 15 \times \frac{5}{6} = x$.

8. 12 joists, 16 ft. long and 4 in. square, contain x board feet.

Find the contents in board feet of lumber measuring as follows:—

9. 6 boards, 16 ft. long, $1\frac{1}{4}$ thick; width in inches: 8, 10, 12, 13, 14, 9.

10. Fifteen 3×5 joists, 18 ft. long.

11. A stick of timber 18 ft. long and 12 in. square.

247.—To find the Surface of a Cube.

Written.

1. How many faces has a rectangular prism? 2. What name is given to a rectangular prism when all its faces are equal?

3. Find the entire surface of a 5-inch cube.
Explain the statement: $5^2 \times 6 = x$.

4. The entire surface of a cube is 150 sq. in. How long is the cube? Explain the statement: $\sqrt{\frac{150 \text{ sq. in.}}{6}} = x$.

Find the entire surface of—

How long a cube has—

5. A 9-in. cube

8. An entire surface of 384 sq. in.?

6. A cube 10 in. long

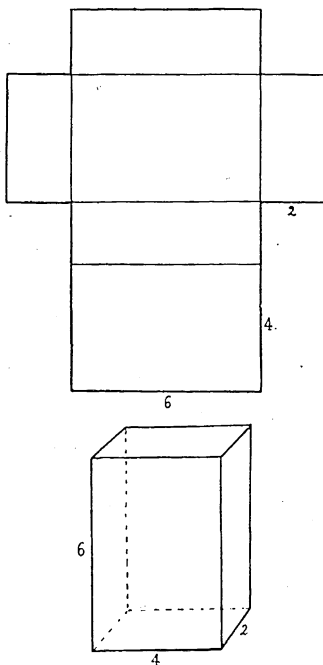
9. An entire surface of 600 sq. ft.?

7. A 16-in. cube

10. An entire surface of 294 sq. in.?

248.—Of Rectangular Prisms.

Written.



1. Compare with each other the ends of a square prism. 2. Compare its four sides.
3. Find the entire surface of a square prism 8 in. long and 3 in. wide. Explain the equation:—

$$2 \times (3 \times 3) + 4 \times (8 \times 3) = x.$$

4. Compare the opposite faces of any rectangular prism. 5. Find the entire surface of a prism measuring 6 by 4 by 2. Explain the statement:—

$$2 \times (2 \times 4) + 6 \times (4 + 4 + 2 + 2) = x.$$

6. Explain the figures at the left.

Find the entire surface of prisms—

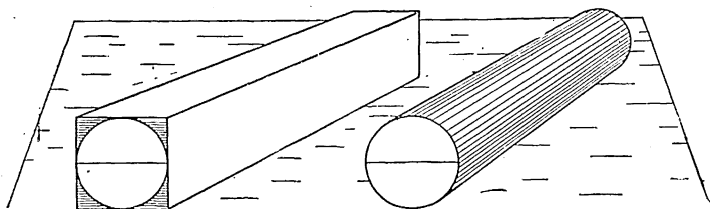
7. 10 in. long, 6 in. wide, 4 in. thick.
8. 12 ft. long, 9 ft. wide, 6 ft. high.
9. 20 ft. long, 14 ft. wide, 10 ft. high.
10. 16 by 18 by 4; 20 by 1 by 1.
11. 12 by 9 by 8; $2\frac{3}{8}$ by $\frac{1}{2}$ by 16.
12. 12 by 12 by 6; $\frac{2}{3}$ by $\frac{3}{4}$ by $7\frac{1}{2}$.

249.—To Find the Contents of a Cylinder.

Written.

1. Mention several common objects that are perfect cylinders; that is, of uniform diameter, and with ends (or bases) that are equal parallel circles.
2. How might a cylinder be turned from a square prism of the same diameter? 3. Recalling the formula for the area of a circle, 0.7854 of D^2 (p. 129), what part of the prism would be shavings, and what part cylinder?

4. Give the contents of the largest cylinder that may be turned out of a square prism 25 in. long, 4 in. wide.



Explain the statement: — $4^2 \times 25 = x =$ contents of \sim .
 0.7854 of $x = y =$ contents of \sim .

5. A circle is $\frac{x}{10000}$ of a square of equal diameter.

A cylinder is $\frac{x}{10000}$ of a square prism of equal diameter and length.

6. Find the contents of a cylinder 10 ft. long and 2 ft. in diameter.

Explain the statement: — 0.7854 of $(2^2 \times 10) = x$ cu. ft.

7. A cylindrical pail 6 inches in diameter inside and 12 inches deep contains x cubic inches. Forecast the result, observing that 0.7854 is a little more than $\frac{3}{4}$; thus, $\frac{3}{4}$ of $6^2 \times 12 = 324 +$.

8. A cylindrical tank is 10 ft. deep and 8 ft. in diameter.

9. A well is 32 ft. deep and 5 ft. in diameter.

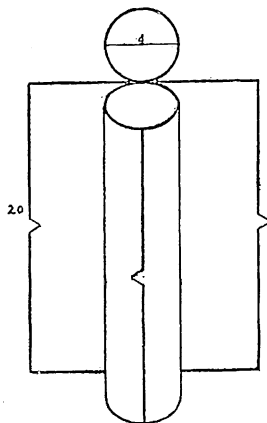
10. A gallon contains 231 cu. in. To hold a gallon, a pail measuring 33 sq. in. on the bottom must be x inches deep.

**250.—To Find
the Surface
of a Cylinder.**

Written.

1. In form, the ends of a cylinder are equal \sim s. The rest of the surface is the *convex* surface.

2. Suppose the diameter of a cylinder to be 4 inches; its circumference = x , or $3.1416 \times D$ (§ 235).



3. The circumference of a cylinder is 8 inches; its diameter is x , or $0.31831 \times C$.

4. A cylinder is 20 inches long and 4 inches in diameter. Find the area of its ends. Explain the statement:—

$$(0.7854 \text{ of } 4^2) \times 2 = x.$$

5. Roll an oblong paper to form a cylinder. Give the length and circumference of the cylinder thus made.

6. Unroll the paper and give the dimensions of a rectangle equivalent to the convex surface of the cylinder. Explain the diagram at the left.

7. The convex surface of a cylinder $= C \times L$. Explain. [L = length.]

8. A cylinder is 25 inches long, 4 inches in diameter. Its convex surface is x . Explain: $(3.1416 \times 4) \times 25 = x$.

9. A cylinder is 20 inches long, 5 inches in diameter. Entire surface?

10. If we allow 17 square inches for seams and the flange of the cover, how many square inches of tin are actually used in making a coffee can 6 inches in diameter and 8 inches deep? Show why there must be an allowance for waste.

251. Oral Review.

1. A bookcase has 10 feet of space right and left and 6 feet up and down. It is 10 inches or $\frac{x}{y}$ feet deep, and the cubical contents $= z$.

2. The sides that make the right angle of a triangle are each 10 feet. The area is x .

3. Give approximately the area of the surface of a lead pencil $\frac{1}{4}$ inch in diameter and 8 inches long.

4. An old tree is 22 feet round; how far is it through?

5. Which takes more room, a cord of wood or a 5-foot cube?
6. $\frac{1}{2}$ a circle = a rectangle having the radius for one side and π for the other.
7. A 20-foot log averages 1 square foot in the cross section. The cubic contents are x .
8. How many cubic yards of earth will a bin hold that is 3 ft. \times 10 ft. \times 9 ft.?
9. About how many cubic yards does your schoolroom contain?
10. What is the approximate capacity of a well 40 feet in depth and 7 feet in area of opening?

**252.—Review
Problems.**

Written.

1. Give the dimensions of three dissimilar rectangles each containing 36 square inches. Give the perimeter of each.
2. A square rod contains x square feet. The wall of a rectangular cellar encloses 2 square rods. One of its dimensions is 20 feet, the other x .
3. The boards of an old floor are 18, 14, 12, 10, and 6 inches wide. If used in equal proportion, what is the average width? To cover 2 squares, how many running feet would be required?
4. A panelled ceiling contains 72 squares $1\frac{1}{2}$ feet wide. It is 12 feet on one side, x on the other.
5. Divide the area of a square on the diameter of a circle by the area of the circle. The quotient is x .
6. Explain the formula $C = \pi \times 2 R$.
7. A board 6 feet by 6 inches contains 324 cubic inches actual measure. How thick is it?
8. How many oranges 3 inches in diameter will go into a box $2 \times 1 \times 1$ feet if packed in equal rows?
9. On a scale of 1 inch to 1 mile, represent a tract of land 2 miles by 3 miles. Divide into square miles by dotted lines. Draw a mile square in the middle, and divide the rest into 4 equal tracts.
10. Each of the four contains x acres.

253. — To find a
Missing Factor.

Written.

1. x times 6 = 144; $18 \times x = 198$.
2. Multiplicand = 25; product = 400.
How is the multiplier found?
3. $186 \div x = 31$.
4. Dividend and quotient being given, how is the divisor found?
5. When product and multiplier are given, how is the multiplicand found?
6. What is the area of a rectangle 12 feet long and $6\frac{1}{2}$ feet wide?
7. A rectangle containing 108 square inches is 9 inches wide.
How long is it? ($9 \times x$ sq. in. = 108 sq. in.)
8. A lot of land is 200 feet long and contains 24,000 square feet.
How wide is it?
9. A sidewalk 50 feet long requires 50 square yards of concrete.
How wide is the walk?
10. One-half an acre of land is taken for a new street 40 feet wide. How long is the street?
11. The area of a triangle is 325 square inches; its base is 25 inches. What is its altitude?
12. The altitude of an isosceles triangle is 14 feet; its area is 126 square feet. What is its base?
13. At 30¢ a board foot a mahogany board one inch thick and 12 feet long cost \$2.70. How wide was it?
14. The area of a rhomboidal field is 12 acres. Its length being 20 rods, what is its altitude?
15. A square contains 400 square inches. How long is it?
16. The perimeter of a square is 1000 feet. Its area?
17. The radius of a circle is 5 feet; its area is 78.54 square feet.
What is the circumference? ($\frac{1}{2} C \times R = A$)
18. What is the area of a circle 100 feet in diameter?
($D^2 \times 0.7854 = A$.)
19. What is the diameter of a circle containing 7854 square feet?
($x^2 \times 0.7854 = 7854$ sq. ft.)
20. A circle contains 28.2744 square inches. What is its diameter?

254.—Contents and
Two Dimensions of a
Solid given, to find the
Third Dimension.

1. $13 \times 7 \times \$x = \910 .

2. I hired 15 men at \$2.50 per day each. At the completion of the work I paid them in all \$150. How many days did they work?

3. A box on my table holds 432 cubic inches. It covers 72 square inches of the surface of the table. How high is the box?

4. The area of the floor of your schoolroom is 900 square feet. The room contains 10,800 cubic feet. How far is the ceiling from the floor?

5. A packing box is 48 inches long and 30 inches wide. How deep must it be to hold 10 cubic feet?

Statement: $\frac{10 \times 1728}{48 \times 30} = \text{depth.}$

Explain the statement, and show a short solution.

6. A closet 8 feet high and 27 inches deep will contain 72 cubic feet. How wide is it?

7. A pile of 198 cords of 4-foot wood covers 16 square rods. How long is it? How high is it?

Explain the statements:—

A. $\frac{16 \times 272\frac{1}{4}}{4} = x.$

B. $\frac{198 \times 128}{16 \times 272\frac{1}{4}} = y.$

8. A cylindrical oil-tank holds 10 gallons. Standing on the floor it covers 77 square inches. How high must it be?

9. A bookcase holding 32 cubic feet covers a wall space of 24 sq. ft. How far must it project into the room?

10. I have room in my stable for a grain bin 8 ft. by 4 ft. How deep shall I make it to have it hold 72 bushels?

11. A grindstone 4 ft. in diameter contains 6.2832 cu. ft. How thick is it? Explain the statement: $6.2832 \div (4^2 \times 0.7854) = x.$

12. In digging a trench 3 ft. wide and $4\frac{1}{2}$ ft. deep 330 cu. yds. of earth were removed. How long was the trench?

255.—Miscellaneous Problems.

Written.

1. I buy a corner lot 120 ft. by 50 ft. and use the earth obtained by digging a cellar 60 ft. by 30 ft. by 10 ft. to raise the grade how many feet?
2. A circular standpipe 75 ft. high is 25 ft. in diameter. When $\frac{2}{3}$ full, how many gallons of water does it contain, reckoning $7\frac{1}{2}$ gallons to a cubic foot?
3. A speculator buys a field 600 ft. long and 500 ft. wide for \$2500. He runs a 40-ft. street through the centre in each direction at an expense of \$425 for labor. He sells the land at 20 cents a square foot. How much does he make or lose?
4. At \$3.75 per square yard what will be the cost of paving $\frac{3}{4}$ of a mile of street 81 ft. wide?
5. A reservoir supplies a town with 4,573,800 gallons of water daily. If its surface area is 7 acres, how much will the water be lowered in a week, provided one-half as much runs in as runs out? Call 1 cu. ft. equal to $7\frac{1}{2}$ gal.
6. In a house of 36 windows a glazier finishes drawing the sash of 4 windows in 3 h., spending twice as long on the inside as on the outside. He can do the outside of them all in 1 day. How long is a day's work?
7. A water glass has two bands round it, each containing ten figures. It takes three seconds to cut each figure. What will it cost to decorate $2\frac{1}{2}$ gross at four dollars an 8-hour day?
8. Find 80% of as many articles as can be bought for \$200 at $16\frac{2}{3}$ ¢ each. If sold at a profit of 100%, how many would be sold for \$2?
9. What will settle a debt of \$127.50 that has been drawing 9% interest for 248 days?
10. $18\frac{3}{4}\%$ of $\frac{12\frac{1}{2} \times \$36 \times 16\frac{2}{3} \times 75}{8\frac{1}{3} \times 37\frac{1}{2} \times 37\frac{1}{2}} = x$.
11. From a lot of land 40 rods square I sold 40 square rods. What is the remainder worth at \$230 an acre?

256.—EXAMPLES FOR PRACTICE.

For dictation.

1. Give the perimeter of a 6-inch square.
2. Of $\frac{1}{2}$ a 6-inch square.
3. What is the ratio of the diameter of a circle to its circumference?
4. 8 feet 3 inches is what part of a rod?
5. How many square feet in $\frac{1}{2}$ a square rod?
6. Two angles of a triangle measure 30° each. What does the third angle measure? Of what kind is the triangle?
7. How many cubic inches in a cube $\frac{2}{3}$ of a foot long?
8. Find the entire surface of a 6-inch cube.
9. Of $\frac{1}{2}$ a 6-inch cube.
10. 4 cord feet cost \$5. What will 3 cords cost?
11. Your schoolroom is 12 feet high and contains 10,800 cubic feet. Length of floor?
12. Contents of a 4-inch cube?
13. Of one twice as long?
14. Rods in $3\frac{1}{4}$ miles?
15. Length of a square containing 900 square feet?

At sight.

1. 0.0001 of $24,765 = x$.
2. $\frac{2}{3}$ of 1 rod $= x$ feet.
3. Days from Oct. 17 to Dec. 25, inclusive.
4. Average temperature for a week, if the thermometer read: 7° , -4° , 10° , -6° , -18° , 12° , 20° ?
5. Area of circle 100 feet in diameter?
6. Cost of 1 pound if $\frac{3}{4}$ pound cost \$ $\frac{2}{3}$?
7. What is the exact middle of February, 1900?
8. Area of rhomboid when base and altitude are 24 inches?
9. Area of square $16\frac{1}{2}$ feet long?
10. Number of board feet in a board 12 feet long, 8 inches wide at one end and 10 inches at the other?
11. $\sqrt{144} - \sqrt{81} = x$.
12. $\frac{\sqrt{25} \times 6\frac{1}{2} \times 38}{19 \times 13 \times \sqrt{25}} = x$.
13. 9 yards @ $13\frac{1}{3}\text{¢}$ cost $= x$.
14. Area of triangle $6\frac{1}{4}$ by 3.
15. How many acres is $6\frac{1}{4}\%$ of a square mile?

257.—Practical Exercises in Mensuration, etc.

Written.

1. $6\frac{1}{4}$, 8, $4\frac{1}{2}$, are the dimensions of my coal bin. Reckoning 90 pounds to a cubic foot, what will a bin full cost @ \$5?
2. Quincy granite weighs $165\frac{3}{4}$ pounds to the cubic foot. What is the weight of 6 pieces of curbing 8 inches thick, 2 feet wide, and half a rod long?
3. Find the cost of carpeting a 9-foot hallway 22 feet long with three-quarter carpeting at \$.87 $\frac{1}{2}$. Cut no strip, and allow $1\frac{1}{2}$ feet per strip for matching.
4. How many tons of 15-inch ice may be cut to the acre, a cubic foot weighing $57\frac{1}{2}$ pounds? Apply your knowledge of cancellation.
5. What is the capacity, in 42-gallon barrels, of a cylindrical oil-tank $3\frac{1}{2}$ feet in diameter, 22 feet long? Make a statement and cancel.
6. What is the area of a sector of 120° , its radius being 24 inches?
7. A ball ground 375 feet long and 280 wide is enclosed by a tight board fence 8 feet high. What will the boards cost at \$24 per M.? Add 10% for waste.
8. Bought 12,000 long tons of coal at \$4.00 and sold it at the same price per short ton. What did I gain?
9. What will it cost to polish the visible portions of a shaft of red granite 6 feet by 2 feet by 22 inches at $6\frac{2}{3}$ ¢ per square inch?
10. Draw a 6-inch square, a rectangle 9 inches by 4 inches, and one 3 inches by 12 inches. Compare areas and perimeters. What inference do you draw?

258.—Examples for Practice.
Written.

1. What decimal of a square prism becomes shavings when the largest possible cylinder is turned from it?
2. What number subtracted 88 times from 80,005 will leave 13 as a remainder?
3. A railroad company fences 13 miles of its road at $73\frac{2}{3}$ cents a rod.
4. How many square feet of zinc will line a cubical cistern 5 ft. 10 in. deep?

5. The time of the operatives in a mill was increased from 52 to 58 hours, and their wages increased $\frac{1}{10}$. Was this a gain or a loss to them?

6. Bread sells for 10 cents with flour at \$ 5.00. Flour goes up to \$ 6.50. What should bread sell for on this basis?

7. In a city of 7200 school children there are 2720 cases of tardiness in a year during which there are 400 sessions of the schools. The average attendance is 6800. How often is each child tardy?

8. Find the cost of six 8×10 sills 18 ft. long at \$ 24.75 per M.

9. In a school containing 567 white children every tenth child is colored. How many children in the school?

10. A schoolroom measuring 32 ft. \times 28 $\frac{1}{2}$ ft. \times 13 ft. seats 49 pupils. Each one needs 1800 cu. ft. of fresh air an hour. The room full would last the class x minutes.

259.—Problems for 1. How many sheets of paper folded into
Analysis. 16 leaves will make a 400-page book?

Oral.

2. At \$ 10.50 a week what is the amount of your board bill from noon of Aug. 21 to noon of Sept. 25?

3. What is a £100 Bank of England note worth in New York at its face value?

4. Cost of 8 $\frac{2}{3}$ yds. at \$ 0.37 $\frac{1}{2}$ per yard.

5. Compare a 5-inch square with one half as long.

6. A circle is 10 feet in diameter. How long is an arc of 36° in its circumference?

7. A cubic foot of distilled water at a temperature of 38° F. weighs 1000 ounces. How will you find the weight of a gallon?

8. $\frac{3}{4}$ of an acre produces a crop that sells for \$ 360. How much is this for every 12 sq. rds.?

9. I pay \$ 1.80 for having my cord-wood sawed into 3 sticks. What ought I pay when it is sawed into 4 sticks?

10. A trapezoid is twice as wide at one end as at the other. It measures 12 in. in the middle, x in. at one end, and y in. at the other.

260. DEFINITIONS.

[FOR REFERENCE.]

Acute Angle. An angle sharper than a right angle.

Altitude. Height. Measured by a straight line perpendicular to the line of the base, and extending from it to the highest point.

Angle. The divergence from a common point of two lines having different directions.

Arc. Any portion of a circumference.

Area. The size or total contents of a surface.

Base. The line or surface on which a figure is supposed to stand.

Chord. A straight line joining the ends of an arc.

Circle. A plane surface bounded by a curve every point of which is equally distant from a point within called the centre.

Circumference. The perimeter or boundary of a circle.

Convex Surface. The surface of a solid excluding that of its bases.

Cube. A solid with six square faces.

Curvilinear surfaces are those bounded by curves.

Cylinder. A solid having for its bases equal parallel circles, and having a uniform diameter.

Degree. A 360th part of a circumference.

Diagonal of a Polygon. A straight line connecting the vertices of two angles not adjacent.

Diameter. A line measuring the shortest distance across a circle or square through the centre.

Dimensions. Measurements needed to find contents.

Equiangular. Having equal angles.

Equilateral. Having equal sides.

Figure. A surface bounded by lines or a space bounded by surfaces.

Horizontal. Parallel to the plane of the horizon.

Hypotenuse. The longest side of a right triangle.

Inclined. Neither horizontal nor vertical.

Isosceles triangles have two sides equal.

Line. The limit of a surface. The path of a point.

Oblique lines are neither horizontal nor vertical. Oblique angles are greater or less than right angles.

Oblong. A rectangle whose length exceeds its breadth.

Obtuse angles are greater than right angles.

Parallel. Extending in the same direction, and in all parts equally distant.

Parallelogram. A quadrilateral whose opposite sides are parallel.

Perimeter. The circumference of a surface or the sum of its bounding lines.

Perpendicular. At right angles to another line or surface.

Plane. A plane surface is a flat or level surface.

Point. That which has position, but no length, breadth, or thickness. The end of a line.

Polygon. A plane surface having straight sides, commonly more than four.

Prism. A solid whose sides are parallelograms, and whose bases are equal parallel polygons. Prisms are named from the form of their bases, as *square* prisms, *rectangular* prisms, *triangular* prisms, *hexagonal* prisms, etc.

Quadrant. A fourth part of a circle or of a circumference.

Quadrilateral. A plane surface having four straight sides.

Radius. A straight line extending from centre to circumference of a circle.

Rectangle. A parallelogram having four right angles.

Rectilinear. Bounded by straight lines.

Rhomboid. A parallelogram with oblique angles.

Rhombus. An equilateral rhomboid.

Right Angle. An angle of 90° .

Scalene triangles have their sides unequal.

Secant. A straight line that cuts a curve at two points.

Sector. The part of a circle bounded by an arc and two radii.

Segment. The part of a circle between an arc and its chord.

Semicircle. Half of a circle.

Sextant. One-sixth of a circle.

Solid. A form having three dimensions, — length, breadth, and thickness.

Square. An equilateral rectangle. A plane surface with four equal sides and angles.

Surface. That which has only two dimensions, — length and breadth. The outside of a solid.

Tangent. A line touching a curve at a single point without crossing.

Trapezium. A quadrilateral no two of whose sides are parallel.

Trapezoid. A quadrilateral only two of whose sides are parallel.

Triangle. A plane surface having three straight sides. A *right triangle* has one right angle; an *obtuse triangle* has one obtuse angle; an *acute triangle* has three acute angles.

Vertical. Relating to the vertex.

Vertical lines point towards the zenith and the earth's centre.

Vertex. The point in an angle where the sides meet.

π . A Greek letter pronounced like *p*. It stands for 3.1416—, the ratio of the circumference of a circle to its diameter.

$^{\circ}$, $'$, $''$. Sometimes used for feet, inches, and lines or twelfths of an inch. Compare their use on p. 9.

Computing by Hundredths.

261. — Percentage: One hundred is the common standard of comparison. The merchant gains 10 on every 100, or 10 per cent. The rate of interest is 6%, or 6 on every 100 used. The commission paid is 2 on every 100 spent, or 2 per cent. 30% of the liquor is alcohol, that is 30 parts in every 100.

1. What is meant by saying:

12% of the grain spoiled?

33 $\frac{1}{3}$ per cent of the month was stormy?

14% of the scholars were absent?

Give other illustrations.

2. What is the meaning of *per cent*, the short form of *per centum*?

3. 25 per cent of 200 tons is 50 tons.

50 tons is x per cent of 200 tons.

50 tons is 25 per cent of y tons.

z is 25% of 200 tons.

The rate per cent, or the number of hundredths, is x .

The base, or the number a part of which is to be found, is y .

The percentage, or part of the base required, is z .

4. The following expressions are alike in value:

$$25 \text{ per cent} = 25\% = \frac{25}{100} = 0.25 = \frac{1}{4}.$$

Which are fractions? Decimals? Which is most easily used?

5. Of 72 find 50%; $\frac{1}{4}$; 75%; $\frac{1}{6}$; 16 $\frac{2}{3}$ %; 33 $\frac{1}{3}$ %; 66 $\frac{2}{3}$ %.

- | | |
|---------------------------------------|--|
| 6. $\frac{1}{8}$ of \$48 = x | 10. Base, \$60; rate %, 25; percentage, x |
| 7. 12 $\frac{1}{2}$ % of 96 lb. = x | 11. Rate, 75%; base, 400 yr.; 300 yr. = x |
| 8. 20% of 90 = x | 12. 33 $\frac{1}{3}$ % of his time, or x hours, he sleeps. |
| 9. 50% of 2000 = x | 13. The loss was 62 $\frac{1}{2}$ % of \$1000, or x |

262.—The Whole
Given: a Part of it
to be Found.

$$P = R \times B.$$

A.
 75 % or $\frac{3}{4}$ of $\frac{1350}{\$400} = \4050 .

1. A farm that cost \$5400 was sold for 75% of its value. What was the selling price?

2. Show that $75\% = \frac{3}{4}$.

3. Of the methods, *A* and *B*, which seems preferable? Why?

B.
 4. In general, in multiplying the base by the rate per cent to find the percentage, will it be better to use the decimal form as in *B*, or the

\$5400
 0.75
 270 00
 3780 0
 \$4050.00

equivalent common fraction as in *A*?

5. What part is \$4050 of \$5400? What %?

6. 9 is $\frac{3}{4}$ of what? \$4050 is $\frac{3}{4}$ of x . \$4050 is 75% of x .

7. Of the \$27,000 paid for an estate, $12\frac{1}{2}\%$ was in cash, and the remainder in notes. What was the cash payment?

8. Of 12,650 bushels of grain, 34% was in corn, 28% in oats, and the remainder in wheat. There were x bu. of corn, y bu. of oats, and z bu. of wheat. Explain the statement:

$$[100\% - (34\% + 28\%)] \times 12650 = z.$$

How much is—

Find a discount of—

9. 25% of 3742 tons?

12. 15% on $6\frac{1}{2}$ yards @ \$2.50.

10. $7\frac{1}{2}\%$ of 784 miles?

13. $37\frac{1}{2}\%$ on a \$558 piano.

11. $16\frac{2}{3}\%$ of 5733 acres?

14. $18\frac{3}{4}\%$ on 42 tons @ \$6.50.

15. Compare $\frac{3}{4}$ of \$400 with $\frac{3}{4}\%$ of it.

16. Read: $0.00\frac{5}{6}$; $\frac{5}{6}\%$; $\frac{5}{100}$. Explain: $\frac{5}{100} \times 6 = \frac{5}{600}$ [p. 73].

17. What is $\frac{7}{8}$ of \$64,000?

18. Find $\frac{7}{8}\%$ of 64,000.

19. My property is assessed for \$24,800. Tax rate $1\frac{3}{4}\%$. My tax?

20. A city increases 24% in 10 years; that is from 37,860 population to x .

263.—A Part Given: 1. $\frac{1}{2}$ of my age is 16 years. How old am I?
 the Whole to be Found. 2. 50% of my money is \$80. I have \$ x .

Oral.

$$B = \frac{P}{R}$$

3. $\frac{6}{7}$ of the price was \$36; $\frac{1}{7}$ of it was $\frac{1}{6}$ of \$36, or \$ x , and $\frac{7}{7}$ or the whole of it was $7 \times \$x$ or \$ y .

4. 6% of my salary was \$72; 1% of it was $\frac{1}{6}$ of \$72, or \$ x , and 100%, or the whole, was $100 \times \$x$, or \$ y .

5. The *percentage*, \$18, is 75%, or $\frac{3}{4}$, of the base, which must be \$ x , for 18 is $\frac{3}{4}$ of x .

6. There are 36 cups with saucers in a set of crockery. This is $33\frac{1}{3}\%$ of the set. There are x pieces in the set.

7. Can you find the whole of a number when you know that 75% of it is 150?

8. $\text{Base} \times \text{Rate } \% = \text{Percentage}$. When the product and one factor are given, how is the other factor found?

9. $\frac{5}{8} \times 40 = 25$; hence $25 \div \frac{5}{8} = x$; and $25 \div 40 = y$.

10. 12% of 800 = 96; hence $96 \div 12\%$, or $96 \div 0.12 = x$.

264.—A Part is 1. Compare 2 and 4: 2 is $\frac{1}{2}$ of 4 or 50%
 what Fraction of the of 4. 4 is 2 times 2 or 200% of it.

Whole?

Oral.

$$R = \frac{P}{B}$$

2. What part is 3 of 6? what % of it?

3. 5 is what part of 20? what % of it?

4. 16 is one ~ or $x\%$ of 48.

5. \$24 is two ~ or $x\%$ of \$36.

6. 12 ounces is x fourths of a pound, or $y\%$ of it.

7. 18 cwt. is ~ of a ton, or $x\%$ of it.

8. $\frac{1}{2}\%$ of 2000 lb. is x lb: 800 lb. is as many per cent of it as x lb. is contained times in 800 lb., or $y\%$.

9. $\text{Base} \times \text{Rate } \% = \text{Percentage}$. The product is 48, one factor is 3, the other is x . The product is the *percentage*, one factor is the *base*, the other factor ~, for $R = P \div B$.

10. $16 = x\%$ of 40. 11. $48 = x\%$ of 64. 12. $96 = x\%$ of 144.

265.—To Find
Percentage, Base, or
Rate %.

Written.

To find *P*.

$$\begin{array}{r} \$3125 \\ 0.48 \\ \hline 25000 \\ 12500 \\ \hline \$1500.00 \end{array}$$

To find *B*.

$$\begin{array}{r} \$3125 \\ \hline \text{^}48\text{.)} \$1500\text{^}00. \\ 144 \\ \hline 60 \\ 48 \\ \hline 120 \end{array}$$

To find *R*.

$$\begin{array}{r} 0.48 \\ \hline 3125) \$1500.00 \\ 12500 \\ \hline 25000 \\ \hline 25000 \end{array}$$

2. 23 % of a cargo of coal was thrown overboard to save the schooner. She sailed with 4320 tons, reaching port with x .

3. 72 % or 4536 volumes in a library are not works of fiction. There are x volumes in the library and y volumes of fiction.

4. 6035 persons bought tickets to a fair. This was x % of the 8500 that attended.

5. 625 scholars belong in the Lincoln school: 600 of them are present, or x % of the whole.

6. 37 % or 11,100 tons of an ice crop remained unsold. There must have been x tons in the whole crop.

7. The cargo of the *Sea King* was valued at \$38,475. The value of the cotton was $16\frac{2}{3}$ % of the whole, that of the sugar $37\frac{1}{2}$ %. The miscellaneous part of the cargo was valued at x dollars, or y % of the whole. Take the shortest method.

8. I sold my bicycle for \$85. It cost me \$125. I must have lost x % of the cost.

9. If I had lost but 15 %, I should have sold it for x \$.

10. 192 pages of a book of 432 pages are illustrated. This is x % of the whole.

266. — To Find 1. Which is most profitable, a gain of 3
Any Term when the per dozen, 5 per score, 25 per cent, or 36 per
other Two are Given. gross? Why?

Oral and Written.

2. Compare $\frac{2}{3}$ of something with $\frac{2}{3}\%$ of it.
3. Six wrong out of 24 problems solved is
 x wrong out of a hundred, or $y\%$.

4. The Brooklyns won 7 games in their series with Boston, the
Bostons won 4, and one game was a tie. The winners' per cent
was x .

5. Thirty-six hits in 80 times at the bat is a batting average
of $x\%$.

6. The centre fielder has 80 chances, and makes 4 errors. His
fielding average is $x\%$.

7. The crew pulled 36 strokes to the minute at starting, but fell
off to 30 at the finish. This was a loss of what per cent?

8.

$$\frac{2}{3}\% \text{ of } 600 = x$$

$$\frac{2}{3}\% \text{ of } x = 2$$

$$x\% \text{ of } 1200 = 8$$

9.

$$\frac{3}{4}\% \text{ of } 800 = x$$

$$\frac{3}{4}\% \text{ of } x = 12$$

$$x\% \text{ of } 200 = \frac{1}{2}$$

10.

$$x = 87\frac{1}{2}\% \text{ of } 128$$

$$x\% \text{ of } 144 = 120$$

$$83\frac{1}{2}\% \text{ bad and } x\% \text{ good}$$

11.

$$25\% \text{ of } 5\frac{1}{2}?$$

$$35\% \text{ of } 400?$$

$$8\frac{1}{3}\% \text{ of } 2\frac{2}{3}?$$

12.

$$19 \text{ is } x\% \text{ of } 57$$

$$70 \text{ is } x\% \text{ of } 2100$$

$$16\frac{2}{3} \text{ is } x\% \text{ of } 66\frac{2}{3}$$

13.

$$25 \text{ is } 4\% \text{ of } x$$

$$280 \text{ is } 14\% \text{ of } x$$

$$8 \text{ is } \frac{4}{5}\% \text{ of } x$$

14. I paid $\frac{2}{3}\%$ commission to my agent for selling a farm for
\$1250. How much money did he have left to send me?

15. $2\frac{1}{4}\%$ was paid a collector who earned \$22.50 a month in this
way. What were his annual collections?

16. Of a farm of 320 acres 108 acres are given to wheat, 96 acres
to oats, and the remainder to corn. What per cent of the farm are
the cornfields?

267.—FOR PRACTICE.

Find the value of x .

Oral.				Written.			
	Rate %	Base.	Percentage.		Percentage.	Base.	Rate %
1	$16\frac{2}{3}$	\$9.30	x	1	$32\frac{1}{10}$ m.	$267\frac{1}{2}$ m.	x
2	$12\frac{1}{2}$	x	123 tons	2	170.40	x	$17\frac{3}{4}$
3	x	75 yd.	$18\frac{3}{4}$ yd.	3	$36\frac{1}{2}$ yr.	130 yr.	x
4	$37\frac{1}{2}$	x	57	4	$18\frac{1}{5}$ A.	x	$45\frac{1}{2}$
5	40	9000 m.	x	5	4857.6 ft.	5280 ft.	x
6	x	\$0.50	$\$0.31\frac{1}{4}$	6	\$5.76	x	$\frac{1}{3}$
7	$\frac{2}{5}$	x	14 d.	7	\$5400	\$9600	x
8	$87\frac{1}{2}$	$1\frac{1}{2}$ tons	x	8	x	1500 cd.	15
9	x	160	$106\frac{2}{3}$	9	$20\frac{1}{5}$ sq. ft.	5200 sq. ft.	x
10	$6\frac{1}{4}$	x	15 sq. m.	10	6500 T.	x	$83\frac{1}{3}$
11	75	725	x	11	$14\frac{3}{5}$ d.	365 d.	x
12	x	a century	8 mo.	12	\$13,651.56	\$75,842	x
13	90	608 bu.	x	13	5	36	x
14	$18\frac{3}{4}$	x	18 bales	14	84	x	$\frac{7}{8}$
15	x	75 reams	$6\frac{1}{4}$ reams	15	x	$31\frac{1}{4}$	$16\frac{2}{3}$
16	$\frac{3}{4}$	\$12,000	x	16	$328\frac{8}{25}$	x	$9\frac{1}{2}$
17	$31\frac{1}{4}$	x	35 cords	17	\$349.06	\$9006	x
18	x	726 gal.	605	18	\$18	x	$\frac{2}{5}$
19	$\frac{5}{6}$	\$120,000	x	19	x	\$8100	$\frac{9}{10}$
20	$\frac{7}{8}$	x	$3\frac{1}{2}$ lb.	20	x	$\frac{3}{4}$	$\frac{3}{4}$

268.—Profit and
Loss. Problems.

Written.

1. Sold a house that cost \$5000 at a profit of 30%. Proceeds of sale?

2. A merchant's sales for January amounted to \$28,000, but 12% was lost in bad debts.

The net proceeds of the sales for the month were x dollars.

3. Gained \$12 or 20% in selling a Century Dictionary. It cost me x dollars, and I sold it for y dollars.

4. A sewing machine cost me \$24. I sold it for \$32. I gained $x\%$.

Gain or loss is always a part of the cost.

Explain the statement: $\frac{32 - 24}{24} = x\%$.

5. A conductor's wages were \$72 a month. They were reduced to \$60. This was a cut-down of $x\%$.

Explain the equation: $\frac{72 - 60}{72} = x\%$.

6. Cost, \$8000; selling price, \$6000; loss per cent, x .

7. Cost, \$6000; selling price, \$8000; gain per cent, x .

8. Which is more profitable, to buy cloth for \$3 and sell it for \$3.50, or to buy it for \$4 and sell it for \$4.80?

9. Gas is reduced from \$2 to \$1.60 per 1000. How much do I save on \$45 worth of gas?

10. Last winter my coal cost me \$6 a ton. This winter I pay \$6.50. This is an increase of what per cent?

269.—Amount and Difference. 1. Bought wood at \$4 a cord and sold at a gain of 20%. What did I sell it for?

Written.

a. $\$4 + 20\%$ of $\$4 = x$.

b. 120% of $\$4 = x$.

Explain the two statements. Which suggests the shorter solution?

2. Sold a typewriting machine that cost me \$80 at a loss of 10%. What did I receive for it?

a. $\$80 - 10\%$ of $\$80 = x$.

b. 90% of $\$80 = x$.

Explain the two statements.

3. Any number is x fifths, y tenths, z hundredths, $w\%$ of itself.

The Base added to the Percentage is the Amount.

The Base less the Percentage is the Difference.

4. Sold a dwelling-house for \$7500 at a profit of 25%. It cost me x dollars.

5. An epidemic decimated a southern village, leaving it with but 639 inhabitants. How many died?

6. A farmer who owned 390 acres had increased his farm 30% within two years. How much did he own at first?

7. A speculator lost \$3000 or 6% of his property. What was it then worth?

8. I sold a yacht for \$800 at a loss of 60%. Required its cost.

9. In a certain class 15% sing soprano, 45% sing tenor, 30% sing alto, and 6 sing bass. How many are there in the class?

10. A piano that cost \$450 was sold for \$292.50. What was the per cent of loss?

270.—Problems.

For dictation.

1. What is 2% of 250?
2. 30% of a man's age is 15 years. How old is he?
3. What per cent of the day has passed at 9 A.M.?
4. Cost, \$2; gain %, 16; selling price?
5. Selling price, \$6; gain, 20%; cost?
6. Gain, \$3, or 5%; cost? Selling price?
7. Selling price, \$18; cost, \$15; gain %?
8. Every 16th coin out of a collection of 176 silver dollars was counterfeit. This is a loss of what per cent?
9. 10 per dozen is what per cent?
10. The square root of 625 is 10% of what?

271.—Problems.

At sight.

1. In $\frac{3\frac{5}{12}}{11}$ what per cent is the numerator of the denominator? In $\frac{5}{6}$?
2. What is $\frac{2}{11}$ of a rod in feet? In inches?
3. 231 cubic inches = 1 gallon. Separate 231 into its prime factors. Give the dimensions of a tin pan that will hold a gallon.
4. What part of a year is in the longest months?

5. What is $\frac{1}{3}\%$ of 21,000?
6. What per cent of the rotation of the earth is accomplished at 10 A.M.?
7. $33\frac{1}{3}\%$ of 60% of $\frac{1}{2}$ of the money remained. How much did the thieves take?
8. My property is assessed for \$2500. The rate of taxation is $2\frac{1}{2}\%$. What is my tax?
9. What per cent of the surface of a 4-inch cube is on five sides of it?
10. Bought thread at 4 cents a spool and gained 300%. It sold for x cents.

272. — Problems.1. $8\frac{1}{3}\%$ of a yard = $x\%$ of a foot.*At sight.*2. Three sides of a square = $x\%$ of its perimeter.

3. $x\%$ of the day has passed at 9 P.M.
4. $(\frac{1}{3} + 30\% + \frac{2}{3})$ of 64 is 25% of x .
5. 16 is $\frac{2}{3}$ of x and $\frac{2}{3}\%$ of y .
6. A quire is $x\%$ of a ream.
7. Gave \$24 to James, and \$30 to Lucy. Lucy had $x\%$ more than James, and he had $y\%$ less than Lucy.
8. Paid the price of a pound for 14 ounces. I thus lost $x\%$.
9. In a series of ball games the Alphas won 40% and the Omegas 50%. Two games were drawn. How many were played?
10. $\sqrt{9} = x\%$ of $\sqrt{144}$.

273. — Problems.*For analysis.*

1. To 20 gallons of alcohol add 5 gallons of water. What per cent of alcohol in a quart of the mixture?
2. Sold a mine for \$72,000, and gained 20%. It cost me x dollars. $\$72,000 \div (\text{cost } \% + \text{gain } \%) = \text{cost}$.
3. I purchased a patent for \$8000. The seller lost 84% of its original value, which was x dollars.

4. I paid \$125 for what I thought was 4-foot wood. It proved to be but 45 inches long. What deduction should be made in the settlement?

5. Sold telephone stock for \$25,000, at an advance of 25% on what I paid for it. What did I gain?

274.—Profit and Loss. 1. \$12 or $12\frac{1}{2}\%$ (of cost) is gained; cost = \$ x .

Oral or written. 2. \$8 or $16\frac{2}{3}\%$ is lost; cost = \$ x .

3. \$24 = cost; $33\frac{1}{3}\%$ is gained; selling price = \$ x .

4. \$35 = cost; $14\frac{2}{7}\%$ is lost; selling price = \$ x .

5. \$36 = selling price, which includes the cost and a gain of 20% of the cost. \$36 = cost + $\frac{1}{5}$ of cost, or $\frac{6}{5}$ cost; \$36 is $\frac{6}{5}$ of \$ x .

6. \$28 = selling price, which is the cost less a loss of 20%. What part of the cost is the loss? The selling price is $\frac{x}{5}$ of the cost; \$28 is $\frac{x}{5}$ of \$ y .

7. Bought a bicycle for \$80, and sold it for \$100. My gain per cent was x .

8. If I had sold it for \$60 I should have lost \$ y or x per cent of cost.

9. An importer bought silk at \$2.50 a yard and sold it to a retailer for \$3, who sold it to the wearer for \$3.50. What per cent of profit did each make?

10. Sold a watch for \$119 and gained $16\frac{2}{3}\%$. How much should I have gained or lost if I had sold it for \$100?

11. A thrifty clerk resolves to live on 60% of his salary. He spends \$48 more than he intended, but still saves \$300. What was his salary?

275.— Gain and
Loss.
Written.

1. Which is more profitable, buying meat at 16¢ and selling at 19¢, or selling potatoes at 64¢ that cost me 56¢?
2. Butter sold at 28¢ yields no profit. What would be gained on \$140 worth sold at 30¢?
3. Milk bought at 20¢ a gallon is sold at 8¢ a quart. What per cent is gained if 25% of the quantity bought spoils?
4. Had I better buy my winter's coal to-day at \$4.00 a ton cash, or wait 6 months and pay \$4.25? I can get 12% interest for my money.
5. A 5% increase in wages means \$200 more a month for the employer to pay. What was his annual pay roll before and after the increase?
6. Mr. H. earns \$1200 a year selling carriages at 15% commission, all expenses paid. The manufacturer makes a net profit of 14 $\frac{2}{3}$ %. If 50 carriages are sold, what is their average cost?
7. An unscrupulous dealer buys 50 gallons of alcohol and adds 14 gallons of water, and sells the mixture at 10% below actual cost. What per cent does he gain?
8. I am offered a 10% discount on a suit of clothes marked to sell at \$60. I know that even then the dealer will make 12 $\frac{1}{2}$ %. I offer \$50 and get the suit. What per cent does the dealer gain?
9. I sell $\frac{5}{6}$ of a lot of land at $\frac{3}{4}$ the cost and get \$200 for the remainder. The original cost being \$1200, what is my per cent of loss?
10. What per cent is gained by selling coal at the rate of $\frac{1}{3}$ of a ton for what 1000 pounds cost?
11. A farmer's sheep cost him \$200. One out of every seven dies, and he sells those that remain for \$275. What was the gain per cent, the cost of keeping being \$40?
12. A merchant sold a stock of goods for \$3042 and gained 17%. What % would he have gained or lost had he sold it for \$2392?
13. For what should he have sold it to gain 100%?

Interest.

276.—Bankers'
Method.

Oral.

1. I pay 6 cents for a year's use of a borrowed dollar: what is the rate of interest?
2. What does the expression "6 per cent interest" mean?
3. A year's interest is what per cent of the principal? 4. What part of a year is 2 months? 2 months' interest is $x\%$ of the principal.
5. 20 months' interest is $y \times x\%$ of the principal, or $z\%$ of it.
6. 200 months' interest is what per cent of the principal?
7. How is 1% of a number found? 10% of it? 100% of it.

*At 6% the interest of any principal
for 12 months = 6% of it;
for 2 months = 1% of it;
for 20 months = 10% of it;
for 200 months = 100% of it.*

To find the interest at 6% of \$380 for 2 yr. 7 mo.

Process.

Explanation of Process.

Interest for 31 mo. of \$380.

"	"	20 mo. =	38.00
"	"	10 mo. =	19.00
"	"	1 mo. =	1.90
"	"	31 mo. =	\$58.90

8. 2 yr. 7 mo. = x mo.

9. 20 mo. int. = $\frac{1}{10}$ of P. How is this found?

10. 10 mo. is what part of 20 mo.? How then is 10 mo. interest found?

11. How is 1 mo. interest found? The total interest?

12. Into what convenient parts would you separate the time, if you are to find the interest for 26 mo.? 37 mo.? 3 yr. 7 mo.? 5 yr. 8 mo.?

- 277.**—Bankers' Method. *Explain the following process of finding the interest at 6% —*
 Rate, 6%. 1. Of \$ 725 for 3 yr. 11 mo.
 Written. 2. Of \$ 278 for 1 yr. 7 mo.

Interest for 47 mo. of \$725.	19 mo. \$278
" " 20 mo. = 72.50	20 mo. = 27.80
" " 20 mo. = 72.50	1 mo. = 1.39
" " 5 mo. = 18.125	19 mo. = \$26.41
" " 2 mo. = 7.25	
" " 47 mo. = \$170.375	

- Find the interest* *What shall be paid for the use*
3. Of \$ 280. for 2 yr. 8 mo. 7. Of \$ 649. for 7 yr. 8 mo. ?
 4. Of \$ 640. for 3 yr. 7 mo. 8. Of \$ 750. for 8 yr. 4 mo. ?
 5. Of \$ 95. for 4 yr. 11 mo. 9. Of \$ 295.75 for 5 yr. 11 mo. ?
 6. Of \$ 73.50 for 1 yr. 4 mo. 10. Of \$ 641.86 for 3 yr. 3 mo. ?

- 278.**—The Time in Days. 1. How many days in an interest month?
 Rate, 6%. 2. In an interest year? 3. The interest for 2 mo. or 60 days is $x\%$ of the principal; for 6 days it is $\frac{1}{10}$ of $\frac{1}{100}$ or $\frac{1}{1000}$ of the principal. To find $\frac{1}{1000}$ of a number we ~.

To find 75 days' interest of \$ 720. at 6%.

Process.

Interest for 75 d. of \$720.
" " 60 d. = 7.20
" " 15 d. = 1.80
" " 75 d. = \$9.00

*At 6% the interest of any principal
 for 60 days = $\frac{1}{100}$ of it;
 for 6 days = $\frac{1}{1000}$ of it.*

4. 15 days is what part of 60 days?

Explain the process of finding the interest at 6% —

5. Of \$196 for 115 days.

Int. for 115 d. for \$196.

$$\begin{array}{r} 60 \text{ d.} = \$1.96 \\ 30 \text{ d.} = 0.98 \\ 20 \text{ d.} = 0.6533 + \\ 5 \text{ d.} = 0.1633 + \\ \hline 115 \text{ d.} = \$3.7566 \end{array}$$

6. Of \$119 for 89 days.

Int. for 89 d. for \$119.

$$\begin{array}{r} 60 \text{ d.} = \$1.19 \\ 20 \text{ d.} = 0.3966 + \\ 6 \text{ d.} = 0.1190 \\ 3 \text{ d.} = 0.0595 \\ \hline 89 \text{ d.} = \$1.7651 \end{array}$$

What shall I pay at 6% for the use of — Find the interest at 6% of —

7. \$780. for 67 d. ?

12. \$94. for 200 d.

8. \$640. for 93 d. ?

13. \$762. for 5 mo. 14 d.

9. \$92. for 3 mo. 12 d. ?

14. \$815. for 86 d.

10. \$87.50 for 117 d. ?

15. \$924. for 8 mo. 11 d.

11. \$106. for 2 mo. 17 d. ?

16. \$17.84 for 17 d.

279. — Bankers' Method.

1. 6% interest = \$18; 1% interest = \$x;
5% interest = 5 × \$x or \$y.

At any Rate.

2. 6% interest = \$42; 7% interest = \$x.

3. What is 10% interest when 6% interest is \$90 ?

4. What is 4% interest when 6% interest is \$72 ?

5. Having found the interest on any sum of money at 6%, how shall we find it at 1% ? At any rate ?

To find the interest of \$105 for 75 d. at 5%.

Process.

6% int. for 75 d. of \$105.

“ “ 60 d. = 1.05

“ “ 15 d. = 0.2625

6) \$1.3125 = 6% int.

0.2187 + = 1% int.

\$1.0938 = 5% int.

6. Show how the interest at 6% is found.

7. At 1%; at 5%.

8. What if the rate had been 7% ? 10% ? 12% ?

9. Find the interest of \$280 for 72 d. at 8%.

10. Which is easier, to find 8% interest by taking 8 times 1% interest, or by adding 2% interest to 6% interest ? Try both ways before deciding.

Find the interest of—

11. \$ 640. for 1 yr. 8 mo. at 1%. ($\frac{1}{6}$ of 6%.)
12. \$ 270. for 3 yr. 10 mo. at $1\frac{1}{2}\%$. ($\frac{1}{4}$ of 6%.)
13. \$ 382. for 1 yr. 9 mo. at 2%. ($\frac{1}{3}$ of 6%.)
14. \$ 927. for 6 mo. 4 d. at 3%. ($\frac{1}{2}$ of 6%.)
15. \$ 864. for 2 mo. 7 d. at 4%. (6% - 2%.)
16. \$ 318. for 1 mo. 13 d. at 5%. (6% - 1%.)
17. \$ 725. for 29 d. at 7%. (6% + 1%.)
18. \$ 649. for 67 d. at $7\frac{1}{2}\%$. (6% + $1\frac{1}{2}\%$.)
19. \$ 84. for 54 d. at 8%. (6% + 2%.)
20. \$ 125. for 276 d. at 9%. (6% + 3%.)
21. \$ 6000. for 118 d. at 10%. ($10 \times 1\%$.)
22. \$ 1525. for 63 d. at $\frac{1}{2}\%$. ($\frac{1}{12}$ of 6%.)
23. \$ 2500. for 93 d. at $2\frac{1}{2}\%$. [$(\frac{1}{3} + \frac{1}{12})$ of 6%.]

280.—The One Dollar Method. 1. What two methods of computing interest have previously been presented?

2. In which one is the interest at 6% found first? 3. What is "a 6% method" of calculating interest? 4. In how many states is 6% the rate of interest established by law? [App., p. 15.]

5. What is the interest of \$ 1 for a year at 6%? 6. For 2 mos., or $\frac{1}{6}$ of a year? 7. At 6%, how long may I use a borrowed dollar for a cent of interest?

8. What is the interest of \$ 1 for 6 days, $\frac{1}{10}$ of 2 months? 9. A mill pays 6% interest on a dollar for how long?

10. The interest of \$1.00

For 2 years = \$ 0.

For 8 months = 0.

For 18 days = 0.

For 2 yr. 8 mo. 18 d. = \$ 0.

At 6%, the interest of \$ 1.00

For 1 year = \$ 0.06

For 2 months = 0.01

For 6 days = 0.001

11. Compared with the interest of \$ 1, what will be the interest of \$ 75? Of \$ 183? Of \$ 420? Of \$ 725? Of \$ 84.75? [p. 20.] Of x dollars?

12. The interest of \$1.00

For 3 yr. = \$0.

For 10 mo. = 0.

For 24 d. = 0.

Total = \$0.

13. The interest of \$1.00

For 4 yr. = \$0.

For 7 mo. = 0.

For 10 d. = 0.

Total = \$0.

14. The int. of \$1.

For 3 yr. = \$0.

For 5 mo. = 0.

For 15 d. = 0.

Total = 0.

15. The int. of \$1.

For 2 yr. = \$0.

For 9 mo. = 0.

For 22 d. = 0.

Total = 0.

16. The int. of \$1.

For 5 yr. = \$0.

For 11 mo. = 0.

For 17 d. = 0.

Total = \$0.

281.—Interest by
the One Dollar
Method.
At Any Rate.

Process.

Interest of \$1 at 6%.

For 3 yr. = \$0.18

" 7 mo. = 0.035

" 19 d. = 0.003 $\frac{1}{2}$

I. $\$0.218\frac{1}{2}$
48.96

816

39168

4896

9792

II. $6\overline{)10.68144}$
1.7802 at 1%

III. \$12.46 at 7%
48.96 = Prin.

IV. \$61.42 = Amount.

I hire \$48.96 at 7% for 3 yr. 7 mo. 19 d.
What shall I pay at settlement?

1. Explain these four steps of the process:

I. Finding the interest of \$1 at 6%.

II. Finding the interest of the given principal at 6%.

III. Finding the interest at the given rate.

IV. Finding the amount.

2. In II. why is the smaller number used as a multiplier?

NOTE.—The work should be carried to four decimal places, and results given to the nearest cent.

3. What will discharge a debt of \$475 which has been drawing 5% interest for 2 yr. 11 mo. 24 d.?

4. Find the amount of \$7000 at 4% for 3 yr. 3 mo. 13 d.

5. I hold two notes of \$731 each, one at 5% interest, the other at 8%. They have been running 4 yr. 8 mo. 17 d. What shall I receive at settlement?

Find the amount under the following conditions:

6. Principal, \$84.75; rate of interest, 4%; time, 3 yr. 15 d.
7. Principal, \$942; rate of interest, 5%; time, 4 yr. 1 mo. 7 d.
8. Principal, \$193; rate, 7%; time, 18 mo. 27 d.
9. Principal, \$64.50; rate, 8%; time, 5 yr. 5 mo. 5 d.
10. Principal, \$712.10; rate, 9%; time, 7 yr. 4 mo. 29 d.
11. 4 yr. 6 mo. 21 d., \$425.50, 3%.

282. — **Time between Dates.** 1. To find the time in years, months, and days from June 24, 1893, to Aug. 13, 1896. Explain each process.

Process A.

Process B.

From June 24, '93, to June 24, '96 = 3 yr., or 6/93 to 6/96 = 3 yr.

From June 24, '96, to July 24, '96 = 1 mo., or 6/24 to 7/24 = 1 mo.

From July 24, '96, to Aug. 13, '96 = 20 d., or 7/24 to 8/13 = 20 d.

2. To find the time from Sept. 14, 1891, to Mar. 11, 1895. Explain each process.

Process A.

Process B.

From Sept., '91, to Sept., '94 = 3 yr., or 9/91 to 9/94 = 3 yr.

From Sept. 14 to Feb. 14 = 5 mo., or 9/14 to 2/14 = 5 mo.

From Feb. 14 to Mar. 11 = 25 d., or 2/14 to 3/11 = 25 d.

3. What advantage has process *B* over process *A*? 4. Why is it well to know the months by their numbers as well as by name?
5. In process *B*, what is found at the left of the inclined line? At the right of it?

6. Napoleon was born Aug. 15, 1769, and died May 5, 1821. How long had he lived?

Process.

8/1769 to 8/1820 = 51 yr.

8/15 to 4/15 = 8 mo.

4/15 to 5/5 = 20 d.

Explain the process.

7. Find the time from May 22, 1890, to June 12, 1893.

8. Find the time from Dec. 25, 1893, to Mar. 3, 1896.
9. How long from 4/18, '96 to 3/11, '99?
10. Find your exact age to-day.

**283.—Interest: Choice
of Methods.**

Written.

METHODS OF COMPUTING INTEREST.

- I. A general method, page 106.
- II. The bankers' method, page 160.
- III. The one-dollar method, page 163.

Any of these three methods may be used exclusively, but as no one method is always the best, it is well to learn to choose the one that will give an accurate result most quickly.

Solve the following problems by each method, compare results, and tell which method you prefer, and why.

1. Find the interest of \$ 360 at 7% for 207 d.
2. What is the amount of \$ 75 at 8% for 3 yr. 4 mo.?
3. What shall be paid for the use of \$ 723.60 for 85 days at 10% interest?

What is the interest under the following conditions?

Principal.	Time.	Rate.	Principal.	Time.	Rate.
4. \$ 648.	111 d.	4%	9. \$ 432.	1 yr. 8 mo. 7 d.	3%
5. \$ 324.	167 d.	5%	10. \$ 767.80	3 yr. 11 mo. 9 d.	5½%
6. \$ 750.	200 d.	9%	11. \$ 50.40	10 mo.	4½%
7. \$ 427.	93 d.	12%	12. \$ 87.75	114 d.	6%
8. \$ 865.	48 d.	4%	13. \$ 137.77	4 yr. 9 mo. 25 d.	7½%

14. Interest is the product of what three factors?
15. Which method of finding interest is best when principal, rate, or time is divided by 4? By 4½? 6? 9 or 12? Why?
16. Which method uses the aliquot parts of the time?
17. Which are "6% methods"? Why so called?
18. Which is the best method when there are years, months, and days in the time, and when cancellation is impossible?
19. What is the interest of \$ 400 at 10% for 2½ years?

284.—FOR FREQUENT PRACTICE.

At sight.

1. Find 200 months' interest of \$ 87.56 at 6%.
2. Find 20 months' interest of \$ 300 at 3%.
3. Interest of \$ 500 for $\frac{1}{3}$ yr. at 10%?
4. Interest of \$ 1 for 6 d. at 6%?
5. Interest of \$ 569 for 60 days at 6%?
6. Interest of \$ 100 for 4 yr. at 8%?
7. $\frac{2}{3}$ yr. interest of \$ 400 at 9%?
8. 8 mo. interest of \$ 200 at 9%?
9. 22 mo. interest of \$ 500 at 6%?
10. 24 d. interest of \$ 800 at 6%?
11. 0.003 of the principal is the interest for x days at 6%.
12. A principal gains as much as itself at 6% in x months.
13. 11 mo. interest of \$ 400 at 9%?
14. A principal gains 25% of itself in x months at 6%.
15. $\frac{1}{10}$ of principal = x mo. interest.

For dictation.

1. 4 yr. interest of \$ 500 at 5%?
2. $3\frac{1}{2}$ yr. interest of \$ 100 at 8%?
3. 6 mo. interest of \$ 120 at 5%?
4. 7% interest of \$ 500 for 2 yr.?
5. 60 d. interest of \$ 567 at 6%?
6. 20 mo. interest of \$ 1200 at 6%?
7. 10% interest of \$ 1 for 36 d.?
8. 6% interest of \$ 1 for 17 mo.?
9. 6% interest of \$ 1 for 112 days?
10. 5 mo. interest of \$ 240 at 10%?
11. At 6%, what part of the principal = 50 mo. interest?
12. 11 days' interest is what part of a year's interest?
13. 28% of the principal is how many years' interest at 5%?
14. Find the interest of \$ 600 at 8% for 4 mo. For 5 mo. For 15 mo. For 15 d.
15. \$ 372 is the interest of \$ 372 for how long at 6%?

285. — Interest: NOTE. — The method to be employed in the
Choice of Methods. solution of the following problems is shown
Written. by the Roman numerals *I*, *II*, or *III* (p. 166).

1. What is the interest of \$ 840 for 9 mo. 17 d. at 4% ? *I*.
2. Find the amount of \$ 722 for 156 d. at 12% . *II*.
3. What will settle an account of \$ 425 that has been drawing interest at 5% for 5 yr. 5 mo. ? *III*.
4. May 17, 1893, I borrowed \$ 284 at $2\frac{1}{2}\%$. Aug. 15, 1895, how much interest had accrued ? *III*.
5. In $4\frac{1}{2}$ years how much will be received on a \$ 5000 railroad bond paying 2% semi-annually ? *I*.
6. May 27, 1898, I paid a note of \$ 475.28 that had been drawing 4% interest since Dec. 31, 1894. *III*.

Find by inspection the best method of solving the following problems, and use it in finding the interest. Try to forecast the result.

7. \$ 9000 on interest 7 mo. 24 d. at 4%.
8. \$ 728 draws 5% interest for 20 months.
9. $3\frac{1}{2}\%$ interest of \$ 900 from Jan. 15 to Nov. 2.
10. Principal, \$ 72.59; time, 125 days; rate, $12\frac{1}{2}\%$.
11. What shall I pay for the use of \$ 500 for 50 days at 5% ?
12. \$ 320; $7\frac{1}{2}\%$; July 7, 1845, to August 4, 1859.
13. \$ 720; 8%; October 19, 1890, to May 11, 1893.
14. \$ 472 3% 112 d. 20. 3% \$ 872 4 yr. 8 d.
15. \$ 648 $4\frac{1}{2}\%$ 8 mo. 21. 5% \$ 5000 16 mo.
16. \$ 800 $2\frac{1}{2}\%$ 180 d. 22. $2\frac{1}{2}\%$ \$ 178.91 104 d.
17. \$ 950 9% 248 d. 23. 8% \$ 64.87 295 d.
18. \$ 2000 7% $19\frac{1}{2}$ mo. 24. 1% \$ 3294 $17\frac{5}{6}$ mo.
19. \$ 4000 4% $4\frac{2}{3}$ yr. 25. $4\frac{1}{2}\%$ \$ 700 $41\frac{2}{3}$ yr.

236. — **Exact Interest:** 1. In computing interest for parts of a year we commonly consider 30 days a month and 360 days a year. In taking $\frac{1}{360}$ of a year's interest to find the interest for 1 day, do we take too much or too little, considering the actual length of a year?

2. *Exact or accurate interest* is reckoned for the actual number of days in the given time, and counting 365 days to the year. It is used by the United States government and sometimes in other business transactions. It differs from common interest only as applied to parts of a year. What part of a year is August? February? The last three months of 1896?

3. $\frac{1}{365}$ is what part of $\frac{1}{360}$? Explain this process:—

$$\frac{1}{365} \div \frac{1}{360} = \frac{1}{365} \times \frac{360}{1} = \frac{360}{365} = \frac{72}{73}.$$

4. If 1 day's accurate interest is $\frac{72}{73}$ of 1 day's common interest, what is the accurate interest when the common interest is \$146?

5. If from the common interest I deduct $\frac{1}{73}$ of itself I shall have the exact interest. Explain.

6. Find the accurate interest of \$500 for 90 days at 4%.

7. Find the exact interest of \$1000 at 3% from May 9 to Sept. 4.

Common interest decreased by $\frac{1}{73}$ of itself is exact interest.

NOTE. — The exact number of days must be found; that is, $22 + 30 + 31 + 31 + 4 = 118$.

Find the exact or accurate interest of—

8. \$800; 6%; Aug. 11 to Oct. 9.
9. \$720; 8%; Jan. 4 to Mar. 15.
10. \$1200; 3 mo. 12 d.; 5%.
11. \$1500; 72 d.; 10%.
12. What is the exact interest of \$1000 for 2 yr. 8 mo. 9 d. at 6%? (Find common interest for 2 yr. + exact interest for 8 mo. 9 d.)
13. Find the accurate interest of \$3000 for 3 yr. 9 mo. at 8%.

287. — Wholesale
 or Retail. For Cash
 or on Credit.

Oral.

1. Show the difference between grower or producer, importer, wholesaler, retailer.
2. From which class of dealers do you buy?
3. With whom do wholesalers have to deal?
4. The regular price of a pear-tree is \$ 1.50.

If I get it for \$ 1.25, what is the *discount* or deduction?

5. If I buy a dozen at one time, I pay only \$ 12. What per cent of the highest price is this? What is the rate of discount?

6. A man is trusted for goods billed at \$ 100. He is to pay in 3 months. How long is the *term of credit*?

7. The dealer offers to sell the same goods for \$ 98 *cash*. Why is this? What per cent does he discount?

288. — Trade Discount. 1. My bill is \$ 15, less 10%, as I am "in the same trade." What must I pay?

2. Price per dozen, \$ 2; for 30 dozen I pay \$ 50. Without discount the cost would be \$ *x*. The discount was *y*%. 3. If I had bought 100 dozen, the *net price*, or what I actually pay, would have been only \$ 1.60. Is this a larger or a smaller rate of discount? Why?

4. Discount on a carload of coal is 10%, or \$ 4. What would it be on 20 carloads at double the rate of discount?

5. It is common to have a permanent *list of prices* and to change the rate of discount as may be necessary.

<i>List Price or</i>	}	<i>= Base.</i>
<i>Amount of Bill</i>		
<i>Discount</i>		<i>= Percentage.</i>
<i>Net Price</i>		<i>= Difference.</i>

List price, \$ 40; net price, \$ 32; rate of discount, *x*%.

6. When the discount changes to 25%, the difference in price is *y*.

7. \$ 5 is 20% of list price. The net price is *x*.

8. Discount, 10%; net price, \$ 90; list price?

**289.—Time and
Cash Discounts.**

Oral.—1. A \$4000 house is offered at \$3500 cash. The discount = \$ x . The rate = $y\%$.

2. Bought \$200 worth of flour. If I need not pay for 6 months, what do I save? Explain.

3. Which customer receives the larger discount, one who pays in 3 months or one who pays in a year?

4. January 1 I buy \$400 worth of wool, and am promised a *time discount* of 2% if I pay by April 1. By paying when the goods were bought, I should be allowed 4%. What is a *cash discount*?

Discount is always a part or per cent of the price which it reduces.

Written.—Find the missing terms:—

Amt. of bill.	% off for cash.	Net cost.	Discount for cash in 30 days.	List price.	%.
5. \$2000	5	x	8. \$24	\$600	x
6. \$900	x	\$10	9. x	\$150	1
7. x	2	490	10. \$20	x	4

**290.—Successive
Discounts.**

1. From 100 take 60%, from the remainder 25%, from that remainder 10%, leaving x .

Written.

What per cent of 100 have you deducted?

2. A box of pens is listed at \$1, but a retailer buys it for 50¢, the trade discount being $x\%$.

3. When he buys 100 boxes, he gets a second discount of 20% from the lower price, each box costing \$ y .

4. A third cash discount of 1% makes a box cost \$ z .

5. Have we deducted

Successive discounts are taken from the price as already reduced.

(50% + 20% + 1%) of \$1 or 50% of \$1 + 20% of 50¢ + 1% of 40¢?

Find net prices:—

List.	% off.	List.	% off.
6. \$15.40	20 then 5	9. \$14.85	60, 10, and 2
7. \$49.50	50 then 2	10. \$320.15	20, 5, and 1
8. \$600	45 then 3	11. \$4000	30, 12, and 3

291. — Problems.

1. A library buys its books 35 % off. An invoice of \$ 10,000 calls for how much net?
2. A bill is made—"Terms: cash in 60 days." What discount may be expected for cash at time of sale? (Money at 6 %.)
3. When money is worth 12 %, a dealer gives 4 mo. credit. What discount for cash may be expected?
4. Which would be more profitable in the end—to sell for \$ 100 cash, or to charge \$ 103, giving 6 mo. credit? Is the cash discount here more or less than 3 %?
5. Cash or net price, \$ 760; discount, 40 % of x ; list price, x .
6. An invoice of jams is charged at \$ 2500 on 6 mo. time, or with time discounts of 1 % a month. What amount will pay the bill in 30 days?
7. One buyer gets 30 % off, another gets 25 % and 5 %. Give net cost to each on a shipment of \$ 2000 gross value.
8. A merchant who gives 90 days on all bills allows 5 % for cash. You infer that money is worth to him x % a year.
9. A furniture maker allows 15 % from the list price. Find the net cost on an order amounting to \$ 12,438, including \$ 118 for carting, which is without discount.
10. Tubing listed at \$ 10,000 is billed less 60 % and 2 % for cash. Net price = x .
11. The trade discount on certain goods is 70 %. Large buyers receive a second discount of 10 %, making the total discount x %.
12. List price, \$ 500; net, \$ 425; discount, x ; rate, y %.
13. List price, \$ 488.90; net, \$ 391.12; discount, x ; rate, y .
14. 1800 ft. of moulding at 20 ¢, less 12 % to the trade and 1 % for cash, cost x .
15. A shipment of sugar invoiced at \$ 11,000 is subject to a rebate or reduction of 5 %. Terms: 15 days. $\frac{1}{4}$ % off for cash makes the net cost x .
16. The discounts on a \$ 1000 invoice are not 45 %, but are 30 %, 10 %, and 5 %. Find the net price.

292.—Insurance.

Oral.

1. If the owners of a hundred ships agree to share the loss if one is wrecked, who might profit by the arrangement?
2. A man owns a house worth \$3000. By spending \$30 he can be sure that loss by fire will be made good. Many others do the same, and from their money his loss is paid. What % will he save?
3. An *insurance company* promises security in case of loss to those who have paid a certain per cent or *premium* on the *insurance* of their property. What will it cost to insure goods for \$250 at $1\frac{1}{4}\%$?
4. 100,000 persons pay 25¢ each to an accident insurance company. If it pays \$15,000 in claims for injuries, and \$4000 for expenses, the profit is x . Who is the *insurer*? Who are *insured*?
5. The agreement to make good a loss on certain conditions is printed in a *policy* made by the *underwriter* or *insurer*, and held by the *insured*. The cost of a \$25,000 policy at $\frac{3}{4}\%$ a year is \$ x .
6. By paying an annual premium a person may be assured that at his death or at a certain age, his family, or he himself, will receive a specified sum. How will this money have been obtained?
7. A ship costing \$210,000 is insured for $\frac{2}{3}$ of its value at 2%. If lost, the owners receive x . The underwriters lose y .
8. \$40 pays for five years' insurance on a brick store which cost \$5000. The insurance valuation is \$4000. What is the annual rate?
9. A wooden tenement house two miles from a fire-engine is insured for $\frac{3}{4}\%$, but only for one year. If the valuation is \$4000, the cost for five years is x . The property insured is called a *risk*. Compare the last two risks.
10. Insurance provides for sharing loss due to what causes?
11. A schoolhouse is insured for 5 years at $\frac{1}{2}\%$ premium, which is \$300. The insurance valuation is $\frac{2}{3}$ of the cost of the house. What is the underwriters' loss, if it burns?

293. — Examples.

Written.

1. A stock of goods is worth \$12,000. The premium for a year is 1%, or \$100. What is the insurance valuation? If destroyed, what will the underwriters pay, and what will the owner lose besides the premium?

Supply values for x:—

Valuation.	Rate %.	Premium.	Rate %.	Premium.	Insurance.
2. \$20,000	x	\$100	5. 20	\$12.50	x
3. 3,500	x	42	6. x	62.50	\$5000
4. x	$\frac{3}{4}$	30	7. $\frac{2}{3}$	x	4740

8. Why is property usually insured for less than its full value? A \$7500 house is insured at $1\frac{1}{4}\%$ for \$62.50. The insurance valuation is x .

9. A man is insured in a *mutual company*, sharing all gains and losses; \$60 insured his \$8000 house in full for five years. What was the rate? After five years \$20 with interest was returned to him. This reduced the rate to x .

10. A ship worth \$30,000 is insured for $\frac{5}{6}$ of its value at $2\frac{1}{2}\%$. The possible loss to the owner, including premium, would be x .

11. \$3.75 was the premium on $\frac{3}{4}$ the value of some furniture at 1% a year. What was its insurance valuation?

12. One company offers to take a \$12,000 risk at $1\frac{1}{2}\%$ for five years; another at $\frac{1}{4}\%$ a year. Which is cheaper, and why?

13. \$234.69 is the amount of a policy on some window glass. The premium is 2%; the difference, or \$ x , equals the value of the glass, which is $y\%$ of the amount of the policy.

14. \$2000 is 98% of the amount insured. Premium = 2% of x .

15. If a stock of goods is worth \$6930, what insurance at 1% will include that amount and the premium? $6930 = 99\%$ of x .

16. A block is insured for \$5000 in each of 15 companies at an average rate of $\frac{4}{5}\%$ for five years. Find the annual cost of insurance, not counting interest. What would each company pay if the building suffered \$50,000 damage? Why is it safer to divide a risk among several companies?

294. — Commission. *Oral.* — 1. I send goods to a person in town to be sold. I am the *principal*; he is my *agent*. He sells them for \$100, and I pay him $1\frac{1}{2}\%$ for the service. He keeps \$ x as his *commission*, and sends me the *net proceeds*, which are \$100 — \$ x , or \$ y .

2. What is 2% commission on a sale of 3000 melons at 50¢ each?

3. My agent, a commission merchant in New York, sends me 95% of what he collects, and keeps \$20. He collects \$ x .

4. The *gross proceeds*, or sum collected, is \$150. The net proceeds are \$147, less \$15 expense of transportation. The commission is \$ x , or $y\%$ of —.

Commission for selling is a percentage of the sum collected.

5. A *consignment* of goods to be sold is sent by the *consignor* to the *consignee*. The sum collected minus the 5% commission is \$285. What are the net proceeds? What per cent does the consignee return? $1\% = \frac{\$285}{95}$, or \$3. The gross proceeds are \$ x .

Written. — 6. Four carloads of peaches are consigned to a *factor*, or agent, who is to receive 5¢ a basket. The price realized is \$1, but the charges for freight equal three times the commission. The owner receives what per cent of the gross receipts?

7. Net proceeds of a sale of cocoanuts, \$100; charge for storage, \$5.60; commission, \$4.40; gross receipts, \$ y ; rate, $x\%$.

8. Find the commission on a consignment of rubber shoes sold at 10% off \$300, the consignee retaining 3%.

9. A shipment of strawberries sells for \$138.15, from which the agent pays out \$125 for freight. His 5% commission amounts to \$ x .

10. Commission = $\frac{x}{100}$ of gross, or of net receipts?

Commission + expenses + net proceeds = what?

295. — Commission.

Buying through
Agents.

Oral.

1. If I employ a *correspondent*, or agent, to buy goods for me, I must pay him a percentage of the amount which he expends for me. If I send him \$1000 with which to buy corn, \$1000 is my *remittance* to him.

If he invests it all, how much more must I remit to him to pay his 3% commission?

2. If I had sent his commission when I remitted the amount he was to expend, how much must I have sent for each dollar he was to expend?

3. If I send 100% of the amount he is to expend, do I send him enough to pay his commission? If I send him \$1.03, on what sum will he receive 3% commission? Why should the remittance be 103% of the amount to be expended?

4. Should an agent receive a commission on his own pay, or on only so much as he expends for his employer?

*Commission for buying is
a percentage of the amount
expended.*

5. An agent buys \$2000 worth of copper, charging 1% commission, or \$*x*. What must his employers remit to pay both bills? For every dollar he spends they must remit exactly ---.

6. Remitted \$102.50 to buy umbrellas; commission $2\frac{1}{2}\%$. How much of my remittance will buy \$1 worth of umbrellas and also pay the agent? The agent can buy \$*x* worth.

7. An agent charges $1\frac{3}{4}\%$ for buying chair stock. If he is to spend \$1000, how much must I remit?

8. Remittance, \$400; goods purchased by agent, \$337.50. He pays \$25 for storing and forwarding them. The gross amount expended is *x*. On what sum should he receive his 10% commission?

9. Remittance \$9.45, less 5% commission = amount of purchase. Explain: $\$9.45 \div \$1.05 = x$.

296. — Commission for
Buying.

Written exercise.

1. At 5% commission, how many dollars' worth can be bought for \$126, leaving enough to pay for the service?
2. The principal in a transaction remits \$2050 for a purchase of apples, less $2\frac{1}{2}\%$ commission. \$2050 is $x\%$ of the amount to be invested. 1% of it = y , 100% of it = z , or \sim .
3. The agent above mentioned spends only \$1500. How much of the remittance must he retain?
4. $1\frac{1}{4}\%$ or \$20 is a charge for buying hides. Find the base and the entire remittance.
5. When the commission is $1\frac{3}{4}\%$, the amount expended is what fraction of the remittance due? $\frac{100 \times 4}{101\frac{3}{4} \times 4} = \sim$.

In Selling :

$$\left. \begin{array}{l} \text{Amount collected or} \\ \text{Gross receipts,} \\ \text{Commission} \end{array} \right\} = \text{Base.}$$

$$\text{Commission} = \text{Percentage.}$$

$$\text{Net Proceeds} = \text{Base} - \left\{ \begin{array}{l} \text{Commission} \\ + \\ \text{Expenses.} \end{array} \right.$$

In Buying :

$$\left. \begin{array}{l} \text{Gross amount} \\ \text{expended} \end{array} \right\} = \text{Base.}$$

$$\text{Commission} = \text{Percentage.}$$

$$\text{Remittance} = \left\{ \begin{array}{l} \text{Base} \\ + \\ \text{Commission.} \end{array} \right.$$

297. — Examples.

Written.

1. \$927 is the amount sent to purchase granite and pay the agent 3%. By what must you divide \$927 to find 1% of the base? What is the agent's commission?
2. Remitted \$ x for the purchase of glassware. The commission was \$42, or $3\frac{1}{2}\%$ of \$ y .
3. Sent my agent \$1200 for purchasing wheat. His commission is 3% of the purchase, or \$ x .
4. Forwarded \$287.50, and received in return goods worth \$250. The commission at 1% would have been \$ x . Actual rate, $y\%$.
5. A person bought from my agent \$347 worth of straw. When a 2% commission and \$47 expenses have been deducted, I receive \$ x .

Supply values for x and y (dealings with selling agents):—

	Gross Proceeds.	Expenses.	Rate of Commission.	Commission.	Net Proceeds.
6.	\$ 437	0	2 %	x	y
7.	x	\$ 47.50	1 %	\$ 27.50	y
8.	\$ 250	x	y	\$ 25	\$ 200
9.	x	0	2 %	y	\$ 980
10.	\$ 1200	\$ 100	x	\$ 18	y
11.	\$ 1680	x	4 %	y	\$ 60
12.	x	0	y	\$ 13.52	\$ 437.18

298.—Examples.

Written.

1. \$ 430 is received from a sale of linen. After retaining $1\frac{1}{2}$ % commission and paying \$ 2.50 for advertising the sale, what is the balance to be remitted?
2. What value of goods can be bought on 5 % commission from a remittance of \$ 577.50, allowing \$ 24.50 for advance charges of forwarding the purchase?
3. A correspondent retains $4\frac{1}{2}$ % on the receipts of a certain sale, and after paying \$ 4.37 for carting, etc., remits \$ 200. The gross receipts include \$ x + \$ y + \$ z .
4. A dealer sends to his agent \$ 20,500, including a commission of 2 % on what the agent will spend, which is \$ 100 for insurance and sundries, and the balance \$ x for wool.

Supply values for x and y (dealings with purchasing agents):—

	Remittance.	Am't of Purchase + Freight, Storage, etc.	Rate of Commission.	Commission.
5.	\$ 595.80	x	y	\$ 6.60
6.	\$ 179.76	\$ 148.30 + \$ 27.94	x	y
7.	x	\$ 755	y	\$ 22.65
8.	x	y	5 %	\$ 9.90
9.	\$ 1293.75	x	$3\frac{1}{2}$ %	y
10.	x	\$ 684.10 + \$ 88	$2\frac{1}{4}$ %	y

299. — Problems: 1. On a bill for hardware amounting to Discount, Insurance, \$ 480, I received four successive discounts Commission. of 10 % each. What is the amount to be paid ?

2. My residence is insured for $\frac{2}{3}$ its value in the Provident Insurance Co. at $\frac{1}{2}$ %. The premium is \$ 40. What is the value of my property ?

3. My agent in Mobile bought 40,000 lb. of cotton at $9\frac{7}{8}$ ¢. His commission is $\frac{3}{4}$ % and his expenses are \$ 143.75. What shall I remit him ?

4. A real estate broker sells a farm for \$ 8000 at a 5 % commission. What are the net proceeds of the sale, and what is his commission ?

5. I can buy 1000 bbl. of oil at \$ 1.12 $\frac{1}{2}$ with 3 % off in 30 days, or 5 % for cash. What shall I save by accepting the better offer, money being worth 6 % ?

6. The estimated loss of property at a large fire was \$ 275,000. The insurance received was \$ 180,000. How much must be taken in new risks at an average of 2 % to cover this loss to the underwriters together with \$ 5000 expenses ?

7. I receive from my agent in London a draft for \$ 3860, the net proceeds of a sale of flour at $3\frac{1}{2}$ % commission. What were the gross proceeds ?

8. A drummer earns \$ 2500 annually. \$ 1000 is a guaranteed salary; the remainder is his commission of 5 %. What are his annual sales ?

9. A broker negotiates a loan of \$ 6500 on a real estate mortgage. His commission of 2 % and the expenses of examining title, etc., are \$ 72.37. What does the mortgager receive ?

10. Bought 1000 gross of screws at 27 cents, with a discount of 15, 10, and 5. I sold the lot at cost plus 30 %. What was my gain ?

300.—Promissory
Notes.

May 17, 1895, Edward Rich lends to Thomas Poor \$ 180 to be repaid when the lender asks it, together with interest at 5% ; as evidence of the loan and security for its payment the lender receives from the borrower a *promissory note* like the following :

\$180 —

Manchester, May 17, 1895.

On demand, after date, I promise to pay to the order of _____ Edward Rich _____
_____ One Hundred Eighty _____ Dollars
with interest, at five per cent.

Value received.

Thomas Poor.

1. Who is the *maker* of this note ; i.e. the promissor ?
2. Who is the *payee*, or the one to whom promise of payment is made ?
3. What is the *face* of the note, or the sum named in it ? 4. When is the note payable ?
5. In what way does the maker acknowledge the receipt of the note, or its equivalent ?
6. Why is such a note as this called a *demand note* ?
7. Why is it called an *interest-bearing note* ?

Promissory notes are a kind of property, and may be bought and sold like other property.

8. Show that on May 17, 1896, the above note is worth \$ 189 to the owner or holder. What will the owner gain or lose by selling it for \$ 200, 2 years from date ?

A

Edward Rich.

B

*Pay to the order of
Henry Hall
Edward Rich.*

9. Whenever the payee of a note transfers it to the ownership of another person he first *indorses* it; that is, he places his signature on the back of it. What is an *indorser*? An *indorsement*?

10. The payee of a note may indorse it in blank as in A, or he may make a special indorsement, as in B.

A *blank indorsement* makes the note payable to the holder. A *special indorsement* makes it payable to the person named by the indorser as payee. Copy and indorse the note of Thomas Poor.

Every indorser of a note is responsible for its payment unless the words "*without recourse*" precede his signature.

11. The holder of the note on the preceding page demands payment of Mr. Poor Aug. 27, 1897. What is the amount then due?

12. If the words "one year," "four months," "sixty days," etc., were substituted for "on demand," when would the note be payable?

NOTE. — In some states three extra days after the expiration of the time named in the note are allowed the maker for its payment. They are called *days of grace*. Interest is exacted, however, for the days of grace. [See § 310.]

13. If the note were a four months' note, at what date would it be payable without grace? With grace? When, if it were a 2 months' note? A 6 months'?

Notes *mature*, or are legally payable, on the day when the time named in them expires, or on the third day thereafter, when *grace* is allowed.

All notes that contain the words "with interest" draw interest from date unless otherwise specified. All other notes draw interest from maturity. When no rate of interest is specified, the legal rate is understood. [See Appendix, p. 15.]

301. — Promissory Notes.*Written Exercises.*

Make interest-bearing notes answering to the following conditions, and compute the amount due at settlement.

In finding the day of maturity, allow three days of grace.

	Date.	Face.	Time to run.	Payee.	Rate.	Settled.
1.	3/17, '95	\$ 240	On demand	A. P. Rice	6	9/14, '96
2.	8/12, '96	\$ 800	One year	E. F. Foss	7	12/18, '99
3.	4/21, '96	\$ 725	Four months	Wm. Ward	5	Maturity
4.	6/15, '93	\$ 1800	Six months	Thos. True	4	9/21, '96
5.	1/19, '95	\$ 610	Two years	A. M. Bates	8	12/25, '97
6.	2/24, '96	\$ 280	On demand	R. E. Nye	4½	7/16, '98
7.	1/1, '97	\$ 75	Sixty days	E. B. Hale	12	Maturity

8. Write a note without signature, making yourself payee, and transfer the note, properly indorsed, to your teacher.

9. A note matures Aug. 16, 1897. Its face is \$ 1200, and it has been running at 4% since May 12, 1894. What is paid at settlement?

10. Draw up a note in which Thos. Talbot hires \$ 1700 of Samuel Strong, at 4%, agreeing to make payment on demand. Demand for payment is made May 14, 1895, the note being made March 12, 1893. What amount pays the debt?

302. — Partial Payments of Notes.

1. I pay \$ 80 on a note whose face is \$ 150. What part do I pay?

2. I pay 30% of a note of \$ 1500. What is the partial payment?

3. A note of \$ 300 draws 10% interest. What amount would discharge the note at the end of the first year? Suppose that instead of the note being paid in full at that time a partial payment of \$ 100 were made, what would then be due?

4. Would the \$ 100 pay all the interest due? How much of the face or principal would it also pay?

5. How much of the original \$ 300 does the maker of the note continue to keep? On what sum, therefore, should he pay interest?

6. If the remaining \$ 230 should be used another year, the interest on it at 10% would be \$ x , and the amount due would be \$ 230 + \$ x , or \$ y .

7. If another partial payment of \$ 100 should then be made, a remainder of \$ y - \$ 100, or \$ z , would still be left at interest in the hands of the maker of the note.

8. Give the values of x , y , and z in the solution of the following problem:

On my note, payable to you for \$ 300, I make a partial payment of \$ 100 at the end of each year for three years. What is then due you, 10% interest being charged?

SOLUTION.

I. Of your money I have for use	\$ 300
For a year's use of it at 10% I owe	x
At the end of the year I owe you	<u>\$ 330</u>
I make a partial payment to you of	<u>100</u>
II. This leaves a balance for me to use of	\$ 230
For a year's use of this sum I owe you	<u>23</u>
At the end of the second year I owe you	\$ y
I make a second partial payment of	<u>100</u>
III. I now have of your money only	\$ z
For a year's use of this sum I owe you	<u>15.30</u>
I owe you at the end of the third year	\$ 168.30
If I pay you a third	<u>100</u>
IV. I shall still owe you	\$ 68.30

9. I borrow \$ 500, giving my note at 6%. At the end of each year, for two years, I pay \$ 100. How much remains due?

10. A note for \$ 800 draws 5% interest and is dated May 1, 1894. May 1, 1895, \$ 200 is paid; May 1, 1896, \$ 100 is paid. What is due May 1, 1897?

11. Two partial payments of \$ 400 each are made on a \$ 1000-note, dated Aug. 20, 1895. The rate of interest is 5%. The first payment is made Aug. 20, 1897, and the second, Aug. 20, 1899. What amount remains due?

303. — Partial Payments of Promissory Notes. To find Amount Due.

When partial payments of a note are made, the owner records the amount and date of each on the back of the note.

To find the amount due on the following note Dec. 31, 1901:

\$ 720 — Springfield, Aug. 14, 1895.
 On demand after date I promise to pay to
 the order of _____ Henry Howard & Co. _____
 _____ Seven Hundred Twenty _____ Dollars
 with interest at six per cent.
 Value received. Thomas F. Powell.

INDORSEMENTS ON BACK OF NOTE.

Received on the within note:
 Dec. 26, 1896, \$ 200
 Sept. 14, 1899, 175
 Dec. 31, 1900, 400
 Settled, Dec. 31, 1901.

The Supreme Court of the United States has decreed that, —

Partial payments of notes must first be used to cancel the interest due. Any balance remaining may be used to lessen the principal.

1. Who puts these indorsements on the note? 2. Will any other receipt be requested for the \$200 paid Dec. 26, 1896, and if so, by whom? 3. What needs to be done before the note can be transferred to a third person?

PARTIAL PAYMENTS.

185

SOLUTION.

From date of note to 1st payment.

8/14, '95 to 8/14, '96 = 1 yr.	\$ 0.06	\$ 720
8/14, '96 to 12/14, '96 = 4 mo.	0.02	0.082
12/14, '96 to 12/26, '96 = 12 d.	0.002	1 44
Int. of \$1	\$ 0.082	57 6
Interest due when 1st payment is made		\$ 59.04
Face of note, or 1st principal		720.
Amount due at time of 1st payment		\$ 779.04
1st payment, which cancels interest due, and more		200.
Remainder which continues to draw Int.; 2d Prin.		\$ 579.04

From 1st payment to 2d payment.

12/26, '96 to 12/26, '98 = 2 yr.	\$ 0.12	0.163 $\frac{1}{8}$
12/26, '98 to 8/26, '99 = 8 mo.	0.04	9650
8/26, '99 to 9/14, '99 = 19 d.	0.003 $\frac{1}{8}$	1 73712
	\$ 0.163 $\frac{1}{8}$	34 7424
Interest due at time of 2d payment		57 904
2d principal		\$ 94.48002
Amount due at time of 2d payment		579.04
2d payment cancels interest and part of principal		\$ 673.52
Remainder which still draws Interest; 3d Prin.		175.

From 2d payment to 3d payment.

9/14, 1899 to 9/14, 1900 = 1 yr.	\$ 0.06	0.077 $\frac{5}{8}$
9/14, 1900 to 12/14, 1900 = 3 mo.	0.015	6)249260
12/14, 1900 to 12/31, 1900 = 17 d.	0.002 $\frac{5}{8}$	41543
	\$ 0.077 $\frac{5}{8}$	3 48964
Interest due at time of 3d payment		34 8964
3d principal		\$ 38.80147
Amount due at time of 3d payment		498.52
3d payment cancels interest and part of principal		\$ 537.32
Remainder which still draws Interest; 4th Prin.		400

From 3d payment to settlement.

12/31, 1900 to 12/31, 1901 = 1 yr.	\$ 0.06	Int.	\$ 8.2392
4th principal			137.32
Amount due at settlement			\$ 145.55

4. How much interest was cancelled by the first payment? How much of the principal? 5. How much of the third payment was applied to lessen the sum on which Powell was paying interest?
6. What interest is due Sept. 14, 1899? At settlement?

304. — Partial Payments too Small to cancel Interest Due.

Commonly the partial payment of a note will not only cancel all the interest due, but will also pay a portion of the principal, and thus reduce the sum at interest. It sometimes happens, however, that the payment is too small even to cancel the accrued interest. In such cases interest must not be used to increase the principal, which must never represent more than the money actually and previously due to the creditor, and in use by the debtor.

To find what is due at settlement on a note of \$ 600 dated Aug. 15, 1893, drawing 6%, and indorsed as follows:

Dec. 15, 1894, \$ 25.

Sept. 15, 1896, \$ 200.

Settled Aug. 15, 1898.

SOLUTION.

From date to 1st payment.

8/15, '93 to 8/15, '94 = 1 yr.	\$ 0.06	
8/15, '94 to 12/15, '94 = 4 mo.	<u>0.02</u>	\$ 600
	\$ 0.08	<u>0.08</u>

The payment of \$25 will not pay the accrued interest, \$48.00. Hence we compute the interest —

From date to 2d payment.

8/15, '93 to 8/15, '96; 3 yr.	\$ 0.18	
8/15, '96 to 9/15, '96, 1 mo.	<u>0.005</u>	\$ 0.185
Interest of \$ 1	0.185	<u>600.</u>
Interest due at 2d payment		\$ 111.0
Face of note on interest		<u>600.</u>
Amount due at 2d payment		\$ 711
Sum of 1st and 2d payments (\$ 25 + \$ 200)		<u>225</u>
New Principal on Interest		\$ 486

From 2d payment to settlement

9/15, '96 to 9/15, '97, 1 yr.	\$ 0.06	
9/15, '97 to 8/15, '98, 11 mo.	<u>0.055</u>	2 430
	0.115	<u>4 86</u>
Interest due at settlement		\$ 55.890
Principal due at settlement		<u>486.</u>
Amount due at settlement		\$ 541.89

1. Why do we not add the first interest, \$48, to the principal, subtract the payment, \$25, and compute the interest on the remainder, \$623, as in the example on page 185? 2. Will the two payments together cancel the interest due? How much of the principal besides? 3. How must we proceed when a payment will not cancel the interest due?

4. Show why the debtor might object to the arrangement proposed below:

I loan Mr. James	\$ 600
A year's interest at 5% is	30
He then owes me	<u>\$ 630</u>
He pays me	20
I ask him to pay me interest on	<u>\$ 610</u>

5. What will settle a 5% note for \$1000 Aug. 1, 1898, on which \$50 was paid 2 years after date, and \$500 4 years after date? The date was Aug. 1, 1890.

6. What was due May 1, 1899, on a note for \$3000, on which \$100 was paid at the end of each year for 3 years? The rate was 4%, and the note was given May 1, 1895.

7. Cyrus Drew gave Frank Watson his note for \$800 at 3% interest, Aug. 17, 1892. Dec. 23, 1894, he paid \$300, and May 29, 1896, he paid \$40. What was due six years from date?

305.—Promissory

Notes: Partial

Payments.

Written exercise.

1. A note of \$300 drawing 6% interest and dated May 15, 1892, has a single indorsement of \$200 paid Aug. 15, 1894. What amount will settle the note June 21, 1896?

2. Oct. 19, 1894, Silas Deane pays \$275 on a \$1000 note which he gave Charles Dole May 19, 1892. The note draws 4% interest and is settled Feb. 19, 1896. What is the amount due?

3. A demand note for \$4000, drawing 3% interest, has a \$1000-payment made upon it at the end of the first and of the second year. What will settle it at the end of the third year?

4. The face of a 5% note is \$1200. It is dated Jan. 1, 1894. It has the following indorsements:

May 13, 1895, \$ 400.

Aug. 25, 1897, 300.

What is due at settlement, Aug. 25, 1899?

5. John Haley pays \$400 July 20, 1896, on a note for \$625, dated Oct. 20, 1895. Dec. 36, 1896, he pays \$120. What is due on the note four years from date, the rate of interest being 9%?

6. Horace Swan buys a 160-acre farm in Idaho for \$17 an acre. He gives a 4% note for $\frac{3}{4}$ the cost and pays \$680 cash. At the end of 2 years he pays one half the face of the note, and at the end of 3 years he pays one half the remainder. How much is still due six years from the date of purchase?

306. — Customs or Duties. The expenses of the national government are paid from —

I. The Internal Revenue, chiefly taxes of a fixed sum on the right to make or sell spirituous liquors, tobacco, etc.

II. **Customs or Duties**, which are taxes on goods imported from foreign countries.

1. What need of money has the government? Mention several articles commonly imported.

The **Free List** includes all classes of goods that are exempt from duty. The **Tariff** is the list of dutiable goods with the rate assessed on each kind.

Ad valorem duties are a percentage on the cost of the goods where they were bought, as shown by the **Invoice**. **Specific** duties are fixed according to number, quantity, weight, etc.,—not according to value.

2. Find the duty at 15% on 200 T. of coke invoiced at \$1.50 a ton. Is the duty specific or ad valorem?

NOTE.—Invoices are made out in the money of the country where the goods were bought. When changed to United States money, the duty is computed on the nearest dollar, 50 cents counting as \$1.

Imported goods must be brought to a **Port of Entry**, a place where the government has established a **Custom-house** with officers for the collecting of duties.

3. What is meant by *smuggling*?

4. A gross of leather pocket-books invoiced at \$ 12.50 a dozen pays 30% ad valorem, or \$ *x*.

Tare, Leakage, and Breakage are allowances for boxes, bags, etc., used in packing, and for liquids lost from barrels or bottles, etc.

5. A dealer withdraws from the custom-house 4 T. (2240 lb.) of rice; 8% is tare. On what weight must he pay duty? What will it amount to at $1\frac{1}{2}\text{¢}$ a pound? 6. Is this a specific or an ad valorem duty, and why?

7. Define *gross weight* and *net weight*.

8. An importation of velvet invoiced at 5181.35 francs weighs 400 lb., 10% tare. Which would be more, a duty of \$ 1.50 a pound, or 50% ad valorem?

9. 20 bbl. vinegar of 42 gal. each pay a specific duty of $7\frac{1}{2}\text{¢}$ a gal., leakage 30 gal. Find the whole duty and the per cent of leakage.

10. The duty on cut nails is $22\frac{1}{2}\%$ ad valorem. \$183.75, the cost including duty on a certain importation, is what per cent of the amount of the invoice? They were invoiced at \$ *x*.

11. In 1888 the duty on brussels carpets was 30¢ a sq. yd., and 30% ad valorem. Find the total cost of 200 running yards invoiced at 6s. a yard, $\frac{3}{4}$ yd. wide.

12. The gross cost of a lot of calf-skins is \$ 740, including \$ 20 freight and \$ 120 duty. Find the ad valorem rate.

13. Gross cost \$ 1315, freight \$ 50, duty \$ 165, rate *x*.

14. If the cost of gloves is doubled by importing, what will be the profit on a pair invoiced at 60 fr. a dozen, and sold for \$ 2.25?

307. — Taxes.

The expenses of states, counties, cities, and towns are met by levying taxes annually on adult male citizens, and on all owners of property.

Oral. — 1. Why are some persons taxed more than others?

2. Mention several ways in which taxes collected in your city or town are expended. 3. What need have county and state to raise money by taxes? 4. Find the amount of poll taxes in your city or town.

Property is divided into two classes for taxation: **Real Estate**, regarded as immovable, as land and buildings, including mines, quarries, forests, railroads, etc.; and **Personal Estate**, which is usually movable.

5. Give examples of valuable personal property. 6. How large a tax on property is assessed in your city or town?

Suppose a town requires a certain sum of money for the expenses of a coming year. Officers, called **Assessors**, first estimate the value of the property to be taxed, and then assess each owner in proportion to what he has.

7. If the amount to be raised is \$20,000, and the poll tax amounts \$1600, how large a property tax must be assessed?

Written. — 8. Assessors find the value of real estate in their city to be \$15,000,000, and of personal estate \$5,000,000. They assess a tax for state, county, and city. If the rate is \$15 tax on \$1000 valuation, the total to be collected will be x .

9. The rate is $12\frac{1}{2}$ on 1000. What will be assessed on \$4,000,000 valuation? A man who pays \$125 is assessed on how much?

10. A man's tax, including \$2 poll, is \$122. The rate is $11\frac{1}{2}\%$ or x on \$1000, y on \$1. His property is valued at how much?

11. 900 is what per cent of 60,000? If \$12 is the tax on \$1000, what is the tax on \$1? what the rate per cent?

12. Find the rate when the tax on \$120,000 is \$1770. On \$100 the tax would be x .

13. If $1\frac{7}{10}\% = x$ per thousand, how much can be raised on a valuation of \$15,492,500?

14. Valuation, \$40,000,000; property tax, \$640,000; rate, x .

15. Property taxes in a certain town are \$25,100; an estate of \$15,000 pays \$195. Find the rate and the total valuation.

16. A person is assessed in one town on \$3500 real property, where the rate is $1\frac{1}{4}\%$; in another on \$8500, rate \$14.50 per \$1000; in a third on \$5000 personal estate, @ $1\frac{1}{2}\%$ on a dollar; besides a \$2 poll tax. What does he pay in all?

17. Valuation, \$11,000,000; property tax required, \$121,000. Each dollar must pay a tax of x . If the cost of collecting the tax is 1%, and the poll tax \$1100, what are the net proceeds?

308.—Compound Interest. 1. Jan. 1, 1895, Mr. D. borrows \$500 of Mr. C., agreeing to pay 6% interest at the end of every year. How much interest is due at the end of the first year?

2. Who is entitled, under the agreement, to the use of this \$30 interest for the second year, the debtor or the creditor? 3. If the debtor uses the overdue interest during the second year, instead of paying it to Mr. C., is it just that he should pay for its use? 4. On how much, then, should the debtor pay interest for the second year, including both principal and overdue interest?

Interest reckoned on both the principal and the overdue interest added to the principal as often as due is **Compound Interest**.

NOTE.—When interest is added to the principal, or “compounded,” it is done yearly, unless otherwise stated, as half-yearly, quarterly, or oftener.

5. What is due on a debt of \$600, which has been standing 2 yr. 6 mo. 18 d., interest at 6% compounded annually? Explain the following process.

Principal used for 1st year	\$ 600.
Interest due at end of 1st year	36.
Principal used for 2d year	\$ 636.
Interest due at end of 2d year	38.16
Principal used for 6 mo. 18 d.	\$ 674.16
Interest due at end of 6 mo. 18 d.	22.25
Amount due at settlement	\$ 696.41
First principal	600.
Compound interest	\$ 96.41
Simple interest would have been	91.80
Interest on all overdue interest	\$ 4.61

6. Find the compound interest of \$ 500 for 3 yr., at 5%.
7. What is the compound interest, at 4%, of \$ 2000 for 2 yr. 6 mo. ?
8. Interest compounds semiannually on \$ 400, at 8% a year, for 1 yr. 6 mo. What is the amount due ?
9. Interest compounds quarterly for 9 mo., at the annual rate of 4%. What is due, the principal being \$ 1000 ?

NOTE. — Compound interest is generally computed by means of tables. [See p. 14 of Appendix.]
The collection of compound interest on notes and debts cannot be enforced, even though agreed upon.

10. Principal, \$ 480 ; time, 1 yr. 3 mo. 15 d. ; annual rate, 6% payable half-yearly.

309. — Savings-

Banks Deposits.

1. What advantage does a thrifty person receive from depositing his savings in a bank ?
2. What use do savings-banks make of the money received from deposits ?
3. How is their profit made ?
4. In 1894 there were in this country 1006 savings-banks, with 4,739,194 depositors, and deposits amounting to \$ 1,739,006,705. What was the average to a depositor ?

NOTE. — Savings-banks allow compound interest on all deposits remaining for a full interest term. They generally compound the interest or declare dividends semiannually.

Find what may be withdrawn from the Franklin Savings-bank by depositors under the following conditions : —

5. James Swan ; \$ 400 ; 4% ; 1 yr. 6 mo. ; semiannual dividend.
6. Edw. Wise ; 250 ; 4% ; 2 yr. ; semiannual dividend.
7. Sam'l Rand ; 800 ; 2% ; 9 mo. ; quarterly dividend.
8. John True ; 325 ; 4% ; 7 yr. ; semiannual dividend.

[Use table in Appendix for Exercise 8.]

310. — Promissory

Notes: the Day of Maturity.

1. At the present time, August, 1895, in all but ten states and territories the law allows the maker of a note *three days of grace* for its payment in addition to the time named in the note. Why is this ? [Appendix, p. 17.]

NOTE. — The ten states that allow no grace are Vermont, Connecticut, New York, New Jersey, Illinois, Wisconsin, Idaho, Oregon, Utah, and California.

2. A note matures, or is legally due, on the day when the time named in it expires; or, on the third day thereafter when grace is allowed. What if the three days expire on a Sunday? [Appendix, p. 17.]

3. When a note is payable "— *months* after date," calendar months are understood, and the note matures (without grace) on the corresponding day of the month, or on the last day of the month when there is no corresponding day. Show that a 2-months note, dated either Dec. 28, 29, 30, or 31, will mature without grace Feb. 28 in a common year, or Feb. 29 in a leap year; and that if grace is allowed, it matures March 3.

4. When a note is payable "— *days* after date," the exact number of days mentioned must be used in finding the day of maturity, plus three days if grace is allowed. Show that a 60-days note dated Feb. 15, 1895, matures Apr. 16/19.

Find the date when notes mature, if—

Payable in	Running	Dated	Payable in	Running	Dated
5. Mass.	2 mo.	Dec. 31	11. Vt.	6 mo.	Aug. 31
6. N. Y.	60 d.	July 10	12. Cal.	60 d.	Feb. 17
7. Vir.	3 mo.	Feb. 28	13. Ill.	90 d.	Aug. 17
8. Penn.	90 d.	Nov. 30	14. N. J.	30 d.	June 7
9. Wis.	4 mo.	July 31	15. Minn.	4 mo.	Oct. 31
10. N. H.	30 d.	Jan. 20	16. Conn.	2 mo.	Feb. 29

311. — Bank Discount: an Allowance to a Bank for the Prepayment of a Note.

[Review pages 180, 181.]

1. What responsibility does the indorser of a note assume?

2. What risk does a bank take in buying notes made by reliable persons and properly indorsed by the payees?

3. Show that banks cannot afford to pay their face value for such notes.

\$ 500 _____ Rochester, N. Y., Aug. 12, 1895.
 Two months after date, I promise to pay to the
 order of _____ Henry Morgan _____
 _____ Five Hundred _____ Dollars
 at the Second National Bank.
 Value received.
 James Shaw.

4. Who must indorse this note before it passes into the ownership of a third person? 5. Who is to pay the note? Where? 6. In what case may Mr. Morgan be called upon to pay it?

7. When does it mature? 8. Is grace allowed? 9. What is its value at maturity? 10. Why is it not worth \$ 500 at date?

11. Aug. 12 the holder of this note offers it for sale to the Second National Bank. The bank officers are satisfied that either Mr. Shaw or Mr. Morgan will pay the note at maturity. The current rate of interest is 6%. According to the custom of banks they compute the interest at this rate on the face of the note for 2 months, the time for which the bank will lose the use of its money. They find this interest to be \$ *x*. They deduct it from the face of the note and pay the seller the remainder, \$ 495. What occurs in connection with this note on October 12th?

The allowance or discount made to the bank by the holder of the note for its payment before maturity is the **Bank Discount**.

The sum received for the note from the bank is the *proceeds* or *avails* of the note.

12. When does the bank get back its \$ 495? When does it receive the \$ 5 paid for the use of its money? If it had lent \$ 495 for 2 months at 6% interest, how much would it have received? The charge of \$ 5 was 1% of *x*, though the money it lost the use of was *y*.

Should Mr. Shaw fail to pay the note before the closing of the bank on the day of maturity, immediate notice is given to the indorser by a notary public, and he is held for payment.

13. I sell a 4-mo. note for \$ 600 payable without grace, to the Marine Bank. Money is worth 5%. What discount from the face do I make to the bank? What are the proceeds of the note?

14. A note for \$1200 is bought by a bank. The money will be repaid in 90 days, with grace. The rate of discount is 4%. What does the bank pay for the note? How much interest will it receive for the use of its money? When will it receive this interest?

15. What is interest? When I sell a note to a bank, is the allowance that I make reckoned on the money that I get to use, or on more or less? How does the bank discount differ from interest? In bank discount is the cost of the accommodation paid in advance, or is it paid when the money used is paid? Who pays the bank discount?

312. — Bank Discount: Most of the notes discounted at banks, or
Notes discounted by brokers or others, are given for short
at Date. times, 30, 60, 90 days, or 2, 3, 4, or 6 months.

NOTE. — Compute bank discount as if it were interest on the face of a note for the time the bank's money is used; and take the shortest method. Allow three days of grace if such is the lawful custom in your state. Answers are given both with grace and without it.

1. What will a bank pay me for a note of \$800 payable in 3 mo., the rate being $4\frac{1}{2}\%$?

2. How much of its money will a bank permit me to use in return for a note of \$720? Money is worth 5%, and the note runs 60 days.

3. What allowance shall Mr. Strong make to the Exchange Bank for its prepayment of a 4-mo. note for \$875 at a discount rate of 3%?

4. A \$600 note dated July 5 is to be paid Sept. 5. Required the proceeds if discounted at 5%.

The *term of discount* is the time for which the bank's money is used. It extends from the day of discount to the day of maturity.

Find the bank discount and the proceeds of notes made under these conditions, and discounted at date:—

	Face.	Time to run.	Rate of Discount.		Face.	Time to run.	Rate of Discount.
5.	\$525	30 d.	3%	11.	\$842	4 mo.	2%
6.	917	3 mo.	4½%	12.	872.50	1 mo.	2%
7.	324	6 mo.	8%	13.	91.82	2½ mo.	2½%
8.	800	90 d.	7½%	14.	6000	4 mo.	4½%
9.	960	60 d.	4%	15.	4297	90 da.	5%
10.	721	75 d.	3½%	16.	845	6 mo.	7%

313. — Bank Discount: When a note is discounted *at date*, the time to Find the Term named in the note (with grace or without, as the case may be) is the term of discount, Discount. showing how long the discounter's money is used. A note, however, may be sold or discounted at any time between date and maturity.

NOTE. — The method of reckoning the time from the day of discount to the day of maturity is not uniform among banks.

The two more common methods are the following:—

I. When the time is less than two months, the exact number of days is counted; but when it is more than two months, the time is reckoned in months and days as on p. 165.

II. The exact number of days is taken in all cases.

In general, the latter method is used, when notes are large, being to the advantage of the bank.

Thus, a 4-mo. note dated June 30 matures in Massachusetts Nov. 2. If discounted July 10, interest may be computed either for 115 days or for 3 mo. 23 da., the difference being \$3½ in favor of the bank on a \$10,000 note.

The student should conform to the custom of his own vicinity. Answers to examples are given for both methods.

The term of discount extends from the day of discount to the day of maturity.

Find date of maturity and term of discount by each method.

Date of Note.	Time to run.	Day of Discount.	Date of Note.	Time to run.	Day of Discount.
1. May 8	60 d.	June 10	5. April 4	4 mo.	June 8
2. Nov. 17	90 d.	Dec. 14	6. Jan. 25	30 d.	Jan. 29
3. Aug. 5	2 mo.	Aug. 29	7. Sept. 19	4 mo.	Nov. 10
4. Mar. 17	3 mo.	May 1	8. Feb. 10	90 d.	Mar. 6

314. — Bank Dis-
count: Notes dis-
counted after Date.

1. I get a 3-mo. note discounted 27 days after date. What is the term of discount?
2. What if it were sold 24 d. before maturity?

3. A 6-mo. note for \$450 dated Aug. 11 is discounted Sept. 4 at 6%. Reckon months and days, with grace. Find proceeds.

4. A 90-d. note for \$1000 is discounted 37 d. before maturity. Find the proceeds, the rate being 5%.

5. A 5-mo. note for \$800 was sold at 3% discount 80 days after it was made. Proceeds without grace?

6. A 90-d. note for \$3000 dated Milwaukee, May 17, 1898, was purchased by the Traders' Bank June 24. What did it yield the holder, money rating at $4\frac{1}{2}\%$?

7. 60 days; \$450; 4%; date June 15; day of discount July 1. What does the borrower have for immediate use out of his note?

8. A 3-mo. note for \$720 is sold in Trenton 7 days after its date at a discount of 9%. What does it yield? How much more or less would it have yielded in Chicago?

Using these data, find the proceeds of notes. Conform to the custom of your own vicinity in allowing grace and finding the term of discount.

Date.	Face.	Time.	When discounted.	Rate.
9. Aug. 21, '96	\$800	90 da.	Oct. 1	5%
10. May 7, '97	\$1218	4 mo.	31 d. after date	7%
11. Feb. 21, '98	\$249	60 d.	Mar. 11	$4\frac{1}{2}\%$
12. July 12, '99	\$1728	6 mo.	2 mo. 11 d. bef. maturity	$3\frac{1}{2}\%$
13. Nov. 16, '95	\$189.75	2 mo.	Dec. 2	4%
14. Dec. 20, '97	\$278	3 mo.	16 d. after date	8%
15. Aug. 17, '97	\$350	72 d.	At date	9%
16. Sept. 12, '98	\$760	5 mo.	Oct. 22	3%
17. Aug. 19, '99	\$5000	60 da.	Sept. 19	$2\frac{1}{2}\%$

315. — Bank Discount on Interest-bearing Notes.

Notes that are discounted commonly bear no interest. When an interest bearing note is discounted, *the maturity value of the note must be made the base of discount.*

\$800---

Hartford, Ct., Nov. 12, 1895.

Six months after date, I promise to pay to the order of ----- Edna French -----

----- Eight Hundred ----- Dollars with interest at four per cent, at the Etna Bank.

Value received.

Mary Wright.

1. Is this note entitled to grace? 2. When does it mature? 3. How much must Mrs. Wright pay Mrs. French on that day to carry out her agreement? 4. Will transferring it to a third person affect the amount to be paid? 5. What is the maturity value of the note?

6. If this note is discounted 4 mo. before maturity at a discount rate of 6%, what will be the proceeds? Explain the process.

Process.

102% of \$800 = \$816, maturity value; 98% of \$816 = \$799.68, proceeds.

7. A 4-mo. note for \$1200, drawing 9% interest, is discounted at 4% 3 months before maturity, without grace. Required the proceeds.

8. Face, \$500; rate of interest, 5%; time to run, 60 d.; term of discount, 48 d.; rate of discount, 3%. Proceeds?

9. A 90-d. note for \$720, dated May 15, 1896, drawing 8% interest, is discounted June 12, 1896. Proceeds?

10. Write an interest-bearing note, and find the proceeds.

316. — Stocks.

When a business requires more capital, or money invested, than is furnished by a few partners, a **Stock Company** or **Corporation** may be formed with any number of partners, who choose a **Board of Directors** to conduct the business as one person.

The capital or stock furnished is then divided into equal shares of \$100 each, unless otherwise stated, as \$10, \$50, \$1000, etc.

1. A corporation is formed with capital \$250,000, shares each \$100. How many shares are there? If there were only 500 shares, each would be \$x.

Each owner or **Stockholder** receives a **Certificate of Stock** bearing the seal of the corporation and giving the number and value of his shares. Shares are often sold, so that the shareholders are constantly changing.

Incorporated under the Laws of the State of New York.

No. 212.

12 shares.

EASTERN PACKING COMPANY.

This certifies that Julius Holm
is entitled to Twelve shares of One
Hundred Dollars each of the full paid Capital Stock of
the

EASTERN PACKING COMPANY.

Transferable only on the books of the company in person or by attorney
upon surrender of this certificate.

New York, July 12, 1895.

Carl Jacobs,
Secretary.

M. S. Kendall,
President.

2. From whom does the Eastern Packing Company get the right to carry on business as a corporation?

3. Who is the owner of the certificate shown at page 199 ?
4. What is the value of the shares represented ?
5. A person owns 800 shares in a gas company valued at \$ 50 each. The total capital is \$ 80,000. If each share has a vote, what part of the whole would he control ?

The income or profit on the business is called a **Dividend** because divided and paid yearly, half-yearly, or quarterly, as a certain % on the **par value** or **face value** of each share.

6. Compare the rate of income from a \$ 1000 share paying 6% annually, and a \$ 100 share paying 2% quarterly.

317. — Stocks :

Market Value.

The **Market Value** of stocks as quoted in the newspapers varies from day to day. It is **above par** or **below par** accordingly as the dividends are large or small, and the business prosperous. The stock is then said to be at a **premium** or at a **discount**.

NOTE. — Look up stock quotations in some newspaper.

1. If a share is quoted at 105, the premium is \$ x . It sells for $x\%$ above par.
2. When a share sells for 15% below par, the stock is quoted at what price ?
3. Find the cost of 25 \$ 50 shares of mining stock offered at 10% discount.
4. Sold 30 shares at 108; with the proceeds bought 10 at 112 and 20 at 105. Balance = x .
5. Bought insurance stock at 60% premium; paid \$ 9600. How many shares did I get ?
6. If 17 shares cost \$ 1725.50, one cost x , and the premium is y ?
7. What is the annual income on 37 shares paying $2\frac{1}{2}\%$ quarterly ?
8. Suppose a person buys a share of bank stock at par. What does he pay ? 9. If it yields 6% dividend, what rate of income does he get on his investment ?

10. Suppose he buys a share of railroad stock that yields 9%. If he pays the market price of \$ 150, what rate of income will he get on his investment?

11. If 23 shares are bought at 150, they cost x . The dividend is 6%. $\frac{6}{100} \times 23 \times \$100 = y$. y is what per cent of x ? The rate of income is what?

15. Sometimes an **Assessment** is levied on the stockholders to pay the debts of the company. If the capital stock is \$ 200,000, how much must be assessed on each share to pay a debt of \$ 10,000? An owner of 40 shares would have to pay \$ x .

318.—Bonds.

When corporations, or national, state, and city governments borrow large sums of money, they usually give a series of **Bonds**, or promissory notes for one or more hundreds or thousands of dollars each, and to run several years at a fixed rate of interest.

Whenever a payment of interest is due, it is usually collected by detaching from the bond a **Coupon**, or small certificate of the amount of interest then due, which is paid by the treasurer of the corporation or government.

If interest and principal are not paid, the bondholders may lawfully take the property of those who gave the bonds.

One of several interest coupons attached to a bond.

THE NORTHERN LOAN ASSOCIATION

— Of St. Paul, Minnesota —

Will pay to bearer at the office of
the Company...Thirty five...Dollars
on the...15...day of...April...1899,
being interest on coupon bond No. 137.

Edward James,

Secretary.

1. If such a coupon is attached to a 7% bond, what is the face value of the bond?

2. When is the next payment of interest due?

3. Where can it be collected, and by whom?

4. If the bonds sell at 140, what would this one cost? 5. What would the annual rate of income be?

6. My $4\frac{1}{2}\%$ bonds yield me \$180 annually. What is their par value? 7. But I get only 3% on what I paid for them, when they were quoted at x .

8. Paid \$1,770 for 16 \$100-bonds and ordinary brokerage, $\frac{1}{8}\%$ on par value. They were quoted at x .

9. \$9721.25 was the cost, including brokerage, of some bonds that were at a discount of $3\frac{7}{8}$. They were quoted at x . What was the face of the bonds bought?

10. I bought 10 \$500-bonds paying 4%, for \$4912.50, including brokerage. The market value was x .

11. A U. S. 4-20 bond (paying 4%, and maturing in 20 years) yields $3.478 + \%$ on the investment. Find the premium in dollars.

319.—Stocks.

As shown in the example on page 199, stocks and bonds may be registered on the books of the company or government issuing them, so that they cannot change owners except in writing and at the treasurer's office.

1. Under what circumstances may a stockholder get no dividend for several years? 2. How will this affect the value of the stock? 3. Why is it that the market price of bonds does not vary so much as that of stocks?

4. April 1, 1895, the U. S. government had \$100,000,000 of 5% bonds outstanding, payable Feb. 1, 1904. If these bonds are not paid till maturity, how much will the government pay out in interest on them?

5. When \$50-shares of mining stock are quoted at \$5, they are at a discount of $x\%$. 6. When telephone stock sells at 230, it is at a premium of what per cent?

Trading in stocks and bonds is usually done through **Brokers**, as agents, who charge $\frac{1}{8}\%$ or more on the face value of these securities.

7. When selling, does the brokerage increase or diminish the proceeds? How does it affect the cost when buying?

Find the cost, or proceeds, after paying brokerage.

Number.	Quoted at	Brokerage.
8. In buying 27 shares	105	$\frac{1}{8}\%$.
9. In buying 100 shares	97	$\frac{1}{8}\%$.
10. In selling 4 \$ 1000-bonds	$100\frac{1}{2}$	$\frac{1}{8}\%$.
11. In selling 80 shares	$53\frac{3}{4}$	25 ¢ each.
12. In buying 8 \$ 500-bonds	$98\frac{3}{8}$	$\frac{1}{8}\%$.

13. If you know the amount of money invested, and the income received, how can you find the *rate of income*?

Apply the rule to the following examples:—

14. 4% bonds at 104, and $\frac{1}{8}\%$ brokerage.
15. 15 shares at 91, paying 5%, no brokerage.

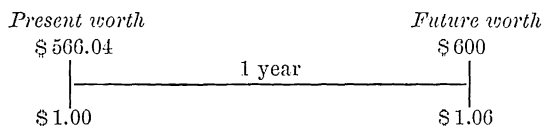
320.—Present Worth 1. Henry Osmond owes me \$ 600, due in of Non-interest-bearing a year without interest. We wish to find Debts due in the Future. what sum he shall pay me to-day, so that neither he nor I shall sustain loss; in other words, to find the *present worth* of a sum of money whose *future worth* is \$ 600.

2. What gives value to money? 3. At 6%, what is it worth to Mr. Osmond to have the use of \$ 600 for a year? 4. The sum I receive from him now, plus the interest on it for a year, at 6%, should amount to how much? 5. Ought an arrangement whereby he has \$ 36 at the end of the year and I have \$ 600 to be mutually satisfactory; and why?

6. Suppose, then, that he pays me \$ 566.04, and retains \$ 33.96. Find the amount of each of these sums at 6% for a year. What do I have at the end of the year? What does he have?

7. What, then, is the present worth of \$ 600, due in a year, without interest?

To find the present worth of a non-interest-bearing debt of \$ 600.



Process.

The future worth of \$ 1 in a year at 6% = \$ 1.06 ; \$ 600 is the future worth of as many dollars as \$ 1.06 is contained times in \$ 600 ; \$ 600 ÷ \$ 1.06 = 566.04 ; hence the present worth of \$ 600 due in a year without interest, is \$ 566.04, and the true discount that should be made for the prepayment of the debt is \$ 600 - \$ 566.04, or, \$ 33.96.

Find the present worth.

8. Debt \$ 500, time 1 yr., rate 5%.
9. Debt \$ 750, time 2 yr., rate 4%.
10. Debt \$ 1200, time $2\frac{1}{2}$ yr., rate 6%.
11. Debt \$ 725, time 6 mo., rate 8%.
12. Debt \$ 86, time 4 mo., rate 9%.
13. Debt \$ 1100, time 5 yr., rate 10%.

321. — Problems in Interest: the Product and Two Factors given to find the Third Factor.

1. The interest of \$ 600 for 4 yr. at 5% is the product of what three numbers ?
2. Make a problem in interest that shall be represented by the equation, \$ 84 = 3 × 7% × \$ 400.
3. Let i = interest, p = principal, r = rate %, t = time in years, and show that $i = p \times r \times t$, or $i = prt$.
4. Find the missing factor: $6 \times 10 \times x = 240$; $180 = 3 \times x \times 20$.
5. When the product and two of its factors are given, how is the third factor found ?
6. When $i = p \times r \times t$, show that $t = \frac{i}{p \times r}$, $p = \frac{i}{t \times r}$, $r = \frac{i}{p \times t}$.

7. The principal is \$ 400, the time 2 yr., the rate 5%. Explain the process of finding the interest, the time, the principal, and the rate by the use of the following formulas:—

$$i = p \times r \times t = 2 \times 5\% \times \$ 400 = \$ 40;$$

$$t = \frac{i}{p \times r} = \frac{40}{5\% \times \$ 400} = \frac{40}{20} = 2 \text{ yr.};$$

$$p = \frac{i}{t \times r} = \frac{40}{2 \times 5\%} = \frac{40}{10\%} = \$ 400;$$

$$r = \frac{i}{p \times t} = \frac{40}{2 \times 400} = \frac{40}{800} = 5\%.$$

8. In what time will \$ 500 earn \$ 60 at 6% ?
9. At what rate will \$ 400 earn \$ 80 in 4 years ?
10. What principal will earn \$ 100 in 5 years at 10% ?

Find the missing element.

	Principal.	Interest.	Time.	Rate.
11.	\$ 3000	\$ 1080	x	8%
12.	x	100.80	1 yr. 4 mo.	9%
13.	780	87.75	2 yr. 3 mo.	x
14.	x	16.00	60 da.	10%
15.	480	40	x	5%
16.	1800	8.10	36 da.	x
17.	1725	x	2 yr. 11 mo. 19 da.	3½%

- 322. — Exchange :**
- | | |
|---|---|
| Payment at a Distance
without sending Money.
Of Small Sums. | 1. Mention some objections to sending coin or paper money by mail or express.
2. To sending United States money to Europe. |
|---|---|

The Postal Service and some Express Companies keep large sums in many offices. If you pay from 1¢ to \$100 to a postmaster or express agent, he can write an *order* directing the postmaster or agent at another office to pay the same sum to any person you name. For this accommodation you pay from 3 to 30¢, no matter what the distance; or, if sending to a foreign country, from 10¢ to \$1, which is the *cost of exchange*.

NOTE. — Post-office orders are payable at an office named; express orders, at any office of the same company.

3. Find the cost of money orders to pay the following debts.

The charge for over \$40 to \$50 is 18 cents; over \$50 to \$60, 20 cents; over \$60 to \$70, 25 cents.

L. Ames of Oakland owes Ch. Adams of Newport \$55; he owes J. Crane \$69.90, and S. North \$50.

4. What must you pay in New York for an international money order for 106 m. payable in Berlin, the rate being 10 ¢ on each \$10 or fraction of \$10?

5. What would an order for 106 fr. cost? What part is cost of exchange?

**323. — Exchange
of Any Sum by
Check.**

If a person, say Edw. Brown, keeps money deposited at a national bank, or with a banking company, he may write a **Check**, as follows:—

\$927²⁴/₁₀₀

New York, July 12, 1895.

THE WEST NATIONAL BANK OF NEW YORK.

Pay to.....Simmons & Newton.....or order

--- Nine hundred twenty-seven and ²⁴/₁₀₀ Dollars.

No. 81.

- Edward Brown.

If *indorsed in blank*, — “Simmons & Newton,” — it may be collected in New York by any person known to the West National Bank.

If *indorsed in full*, — “Pay to the order of James Gray,
Simmons & Newton,” —

it may be paid to James Gray or his order as soon as he has indorsed it.

If sent to any city other than the one on which the check is drawn, it may be deposited in a bank and forwarded to the New York bank for collection, at a cost of 15 ¢ or more except to regular customers.

Most debts are paid in this way.

1. Make a personal check for \$382.19, signed by A. Snow, payable to E. S. Paine's order. 2. Make a proper indorsement and explain the effect. 3. If the words "or order" were omitted, to whom would it be payable?

4. Is a check *payable* in more than one place? 5. If a Baltimore check is sent to pay a debt in St. Paul, does the creditor know at *once* that the debt is paid?

324. — Exchange: If a party, say Howe & Co. of Albany, of Any Sum by cannot draw a check, or if the creditor in Bank Draft. Syracuse will not accept one, a draft may be bought at a bank, for a small cost of exchange, like an order at a post-office. Thus:—

\$234---

Albany, July 12, 1895.

TENTH NATIONAL BANK.

Pay to the order of Howe & Co.

..... Two hundred thirty-four Dollars.

To the

*Farmers' National Bank,
Syracuse, N. Y.*

*Patrick Matthews,
Cashier.*

1. Of what bank is Matthews cashier? 2. Where is the draft payable? 3. How can Howe & Co. make it payable to their creditors?

4. Make a draft from the National Exchange Bank of Peoria to the Traders' Mutual Bank of Chicago. The Western Machine Co. pays \$400 for it.

5. Why is a bank draft more likely to be "good" than a private check?

NOTE. — A bank draft on New York may be cashed almost anywhere in the United States. Drafts on another city will usually be paid before collection in the region of which it is a commercial centre.

325.—Exchange: Money may be sent for, as well as sent. Collecting Debts by Thus, if due to a Chicago firm from Haines Sight or Time Drafts. & Co. of Topeka, payment may be requested as follows:

<p><i>\$1500⁵⁰</i></p> <p><i>At sight</i> [this makes it a sight draft]</p> <p>[or] <i>Thirty</i> days after date [this makes it a time draft]</p> <p>Pay to the order of <i>Ourselves</i></p> <p>[or say] <i>A. H. Wells</i></p> <p><i>Fifteen hundred and $\frac{50}{100}$</i> Dollars,</p> <p>Value received, and charge to the account of</p> <p><i>To Haines & Co., Topeka, Kas.</i></p>	<p><i>Chicago, July 12, 1895.</i></p> <p><i>S. J. Richards & Co.</i></p>
---	--

1. Read the above as a **Sight Draft**, then as a **Time Draft**, and explain the difference. 2. From whom is payment requested; that is, who is the *drawee*? 3. Who is the creditor? The payee? The *maker* or *drawer* of the draft?

Such drafts, if known to be "good," may be sold to a bank by allowing a percentage for cost of exchange, or they may be forwarded through banks for collection.

If the drawee on receiving a time draft agrees to pay it, he writes across the face of the draft the word "*Accepted*," followed by his signature and the date. This converts the draft into an *acceptance* and gives it precisely the force of the drawee's promissory note, owned by the payee.

4. Suppose the above, as a 30-day draft, to have been accepted, and offered for discount July 17. What is the day of maturity? 5. Find the discount at 6%, and the proceeds.

6. Suppose it had been bought by a bank the day it was drawn, and $\frac{1}{2}\%$ charged for exchange besides the time discount. The proceeds would have been x .

7. F. Alto of New Orleans draws at sight on R. Fay of Waco, Tex., Aug. 3, 1895, for \$ 500. Make the draft.

8. Suppose the debt not due till Nov. 1. Make a proper time draft dated Aug. 3.

9. If discounted Aug. 18, the proceeds would be x .

10. If discounted Aug. 3, less an additional $\frac{1}{2}\%$ for exchange, the proceeds would be y .

11. Paine of Macon, Ga., owes Drew of Atlanta \$ 4000, due Jan. 1. Paine accepts a draft Oct. 1 and discounts it himself for \$ x .

12. A Charleston bank buys a draft on Richmond for \$ 2100, charging $\frac{1}{2}\%$ exchange. If it had taken the draft for collection only, the charge would have been 25¢. To the maker what is the difference in money? In which case is payment made more quickly?

326. — Foreign Exchange.

Payment in foreign countries without sending money is **Foreign Exchange**. Drafts are also called **Bills of Exchange**.

1. Define **Domestic Exchange**, and mention several kinds.

Bills of Exchange are drawn by one banker or broker on another. The rate of exchange varies from day to day, a pound sterling costing a little more or less than its bullion value; 4 marks, a little more or less than \$ 0.952; while \$ 1 will buy about 5.18 francs.

2. Find the cost of a sight draft on London for £ 800, exchange at \$ 4.90.

3. A sixty-day bill can be bought at \$ 4.87. What is the face of one that costs \$ 9788.70?

4. A debt of \$ 4199.24 is due in London to a firm in Philadelphia. For what amount sterling must a draft be made, if sold in Philadelphia at \$ 4.88?

5. Find the face of a bill on Hamburg costing \$871.325, exchange at $\$0.95\frac{3}{4}$ (= 4 m.).

6. What must be paid for a draft on Paris for 2000 francs, exchange being quoted at fr. 5.16 (= \$1)?

**327. — Review
Exercises.**

Written.

1. $(17\frac{1}{2})^2 - \sqrt{361} = 10\%$ of what?

2. How much pays this bill?

$3\frac{7}{8}$ yd. silk, at \$2.50;

2 pr. blankets, at \$7.37 $\frac{1}{2}$;

10% off to the trade and 2% for cash.

3. Add horizontally: 3.75, 23.08, 176.97, $0.83\frac{3}{4}$, 12.37 $\frac{1}{2}$.

4. \$156.91, \$73.99, \$1439, \$76.84, \$97 $\frac{1}{2}$, \$42.97, \$198 $\frac{1}{2}$.

Find cost, but write only the products for adding: —

5. $16\frac{2}{3}$ at 80¢

48 at $66\frac{2}{3}$ ¢

75 at $12\frac{1}{2}$ ¢

24 at $62\frac{1}{2}$ ¢

$37\frac{1}{2}$ at 10¢

7. 901 at 25¢

1200 at 75¢

12 at \$1.16 $\frac{2}{3}$

39 at \$2.33 $\frac{1}{3}$

42 at $83\frac{1}{3}$ ¢

9. $\frac{11}{16}$ at 20¢

$1\frac{6}{12}$ at $37\frac{1}{2}$ ¢

$4\frac{3}{8}$ at 25¢

100 at $2\frac{7}{8}$ ¢

$\frac{5}{12}$ at \$1.25

6. 6400 at $87\frac{1}{2}$ ¢

279 at $11\frac{1}{9}$ ¢

108 at $8\frac{1}{3}$ ¢

144 at $6\frac{1}{4}$ ¢

1608 at $33\frac{1}{3}$ ¢

8. $2\frac{2}{3}$ at 21¢

$4\frac{7}{8}$ at 50¢

$\frac{7}{12}$ at \$1.00

$10\frac{3}{4}$ at 28¢

$16\frac{1}{2}$ at $33\frac{1}{3}$ ¢

10. $\frac{5}{8}$ at 90¢

48 at $18\frac{3}{4}$ ¢

1000 at $2\frac{7}{8}$ ¢

$\frac{7}{12}$ at \$2.50

$1\frac{1}{12}$ at $87\frac{1}{2}$ ¢

Find total interest due on five notes, as follows. Write only results for adding.

11.

\$500, $2\frac{1}{2}$ yr., 4%.

630, 60 d., 8%.

180, 30 d., 12%.

900, 4 mo., 9%.

200, 15 d., 6%.

12.

\$720, 45 d., 6%.

376, 30 d., 3%.

200, 12 d., 4%.

820, 10 m., 6%.

1800, 10 d., 4%.

13.

\$700, 4 yr., $4\frac{1}{2}$ %.

300, 90 d., 6%.

400, 63 d., 3%.

480, $2\frac{1}{2}$ yr., 5%.

150, 54 d., 6%.

14. Bought 500 tons Franklin coal, at \$ 5.62½. Sold at an average advance of 22%, but lost 5% in bad debts. Required net gain. Allow 25¢ a ton storage, and \$ 32.60 for other expenses.

Find proceeds of notes without grace:—

15.	17.	19.
\$ 60, 3 mos., 8%.	\$ 850, 47 d., 4%.	\$ 1900, 63 d., 4%.
16.	18.	20.
\$ 260, 90 d., 6%.	\$ 946, 62 d., 3%.	\$ 1217, 14 d., 8%.

**328. — Commission,
Insurance,
Taxes, Duties, etc.
Written.**

1. Estimate the commission on a sale of \$ 5000, at 5%.

2. On an investment of \$ 5000, less the commission, at the same rate.

3. Insured a mill, valued at a quarter of a million of dollars, for \$ 200,000, at the following rates for 5 years: \$ 50,000 in each of three companies, at $\frac{3}{4}\%$; \$ 20,000 at $\frac{5}{8}\%$; and the remainder at $\frac{7}{8}\%$. What per cent of the value of the property is my annual premium?

4. \$ 40,000 is to be raised by taxation for a schoolhouse. The assessed valuation of the town is \$ 6,400,000. My property is assessed at \$ 25,000. What shall I pay towards the cost of the schoolhouse?

5. An importation of 380 yd. of cloth, invoiced at 14 shillings, pays an *ad valorem* duty of 20%. It sells at \$ 4 a yard. Loss on the lot? (\$ 4.8665 = ?)

6. Received \$ 6000 for the purchase of wheat. Charged 2% commission, and paid 90 cents a bushel. How many bushels did I buy?

7. A lawyer collects a debt for me at a commission of \$ 100 guaranteed and 5%. He sends me a check for \$ 3880. What was the amount of the debt?

8. Imported 40,000 lb. sugar, invoiced at $2\frac{3}{4}\%$, at 40% *ad valorem*, and $\frac{1}{8}\%$ per pound, specific. Required the cost of the importation.

9. Bought 19 shares of 7% manufacturing stock, at $128\frac{3}{4}$. Received a semiannual dividend, and then sold for 126. No brokerage. My gain or loss?

10. Sold \$50,000 worth of hides, at 2% commission, and with the net proceeds bought cotton at the same commission. Required my total commission.

- 329. — Interest and Bank Discount.** 1. What is meant by principal? interest? rate? amount? proceeds? term of discount? days of grace? bank discount? promissory note? indorser? indorsement? partial payment? exact interest? protest?

Written.

2. How large a check will pay a note of \$385, that has been drawing 5% interest from June 10, '95, to May 5, '96?

3. Bought an acre of land for \$3500, and sold it at once for 10¢ a foot, taking a 4-mo. note, which I have discounted immediately at 7%. My profits? Allow grace.

4. Find the exact interest of \$725, at 4%, for 125 days.

5. A note for \$1200, dated Aug. 19, 1895, bears 8% interest. It has one indorsement of \$800, June 19, 1896. What will settle the note 3 years from date?

Find the missing term: —

6. Principal, \$800; time, 50 d.; rate, 3%; interest, \$*x*.
7. Principal, \$*x*; time, 4 yr.; rate, 10%; interest, \$328.
8. Principal, \$600; time, *x* yr.; rate, 4%; interest, \$60.
9. Principal, \$425; time, 3 mo.; rate, *x*%; interest, \$8.50.
10. What is the present worth of a non-interest-bearing debt, whose future worth in 8 mo. is \$960? Money at 9%.
11. Face of 90-day note, \$696; discounted with grace 25 days after date at $4\frac{1}{2}\%$.

12. Required the proceeds of \$ 500 note for 60 days, dated Dec. 15, 1896, discounted Jan. 12, 1897, at 5%.

13. What may Charles French draw from a savings-bank paying 4%, in semiannual installments; his deposit, \$ 200 for 18 months?

14. Compare the interest, exact interest, compound interest, bank discount, and true discount of \$ 600 for 1 year 4 months, at 6%.

15. What amount will draw 3-mo. interest to Jan. 15? [\$ 309.]

Savings-bank Deposits.

June 12 \$ 20

Oct. 14 30

Oct. 17 15

Withdrawals.

Aug. 13 \$ 10

Oct. 16 20

Nov. 3 5

330.— Proportion: [Review sections 92, 93; 146–149.]
an Equality of Ratios. 1. Define ratio; antecedent; consequent; couplet.

2. $15 \div 3 = 5$; $\frac{15}{3} = 5$; $15:3 = 5$. Give three different names to 15, as used in these expressions; to 3; to 5.

3. Will the *inverse* ratio, $3 \div 15 = \frac{1}{5}$, show the relative size of the numbers equally well?

4. What is the direct ratio of 12 and 36? The inverse ratio?

5. Compare the following ratios or fractions:—

12:3 and 16:4; 5:20 and 6:24; $\frac{3}{18}$ and $\frac{1}{6}$.

Two equal ratios make a **Proportion**; thus:—

12:3 = 16:4

5:20 = 6:24

$\frac{3}{18} = 1:6$

$\frac{12}{3} = \frac{16}{4}$

$\frac{5}{20} = \frac{6}{24}$

$\frac{3}{18} = \frac{1}{6}$

4 = 4

$\frac{1}{4} = \frac{1}{4}$

$\frac{1}{6} = \frac{1}{6}$

Changing these ratios to fractional form, and then to smallest terms, we find them to be equal. The four numbers are therefore in **proportion**, and we say, for example, that 12 is to 3 as 16 to 4; or that the ratio of 5 to 20 is the same as the ratio of 6 to 24; or that 3 bears the same relation to 18 that 1 does to 6.

Test the following proportions by seeing whether the ratios are equal:

- | | |
|--------------------------------------|---|
| 6. $6:48 = 5:40$ | 11. $\frac{2}{3}:\frac{3}{4} = 8:9$ |
| 7. $32:4 = 96:12$ | 12. $7:42 = 9:54$ |
| 8. $5:3 = 35:21$ | 13. $2\%:4\% = 12\frac{1}{2}\%:25\%$ |
| 9. $2\frac{1}{2}:5 = 4\frac{1}{2}:9$ | 14. $3\text{ yd.}:4\text{ yd.} = \$0.75:\$1.00$ |
| 10. $8:10 = 20:25$ | |

The first and last terms of a proportion are the **extremes**; the second and third terms are the **means**.

15. Which terms of a proportion are **antecedents**? Which are **consequents**? Which are **dividends**? Which, **divisors**? Which may be **numerators**? Which, **denominators**?

16. Arranging the proportion $16:8 = 10:5$ in a fractional form, we have $\frac{16}{8} = \frac{10}{5}$; multiplying both sides of the equation by 40, the l. c. m. of the denominators, we have $40 \times \frac{16}{8} = 40 \times \frac{10}{5}$; cancelling 8 on one side and 5 on the other, $5 \times 16 = 8 \times 10$; but 5×16 is the product of the extremes, and 8×10 , the product of the means. What conclusion may we draw regarding a proportion?

In a proportion, the product of the extremes is equal to the product of the means.

17. Test the proportions in Ex. 6-14 by this principle or law.

18. Find the values of x :—

$3 \times 18 = x \times 9$	$3:x = 9:18$	$7:16 = x:80$
$9 \times x = 6 \times 15$	$9:6 = 15:x$	$x:24 = 6:72$

19. How is a missing factor found? How is the missing term of a proportion found?

Find the missing term in these proportions:—

- | | |
|--|---|
| 20. $17:60 = 85:x$ | 26. $\$x:\$9 = 60\text{ lb.}:27\text{ lb.}$ |
| 21. $27:x = 81:100$ | 27. $10\text{ T.}:\frac{1}{2}\text{ T.} = \$x:\$3.75$ |
| 22. $x:16 = 130:26$ | 28. $15:x = 3^2:12^2$ |
| 23. $324:243 = 12:x$ | 29. $24\text{ lb.}:x\text{ lb.} = 2^3:8^3$ |
| 24. $\frac{1}{1\frac{1}{2}}:x = 8:16$ | 30. $9\frac{1}{2}:16\frac{1}{4} = 38:x$ |
| 25. $\$90:\$48 = 15\text{ yd.}:x\text{ yd.}$ | 31. $7.25:0.5 = x:3$ |

331. — Proportion: Proportion may be applied to the solution of the Rule of Thrée. problems in which three terms are given to find a fourth.

Of the three terms given, two are like numbers and the third is like the required result.

1. Why must the terms of a ratio be like numbers?

A. $16 \text{ yd.} : 2 \text{ yd.} = \$32 : \$4$

B. $3 : 18 = \$6 : \36

2. In A which ratio is larger than 1? 3. How is it in B?

4. Show that the ratios of a proportion must correspond, both being larger than 1, or both smaller than 1.

5. If 16 yards cost \$40, what will 10 yards cost?

Process.

$16 : 10 = \$40 : \$x \text{ or } \$25.$

$\frac{10 \times \$40}{16} = \$25.$

$\frac{10}{16} \times \$40 = \$25.$

6. What is the denomination of the answer, or 4th term of the proportion?

7. Why must the 3d term be dollars also?

8. Will 10 yards cost more or less than 16 yards?

9. Then will the 2d ratio be greater or less than 1? 10. If the ratios of a proportion must correspond, must the 1st ratio be made greater or less than 1?

11. Read the incomplete proportion. 12. How is the missing extreme found?

13. What is the ratio of 16 to 10? Of \$40 to \$25? 14. Perform the problem by analysis.

Solve the following by the "Rule of Three":—

15. When 60 bushels of oats cost \$36, what will 25 bushels cost?

16. What will 18 tons of hay cost if 7 tons cost \$147?

17. If 9 weeks' board costs \$94.50, what will 12 weeks' board cost?

18. If a yacht sails 24 miles in 70 min., how long will she be in sailing 108 miles?

19. If 72 men lay 2 miles of water pipe in 15 days, how many days will 48 men require?

20. If a train runs 1000 miles in 28 hours, how many miles can it run in 120 hours?

21. If $2\frac{3}{4}$ yards of cloth can be bought for \$23.10, what should be paid for $15\frac{7}{8}$ yards at the same rate?

22. If 1 acre yields 22 bu. 3 pk. of corn, how many acres would yield 546 bu.?

23. If a 5-cent loaf of bread weighs 8 ounces when flour is worth \$5, what would be the proportional weight when flour is at \$6?

24. $\frac{7}{9}$ of a yard costs $\$ \frac{5}{6}$; at the same rate $\frac{3}{4}$ yard costs \$ x .

332. — Partnership. 1. James Smith and Charles Butler form a *partnership* under the *firm* name of James Smith & Co., with a *capital* of \$5000, of which Smith furnishes \$4000 and Butler \$1000. What part of the capital does each furnish?

2. They agree to share gains and losses in proportion to the capital each furnishes. They gain \$1500 the first year. How shall it be divided between them?

3. The second year they lose \$700. What should each pay?

4. In the absence of an agreement between the partners, is it just to divide the gain according to the capital furnished?

5. Divide \$9000 profit among three partners who furnish \$20,000, \$30,000, and \$40,000, respectively.

6. The profits were \$4800. Dunlap's share was \$2000. What part of the capital did he put in?

7. Two young men hire a bicycle for \$4 a week. One has it Mondays and Thursdays. What is his share of the expense? The bicycle rests on the Sabbath.

8. Divide \$750 between a girl and her brother in proportion to their ages of 6 and 9. (Their united ages are the capital.)

9. A furnished the store, and B the stock, worth \$6000. Of the profits B had $37\frac{1}{2}$ cents out of every dollar. What was the store worth?

10. A bankrupt firm's assets are \$27,500, and their liabilities \$32,500. What do they pay on a dollar? They owe me \$600. I get \$ x .

11. A's gains are $\frac{1}{3}\frac{5}{2}$ of the total gains. B's capital is \$8400. What is A's capital?

12. Divide \$13,000 among 3 men who furnished capital at the rate of \$ $\frac{1}{2}$, \$ $\frac{1}{3}$, and \$ $\frac{1}{4}$ each.

13. Two partners earn 10% on their capital besides the \$2000 which each takes in weekly allowances. One receives in all \$3500, the other \$2500. What is the total capital?

333.—Powers and 1. Find these powers and roots:

Roots: how a Number	10^2 ; $\sqrt{100}$; 40^2 ; $\sqrt{1600}$; 70^2 ; $\sqrt{4900}$
is Squared.	20^2 ; $\sqrt{400}$; 50^2 ; $\sqrt{2500}$; 80^2 ; $\sqrt{6400}$
	30^2 ; $\sqrt{900}$; 60^2 ; $\sqrt{3600}$; 90^2 ; $\sqrt{8100}$

2. How is a number squared? 3. How are roots related to powers? 4. How many equal factors make a square? A cube?

Extracting the square root of a number, or separating it into two equal factors, is the reverse of squaring one of the equal factors. A careful analysis of the process of squaring a number will enable us to reverse the process and find the square root of a number.

<p>A</p> <p>47 In <i>A</i> we first multiply by the <i>ones</i> as usual.</p> <p>47 In <i>B</i> we begin with the <i>tens</i>.</p> <p>47</p> <p>49 = $7 \times 7 = \text{ones}^2$</p> <p>280 = $7 \times 40 = \text{ones} \times \text{tens}$</p> <p>280 = $40 \times 7 = \text{tens} \times \text{ones}$</p> <p>1600 = $40 \times 40 = \text{tens}^2$</p> <p>2209 = $47 \times 47 = \text{tens}^2 + \left(\begin{smallmatrix} \text{tens} \times \\ \text{ones} \end{smallmatrix} \right) + \left(\begin{smallmatrix} \text{ones} \times \\ \text{tens} \end{smallmatrix} \right) + \text{ones}^2 = t^2 + 2to + o^2 = 2209$</p>	<p>B</p> <p>$40 + 7 = 47$</p> <p><u>$40 + 7 = 47$</u></p> <p>$t^2 = 40 \times 40 = 1600$</p> <p>$t \times o = 40 \times 7 = 280$</p> <p>$o \times t = 7 \times 40 = 280$</p> <p>$o^2 = 7 \times 7 = 49$</p>
--	--

5. In these processes of squaring 47, how many partial products are obtained? 6. What is the largest? The smallest? 7. Compare the remaining two and show how each is found.

$$\begin{array}{r}
 C \\
 47 = 40 + 7 \\
 \underline{47 = 40 + 7} \\
 1600 = 40^2 = t^2 \\
 560 = 2 \times (40 \times 7) = 2 to \\
 \underline{49 = 7^2 = o^2} \\
 2209 = t^2 + 2 to + o^2
 \end{array}$$

$$\begin{array}{r}
 56 \\
 \underline{56} \\
 2500 = t^2 = \text{Part I.} \\
 600 = 2 to = \text{Part II.} \\
 \underline{36 = o^2 = \text{Part III.}} \\
 3136
 \end{array}$$

16. Explain what three parts the square of every number of two digits contains.

8. What is the difference between 7×40 and 40×7 ? 9. What is the square of the tens? The square of the ones? 10. The product of the tens and the ones? 11. What is the product of the tens and ones taken twice, as in *C*?

12. Show that the square of 47 contains the square of its tens, the square of its ones, and twice their product.

13. Square 56, and show what three parts its square contains.

14. In the same way square 45; 28; 32; 44; 83; 96.

15. Without writing the process, square 24; 36; 58; 65; 73.

The square of a number of two digits contains—

The square of the tens, the square of the ones, and twice their product.

334. — Squares and their Roots compared as to the Number of Places they contain.

1. Square 10 and 99 and any number between them. Compare the number of places in the root with the number in the power.

2. Do the same with 100 and 999, and any other number of three figures.

3. How many figures in 1000^2 ? In 10000^2 ?

4. If the root has three figures, the square will have how many? If the square has eight figures, how many has the root?

5. How many figures in the square of 33? 75? 207? 964? 28796? 6. How many figures in the square root of 9409? 381? 27225? 182329? 49434961?

7. How does the number of places in a square compare with the number of places in its root?

8. Square 0.2; 0.02; 0.4; 0.12; 0.25; 0.03; 0.005.

9. Compare the number of decimal places in power and root.

10. Why can the square of a decimal never contain an odd number of decimal places? 11. Which is larger, $\frac{3}{4}$ or $(\frac{3}{4})^2$? $\frac{1}{25}$ or $\sqrt{\frac{1}{25}}$? A fraction or its root?

335.—Extracting Square Roots: finding two Equal Factors of a Number.

1. Write the squares of 10, 20, 30, and so on to that of 100.

2. Write the squares of hundreds, 100, 200, 300, etc., to that of 1000.

To find the square root, or one of the two equal factors, of 2809.

Process.

$$t^2 + 2to + o^2 = 2809 \text{ (50 + 3)}$$

$$t^2 \text{ or } 50^2 = 2500 = \text{Part I.}$$

$$2t \text{ or } 100 \quad 309 = 2to + o^2$$

$$2to \text{ or } 100 \times 3 = 300 = \text{Part II.}$$

$$o^2 \text{ or } 3^2 = 9 = \text{Part III.}$$

3. How many figures in this square?

In its root? 4. Of what three parts

does the square consist? 5. What is

the square of 50? Of 60? 6. Between

what two squares does 2809 come?

7. Then its root is between what two

numbers?

8. What is t^2 , or Part I. of the square? 9. Taking it out of the square, what two parts remain? 10. What are the tens of the root? Twice the tens, or $2t$?

11. $3 \times x = 24$. How is a missing factor found? 12. Assuming 309 to be the product of two factors, $2t$ and o , and calling $2t$ 100, what is o , or the ones of the root? 13. If $2t = 100$ and $o = 3$, what is $2to$, or Part II. of the root?

14. Taking 300, Part II. of the root, out of 309, what remains. 15. What part of the square is this? 16. What, then, is the square root of 2809? Prove it.

In the same way find—

17. $\sqrt{529}$ 18. $\sqrt{676}$ 19. $\sqrt{1156}$ 20. $\sqrt{1764}$ 21. $\sqrt{2025}$

22. $\sqrt{2916}$ 23. $\sqrt{3969}$ 24. $\sqrt{4624}$ 25. $\sqrt{5625}$ 26. $\sqrt{7056}$

A Shortened Process.

$$t^2 + 2to + o^2 = 88^2 36(94)$$

$$t^2 = 81$$

$$2t = 180 \quad \begin{array}{l} 736 = (2t + o) \times o \\ 736 = 2to + o^2 \end{array}$$

$$o = \frac{4}{2t + o = 184}$$

We may shorten the process by omitting ciphers both in t^2 and in dividing by $2t$. In $180)736$, show that—

$$(2t + o) \times o = 2to + o^2$$

$$\text{or } (180 + 4) \times 4 = 720 + 16 \text{ or } 736.$$

Solve by both processes. — 27. $\sqrt{4096}$

$$28. \sqrt{5329} \quad 29. \sqrt{6889} \quad 30. \sqrt{8464} \quad 31. \sqrt{3364} \quad 32. \sqrt{9801}$$

336. — Square Root of Large Numbers, Decimals, and Fractions.

The process on the preceding page may be applied in finding the root of any number.

To find the square root of 601.7209.

Explain each step of the process, telling how we get each number.

Process.

$$\begin{array}{r} 6'01'.72'09(24.53 \\ \begin{array}{l} 4 \\ 201 \\ 176 \\ 2572 \\ 2425 \\ 14709 \\ 14709 \end{array} \end{array}$$

SUGGESTIONS.— We begin at the point and separate the power into 2-figure groups, showing that the root has 4 figures. We first use the left-hand groups, 6'01, to find the first figures of the root, 24. We then annex the third group, 72, and treat the 24 as the tens of the root, and so on. Having found the third root-figure, 5, we consider 245 as the tens of the root, etc.

$$1. \sqrt{283024}$$

$$3. \sqrt{404496}$$

$$5. \sqrt{755161}$$

$$2. \sqrt{299209}$$

$$4. \sqrt{556516}$$

$$6. \sqrt{2137444}$$

To find the root of 0.501.

Process.

$$\begin{array}{r} 0.50'10(0.707 + \\ \begin{array}{l} 49 \\ 1407 \sqrt{1\ 1000} \\ 9849 \\ 1151 \end{array} \end{array}$$

EXPLANATION.— We begin as before at the point and separate the power into 2-figure groups, annexing a zero to fill the second group. As no decimal power can have a partial group, we know that this decimal is an imperfect power. For the third root-figure we annex a cipher-group, and proceed as before, using + or - to mark an approximate root. The work might have been carried farther.

- | | | | |
|--------------------|---------------------|---------------------|-----------------------|
| 7. $\sqrt{0.5625}$ | 11. $\sqrt{0.89}$ | 15. $\sqrt{0.64}$ | 19. $\sqrt{225.9009}$ |
| 8. $\sqrt{0.9216}$ | 12. $\sqrt{19.467}$ | 16. $\sqrt{0.064}$ | 20. $\sqrt{2044900}$ |
| 9. $\sqrt{42.25}$ | 13. $\sqrt{824.9}$ | 17. $\sqrt{1932.4}$ | 21. $\sqrt{76.3876}$ |
| 10. $\sqrt{0.783}$ | 14. $\sqrt{17.035}$ | 18. $\sqrt{783.95}$ | 22. $\sqrt{0.8}$ |

A. $\sqrt{\frac{320}{105}} = \sqrt{\frac{64}{81}} = \frac{8}{9}$

B. $\sqrt{\frac{8}{9}} = \sqrt{0.375} = 0.612 +$

C. $\sqrt{7\frac{1}{9}} = \sqrt{\frac{64}{9}} = \frac{8}{3} = 2\frac{2}{3}$

D. $\sqrt{4\frac{4}{9}} = \sqrt{\frac{40}{9}} = \frac{6.3245 +}{3} = 2.108 +$

E. $\sqrt{6\frac{3}{8}} = \sqrt{6.375} = 2.52 +$

In finding the root of fractions:

I. First change them to simplest form, as in A or C.

II. Use the method in A or C when both terms are perfect powers.

III. Use B or E when both terms are imperfect powers.

IV. D may be used when the denominator is a square.

- | | | | |
|------------------------------|-----------------------------|--------------------------------|-----------------------------|
| 23. $\sqrt{\frac{147}{432}}$ | 25. $\sqrt{6\frac{19}{25}}$ | 27. $\sqrt{82\frac{4}{5}}$ | 29. $\sqrt{151\frac{7}{8}}$ |
| 24. $\sqrt{\frac{5}{16}}$ | 26. $\sqrt{5\frac{2}{3}}$ | 28. $\sqrt{\frac{1905}{6480}}$ | 30. $\sqrt{2\frac{3}{16}}$ |

337.—Extract the Square Root.

Oral.

- | | |
|-------------------------------|---------------------------------|
| 1. $\sqrt{14400}$ | 11. $\sqrt{0.49}$ |
| 2. $\sqrt{\frac{18}{32}}$ | 12. $\sqrt{0.049}$ |
| 3. $(5\frac{1}{2})^2$ | 13. $\sqrt{0.00490}$ |
| 4. $(16\frac{1}{2})^2$ | 14. $\sqrt{625}$ |
| 5. $\sqrt{0.09}$ | 15. $\sqrt{0.625}$ |
| 6. $\sqrt{36 \times 49}$ | 16. $\sqrt{16 \text{ million}}$ |
| 7. $\sqrt{1735^2}$ | 17. $\sqrt{7\frac{1}{9}}$ |
| 8. $(\sqrt{87\frac{1}{2}})^2$ | 18. $\sqrt{30\frac{1}{4}}$ |
| 9. $3\sqrt{81}$ | 19. $\sqrt{272\frac{1}{4}}$ |
| 10. $\sqrt{0.0625}$ | 20. $\sqrt{10\frac{9}{16}}$ |

Written.

- | | |
|---|-----------------------------|
| 1. $\sqrt{94249}$ | 11. $\sqrt{95\frac{1}{16}}$ |
| 2. $\sqrt{0.729}$ | 12. $\sqrt{16^3}$ |
| 3. $\sqrt{13\frac{7}{8}}$ | 13. $\sqrt{2}$ |
| 4. $\sqrt{1008016}$ | 14. $\sqrt{\frac{1}{5}}$ |
| 5. $\sqrt{9834496}$ | 15. $\sqrt{\frac{3}{80}}$ |
| 6. $\sqrt{62742241}$ | 16. $\sqrt{127\frac{1}{3}}$ |
| 7. $\sqrt{2033.1081}$ | 17. $\sqrt{785}$ |
| 8. $\sqrt{\frac{30}{32}}$ | 18. $\sqrt{3444736}$ |
| 9. $\sqrt{\frac{3}{4} + \frac{5}{6} + \frac{6}{9}}$ | 19. $\sqrt{17^2 + 25^2}$ |
| 10. 998001 | 20. $\sqrt{0.741}$ |

338.—To find Any Side of a Right Triangle.

1. Draw a right triangle with base $1\frac{1}{2}$ inches long and perpendicular 2 inches.
2. On each of the three sides as base draw a square.
3. Separate each square into half-inch squares.
4. Compare the squares on the hypotenuse with the sum of the squares on the other two sides.

5. If from the square on the hypotenuse you take the square on the base the remaining area will equal what square?

6. The square of the base is 36; the square of the hypotenuse is 100; the square of the perpendicular is x . Prove this by drawing a triangle with squares on its sides.

In a right triangle,
The square of the hypotenuse equals the sum of the squares of the other two sides.

7. Hypotenuse² = 225
 Perpendicular² = 144
 Base = x

8. $B^2 = 144$; $P^2 = 256$; $H = x$.
 9. $H^2 = 625$; $B^2 = 400$; $P = x$.

10. Explain these formulas:

$$H = \sqrt{B^2 + P^2}$$

$$B = \sqrt{H^2 - P^2}$$

$$P = \sqrt{H^2 - B^2}$$

11. The three sides of a right triangle are respectively 39 in., 65 in., and 42 in. With any two given find the other.

Explain the following process:

$$H = \sqrt{B^2 + P^2} = \sqrt{42^2 + 39^2} = \sqrt{1764 + 1521} = \sqrt{4225} = 65$$

$$B = \sqrt{H^2 - P^2} = \sqrt{65^2 - 42^2} = \sqrt{4225 - 1764} = \sqrt{1521} = 39$$

$$P = \sqrt{H^2 - B^2} = \sqrt{65^2 - 39^2} = \sqrt{4225 - 1521} = \sqrt{1764} = 42$$

Find the unknown sides of the right triangle, drawing a figure and marking the dimensions in each case:—

	Hypotenuse.	Base.	Perpendic.		H.	B.	P.
12.	55	x	33	17.	162	70	x
13.	26	14	x	18.	x	39	27
14.	36	20	x	19.	208	x	93
15.	x	15	60	20.	117	13	x
16.	25	x	13	21.	$6\frac{1}{2}$	$3\frac{1}{2}$	x

**339. — Practical
Application of
Square Root.**

Written.

1. The top of a square table has an area of 576 sq. in. What is its length?
2. What is the length of a square field containing 10 acres? Its perimeter?
3. A rectangle measures 22 ft. by 10 ft. How long is its diagonal?
4. The foot of a 25 ft. ladder is 12 ft. from the side of the house against which it leans. How far from the ground is its top?
5. What is the area of a right triangle whose longest side is 20 ft., and its shortest 8 ft.?
6. Find the diagonal of a 36-inch square.
7. Find the altitude of an equilateral triangle whose side measures 24 ft.
8. What will it cost to fence a square field containing 5 A., at \$1.25 a rod?
9. A pitch-roof house is 22 ft. wide. The ridge-pole is 10 ft. higher than the plate. How long are the rafters if they project 1 ft.?
10. Find the area of an isosceles triangle whose base is 12 ft., and its perimeter 50 ft.

**340. Rectangles and
Triangles.**

Written.

1. What is the length of a square equal in area to a rectangle 24 rd. long and 33 ft. wide?
2. What is the longest straight line that can be drawn on the ceiling of your schoolroom if it measures 32 ft. by 30 ft.?
3. Compare the perimeter of a rectangle 48 in. by 12 in. with that of a square of equal area.
4. How much do I save by crossing along its diagonal a square that contains 1296 sq. rods instead of going round its two sides?
5. How long is an acre of land in the form of a square?

6. How long a guy will support a derrick 48 ft. high if fastened 85 ft. from its base?

7. The hypotenuse of a right triangle measures 90 ft. The other sides are equal. How long are they?

8. What is the shortest *possible* distance that I must walk to go from the center of a 10-acre square field to each corner, and return to the starting-point?

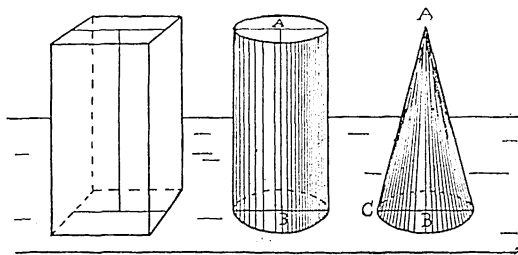
9. The area of a circle $= D^2 \times 0.7854$; then $D^2 = \frac{\text{Area}}{0.7854}$ and $D = \sqrt{\frac{\text{Area}}{0.7854}}$. What must be the diameter of a circle to contain approximately an acre?

10. Two poles are 100 ft. apart. One is 60 ft. high, and the other 80 ft. How long a line will connect their tops?

341.—Contents of [Review section 249.]

Cones.

1. What part of a square prism is a cylinder of equal diameter and height? 2. A square prism contains 10,000 cu. in. A cylinder of the same diameter and altitude contains 0.7854 as much, or x cu. in.



A **Cone** is a solid, having a circle for its base, tapering uniformly to a point, the vertex of the cone. AC is the **slant height**.

3. Construct a hollow cylinder and a cone of equal base and altitude. Using sand or water, show that the cone is $\frac{1}{3}$ of the cylinder.

A circle $= 0.7854$ of a square of the same diameter.

A cylinder $= 0.7854$ of a square prism of the same diameter and altitude.

A cone = $\frac{1}{3}$ of a cylinder or $\frac{1}{3}$ of 0.7854, — that is, 0.2618, — of a square prism of equal base and height.

4. Find the contents of a prism 4 in. square and 12 in. in altitude, and of the largest cylinder and cone that can be turned from it. Explain the process.

Process. $12 \times 4^2 = 192 =$ contents of square prism.

0.7854 of $12 \times 4^2 = 150.7968 =$ contents of cylinder.

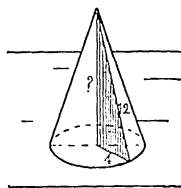
0.2618 of $12 \times 4^2 = 50.2656$, or $\frac{1}{3}$ of 150.7968 = contents of cone.

5. Diam. of base = 15; alt. = 20.

Find the contents of cones: 6. Diam. of base = 8; alt. = 40.

7. Diam. of base = 10; alt. = 36.

8. First find the altitude and then the contents of a cone, the diameter of the base being 8, slant height 12.



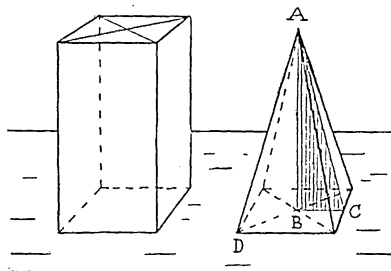
$$H^2 = a$$

$$B^2 = b$$

$$\frac{P}{B} = \sqrt{a - b}$$

A Cone is $\frac{1}{3}$ of a cylinder of equal base and altitude.

342. — Contents of Pyramids. A Pyramid is a solid whose base is a regular polygon, and whose sides are triangles meeting in a common point, the vertex of the pyramid.



AC is the slant height.

Pyramids, like their bases, are square, triangular, hexagonal, etc.

A Pyramid is $\frac{1}{3}$ of a prism of equal base and altitude.

1. A square pyramid 12 ft. high measures 3 ft. along one side of its base. Required its contents.

2. A granite shaft 10 feet high and 20 inches square is surmounted by a square pyramid 2 feet in altitude. The contents of both?

3. Which is more easily measured, the slant height or the altitude of a pyramid? Which line of a triangle is the slant height of a pyramid?

4. The slant height of a square pyramid is 15 inches, and the diameter of the base 10 inches. Find its contents.

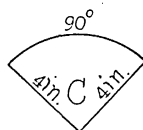
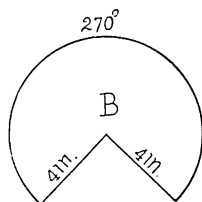
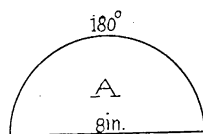
5. The area of the base of a pentagonal prism is $62\frac{1}{2}$ sq. in.; its altitude is 24 in.; the contents?

343.—Of the Convex Surface of Pyramids and Cones. 1. Of what form are the sides of a pyramid? 2. How is the area of a triangle found?

3. Find the convex or lateral surface of a square pyramid 16 in. in slant height, with base 1 ft. square.

4. Altitude of a square pyramid = 20 in.; diameter of base = 10 in. Lateral surface = x .

5. Slant height of octagonal pyramid 24 in.; side of base 6 in.; lateral surface?



6. With a radius of 4 inches, draw any sector, as A, B, or C, on thick paper. Cut it out, and place its radii together to form a hollow cone. 7. What line of the cone is the radius of the sector? The arc of the sector?

8. The product of the circumference of a circle and half its radius equals what? 9. The area of a sector is found like the area of a triangle. Explain from the figures that $\text{base} \times \frac{1}{2} \text{ the altitude}$, or $\text{arc} \times \frac{1}{2} \text{ the radius} = \text{area}$.

10. The circumference of the base of a cone (arc of sector) is 21 inches; the slant height of the cone (radius of sector) is 10 inches. What is the area of its convex surface?

11. Circumference of base 60 inches, slant height $12\frac{1}{2}$ inches. Convex surface?

12. The diameter of the base of a cone is 10 inches, its slant height is 15 inches. Required the convex surface.

13. How many square inches in the entire surface of a cone 8 inches in diameter at the base, and 20 inches in slant height?

The convex surface of a cone or pyramid equals the product of the circumference of its base and $\frac{1}{2}$ its slant height.

14. How many square inches in the entire surface of a cone whose base is 10 inches and whose *altitude* is 24 inches?

15. What is obtained when the perimeter of the base of a cone or of a pyramid is multiplied by $\frac{1}{2}$ its slant height?

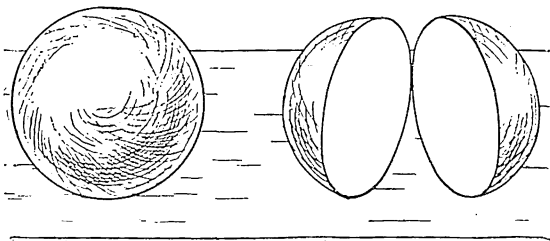
16. If the largest possible cone should be turned out of a square pyramid, what decimal part would become shavings?

344. — Surface of Spheres.

1. Cut away any slice of a sphere, as an apple. What is the form thus exposed?

2. When a sphere is bisected, the plane surfaces thus exposed are *great circles* of the sphere. Would the diameter and circumference of one of these circles be the diameter and circumference of the sphere?

3. It can be proved that the flat surface of a hemisphere is $\frac{1}{2}$ of its curved surface. How many great circles in the curved surface of a hemisphere?



4. How many great circles in the surface of a sphere?

5. How is the area of a circle found?

If $D^2 \times 0.7854$ = Area of a great circle of a sphere,
 then $D^2 \times 0.7854 \times 4$ = Area of 4 great circles, or of the sphere;
 but $D^2 \times 0.7854 \times 4 = D^2 \times 3.1416$ = Area of sphere.

6. What is the area of the surface of a sphere 5 inches in diameter? Explain.

$$5^2 \times 3.1416 = 78.54 \text{ square inches.}$$

The surface of a sphere is 3.1416 times the square of its diameter.

7. How many square inches in the surface of a 12-inch globe?
 8. Calling the diameter of the moon 2000 miles, how many square miles in its surface?

$$D^2 \times 3.1416 = \text{surface of sphere.}$$

But $D^2 \times 3.1416 = D \times (D \times 3.1416),$

and $(D \times 3.1416) = \text{the circumference.}$

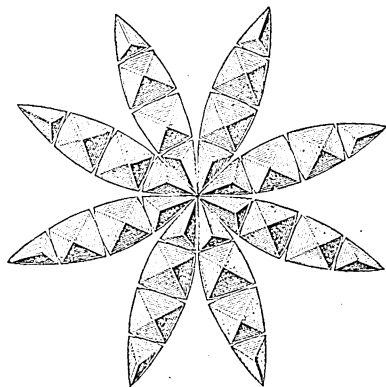
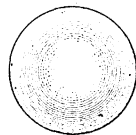
Hence $D \times (D \times 3.1416) = D \times \text{circumference, and}$
 area of sphere = diameter \times circumference.

9. Diameter of sphere = 7 inches. Its circumference = 22 inches. What is its area?

10. Diameter of earth = 8000 miles. Its circumference 25,000 miles. Area of its surface = x miles.

345. — Contents of Spheres.

1. If a sphere should be dissected as in the accompanying illustration, of what would it appear to be composed?



2. What line in the sphere forms the altitude of each pyramid?
 3. What forms the base of each?

4. Taken together, what will the bases of all the pyramids make?
5. How are the contents of any one pyramid found?
6. What should we obtain by multiplying the surface of the sphere (the bases of all the pyramids) by $\frac{1}{3}$ of its radius (the altitude of each of the pyramids)?
7. The surface of a sphere is 113 sq. in., and its radius 3 in. What are its contents?

8. How is the surface of a sphere found? $\frac{1}{3}$ of the radius is what part of the diameter?

9. Read and explain the following:

$\frac{R}{3} \times D^2 \times 3.1416 = \text{volume of sphere}; \text{ but } \frac{R}{3} = \frac{D}{6}; \text{ hence}$

$\frac{D}{6} \times D^2 \times 3.1416 = \text{contents of sphere. By cancelling, we obtain}$

$D \times D^2 \times 0.5236, \text{ or } D^3 \times 0.5236, = \text{Contents of Sphere.}$

10. The diameter of a sphere is 24 inches. Its contents?

The contents of a sphere are equal to 0.5236 of the cube of its diameter.

11. How many cubic miles in the moon, if of 2000 m. diameter?

12. If the largest possible sphere is turned out of a cube, what decimal part becomes shavings and what part sphere?

To be remembered:

$\pi = 3.1416$, used in finding circumferences.

$\frac{1}{4}$ of $\pi = 0.7854$, used in finding $\left\{ \begin{array}{l} \text{area of circle, and} \\ \text{contents of cylinder.} \end{array} \right.$

$\frac{1}{6}$ of $\pi = 0.5236$, used in finding contents of sphere.

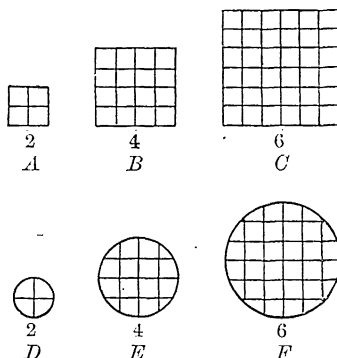
$\frac{1}{12}$ of $\pi = 0.2618$, used in finding contents of cone.

$\frac{1}{\pi} = 0.31831$, used in finding diameters.

346. — Comparison *Similar surfaces* have the same form.

of Similar Surfaces. 1. Are all circles similar surfaces? Are all triangles? Are all equilateral triangles?

Mention other similar surfaces of different sizes.



2. Give values to x and y .

$$4:2 = x. \quad 6:2 = x. \quad 10:4 = x.$$

$$4^2:2^2 = y. \quad 6^2:2^2 = y. \quad 10^2:4^2 = y.$$

3. In the figures at the left, what is the ratio of the *side* of B to the *side* of A ? Of the *area* of B to the *area* of A ?

4. Compare the side of C with the side of A . Compare their areas.

5. What is the ratio of E to D ? Of their diameters?

6. Compare D and F , as to diameters and as to areas.

7. How much more water will pass through a 2-inch nozzle than through a 1-inch nozzle?

8. A faucet with a $\frac{1}{2}$ in. opening will require 4 times as long to fill a tank as one with an inch opening. Explain.

9. If a square lot 60 ft. long costs \$300, what will one of the same shape 3 times as long cost?

10. Compare the strength of a rope 2 in. round with that of one 3 in. round.

11. If it costs \$8.25 to gild a sphere 20 inches in circumference, what will it cost to gild one of 30 inches?

12. A $\frac{5}{8}$ in. faucet fills a tank in 28 min.; a $\frac{3}{4}$ in. faucet in x min.

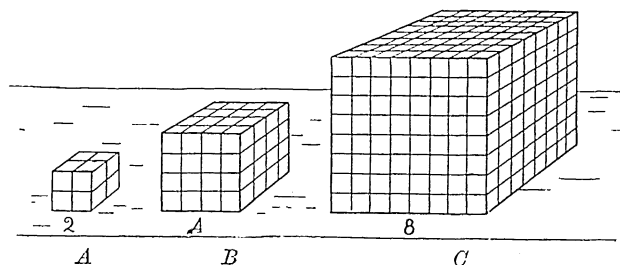
13. The ratio of two similar triangles is 25; the ratio of their altitudes is x .

14. To paint a conical steeple 30 ft. high costs \$35; to paint one 45 ft. high costs, at the same rate, \$ x .

15. A conical tent, 16 ft. slant height, costs \$13.50; one measuring 4 ft. more would cost \$ x .

Similar surfaces have the same ratio as the squares of their corresponding lines.

347. — Comparison 1. Similar solids have the same form. Men-
of Similar Solids. tion several similar solids.



2. Compare the two corresponding edges in A and B . In B and C . In A and C .

3. Compare a face of A with one of B . With one of C .

4. Read and explain the following proportions: —

Edge of A : edge of $B = 2 : 4$.

Edge of B : edge of $C = 4 : 8$.

Surface of A : surface of $B = 2^2 : 4^2$.

Surface of C : surface of $B = 8^2 : 4^2$.

Volume of A : volume of $B = 2^3 : 4^3$.

Volume of B : volume of $C = 4^3 : 8^3$.

Similar solids have the same ratio as the cubes of their corresponding dimensions.

5. C will weigh how many times as much as A ? As B ?

6. If a 2-inch sphere weighs 1 lb., how much will a 6-inch sphere weigh?

7. If a rectangular bin, 5 ft. long, contains 75 bu. of oats, how many bushels will a similar bin, $12\frac{1}{2}$ ft. long, contain?

8. It requires 90 min. to fill a cylindrical tank, $3\frac{1}{2}$ ft. in diameter. At the same rate how many minutes will be required to fill a similar tank, 14 ft. in diameter?

9. If a coil of $\frac{1}{16}$ -in. wire weighs 48 lb., what will a coil of similar wire, $\frac{3}{32}$ -in. in diameter, weigh?

10. Compare a $\frac{1}{2}$ -in. cube with a cubic yard.

348. — Mensuration.*Written.*

1. Find the cost of plastering the walls and ceiling of a room 18 ft. long, 15 ft. wide, and 9 ft. high, allowing $\frac{1}{3}$ of the area of the walls for openings and wood-covered portions. Price $12\frac{1}{2}$ ¢ per square yard.
2. Find the area of a right triangle, base 25 ft., hypotenuse 60 ft.
3. A rhomboidal field contains 5 acres, and measures 50 rods along a straight road. How wide is it?
4. Name the six quadrilaterals and the four parallelograms. Draw a trapezoid and its equivalent rhomboid and rectangle.
5. Required the contents of the surface of a stick of timber 18 ft. long, 4 in. thick, 8 in. wide at one end, 12 in. at the other.
6. The diagonal of a trapezium is $22\frac{1}{2}$ ft., and the perpendiculars drawn from the vertices of the angles opposite it are 16 ft. and 12 ft., respectively. What is the area of the trapezium? Draw it.
7. Find the cost of 28 six-by-four joists, averaging 18 ft. in length, at \$32 per M.
8. What is the area of a walk 3 ft. wide, around a semicircular flower-bed, the straight edge of which measures 12 ft.?
9. There is a difference of 6 in. in the diameter of the wheels of a carriage. The fore-wheel turns 1000 times in going a certain distance. The hind-wheel turns x times and is 4 ft. in diameter.
10. What is the axis of a sphere 4 ft. in circumference?
11. From a sheet of zinc, weighing 16 lb., and measuring 8 ft. by 4 ft., a square was cut, reducing its weight to $11\frac{1}{2}$ lb. How long was the square?
12. How much ground is covered by 12 cords of 4-ft. wood piled 2 ft. high?
13. A granite sphere barely clears a gateway 2 feet wide. Find its contents.
14. Three semicircles are so arranged that their 3 ft. diameters enclose a triangle. Find the area of the whole figure.

349.—Mensuration.*Written.*

1. A span of horses draws a load of brick weighing two tons. The brick are of the ordinary size, $8 \times 4 \times 2$, and weigh 100 lb. to the cubic foot. How many bricks in the load?
2. Find the contents of a cone 6 in. in altitude and 2 in. in diameter at the base.
3. Find the slant height of a square pyramid 12 in. in altitude, base 8 in. on a side.
4. Find the entire area of a hemisphere 15 in. in diameter.
5. How long is the equator, the equatorial semi-diameter of the earth being 3963.296 miles?
6. Compare the perimeters of a rectangular field 60 rd. by 30 rd., of an equivalent square field, and of a circular field of the same area.
7. The inside dimensions of a cellar are 16 ft., 12 ft., and 8 ft. The wall is to be two feet thick. How many cubic yards of earth will need to be removed?
8. A cylindrical 8-inch driven well is 76 ft. deep. How many cubic feet of earth, etc., have been taken out?
9. One fire engine throws a 2-inch stream, another a $1\frac{3}{4}$ -inch stream. Compare the amount of water thrown.
10. One piece of shafting 2 in. in diameter weighs 300 lb. What is the weight of a similar piece $3\frac{1}{2}$ in. in diameter?
11. How deep shall a 12-ft.-square bin be made to hold 1728 bushels?
12. How many cords of wood in a section of a giant pine 18 ft. long and 16 ft. in diameter?
13. If a 2-inch rope breaks with a weight of 8000 lb., what weight might break a similar rope 3 in. in circumference?
14. Give area of basin required for a fountain that throws its spray out 15 feet.
15. A cone of cinder 50 ft. high is 314.16 ft. in circumference at the base. What is the shortest line of ascent?
16. A park 1 m. square is bounded by a 10-ft. walk inside the wall. How many squares of cement cover the walk?

350.—DEFINITIONS.

[FOR REFERENCE.]

Acceptance. The formal agreement by signature of a drawee to pay a draft according to its terms.

Agent or Correspondent. One employed to transact business for another.

Assessment. Money collected from shareholders in stock companies to meet losses or expenses.

Assessors. Officers who estimate the value of taxable property and apportion the tax to be raised.

Average of Accounts. The process of finding the equated time for the payment of the balance of an account.

Bank. A corporation formed to trade in money and securities, or for the custody and loaning of money.

Bank Discount. The allowance made to a bank by the holder of a note for having it paid to him before maturity.

Base. The number of which a percentage is taken.

Bill of Exchange. A general term for foreign or domestic drafts, especially for the former.

Bonds. A series of interest-bearing notes of a government or corporation.

Broker. An agent who buys and sells securities or other property.

Brokerage. A broker's fee or commission.

Capital. Money or other property invested in business.

Charter. A special act of a legislature setting forth the rights and duties of a corporation.

Check. A depositor's order for the payment of money by his bank.

Commission. A percentage paid to an agent for transacting business for another.

Compound Ratio. The indicated product of two or more simple ratios, as $3:2 \times 4:8$, or $\left. \begin{matrix} 3:2 \\ 4:8 \end{matrix} \right\}$.

Cone. A solid having a circle for its base, and tapering uniformly to a point, the vertex of the cone.

Consignor. One who sends merchandise (a *consignment*) to an agent (the *consignee*) to be sold.

Corporation. A company authorized by charter to transact business as a single individual.

Coupons. Interest certificates attached to bonds.

Days of Grace. Three days, in addition to the time named in a note, allowed by law in most states for the payment of a note by its maker.

Discount. An allowance deducted.

True Discount. The difference between the face and the present worth of a debt due at a future time without interest. The interest on the present worth.

Dividend. Profits of business divided among stockholders in proportion to their shares.

Draft. An order sent by one party to another, requesting him to pay a specified sum to the order of some one named.

Sight Draft. One payable when presented to the drawee.

Time Draft. One payable at a specified time after sight or after date.

Drawee. The party ordered to pay a draft.

Drawer. The maker of a draft.

Duties or Customs. Taxes laid by the government on imported goods.

Duty, ad valorem. A tax of a certain per cent of the cost of imports in the country where they are bought.

Duty, specific. A fixed tax levied on imports according to weight, number, or measure.

Equation of Payments. The process of finding when several debts due at different times may be paid at one time without loss to either debtor or creditor.

Exchange. A method of making payments or collections in distant places, by means of orders or drafts, without the actual sending of money.

Extremes. The third and fourth terms of a ratio.

Face of Note, Check, or Draft. The sum for which it is written.

Gross Weight includes the material used in packing.

Imports. Merchandise brought from a foreign country.

Indorsement. A signature on the back of negotiable paper. A record of payment on the back of a note.

Indorser. One who puts his signature on the back of a note, check, draft, etc.

Insurance. Compensation for loss by fire or other disaster.

Annual Interest. Simple interest on the principal, and simple interest upon any overdue interest.

Compound Interest. Interest reckoned on both the principal and the overdue interest added to the principal as often as due.

Exact Interest. Interest computed for parts of a year by taking the exact number of days and reckoning 365 to a year.

Leakage and Breakage. A discount for liquors lost from casks or bottles during importation.

Maker of a Note. The one who makes the promise and signs it. The promissor.

Market Value. Present value in open market.

Maturity. The time when a note, draft, or bond falls due and is legally payable.

Means. The second and third terms of a ratio.

Negotiable Paper. Notes, drafts, or other written obligations that may be bought and sold.

Net Price or Cost. The price or cost after all discounts or charges have been deducted

Net Weight. Weight exclusive of packing material.

Demand Note. One payable at the demand of the holder. **Time Note.** One payable at a specified time.

Interest-bearing Note. One containing the words "with interest."

Promissory Note. A written promise to pay a specified sum of money.

Partial Payments. Payments in part of a note or debt.

Par Value. Face value.

Payee. The one to whom or to whose order a note, check, or draft is payable.

Percentage. The process of computing by hundredths. The part of the base indicated by the rate per cent.

Personal Estate. Property exclusive of land and buildings.

Policy. The written agreement given to the insured by the underwriters.

Poll-tax. A uniform tax on persons of a certain class.

Port of Entry. A city or town containing a custom-house, where U. S. duties are paid.

Premium. The sum paid for insurance. Excess of market value above par value.

Present Worth. The sum that, at the present time, will pay a non-interest-bearing debt due in the future, without loss to either debtor or creditor.

Proceeds or Avails of a Note. The sum for which the note is sold. Its maturity value less the bank discount. **Net Proceeds.** What is left after all charges have been deducted.

Proportion. An expression of the equality of two ratios.

Pyramid. A solid, whose base is a regular polygon, and whose sides are triangles meeting in a common point, the vertex of the pyramid.

Rate per cent. The number of hundredths used in finding a percentage.

Real Estate. Land and buildings.

Remittance. Money or negotiable paper sent to another.

Share. One of the equal parts into which corporation capital is divided.

Slant Height. The shortest distance from the vertex of a cone or pyramid along the outside to the base.

Sphere. A solid having a curved surface equally distant from the centre at every point.

Stocks. Shares in the capital of corporations. Government or corporation bonds.

Stock Certificate. A statement given by a corporation, showing the par value and the number of shares owned by a stockholder.

Stockholders. Owners of the capital or stock of corporations.

Tare. An allowance for the weight of boxes, bags, etc., used in packing goods.

Tariff. The list of dutiable articles with the rate assessed on each.

Taxes. Money raised by government for public uses.

Term of Discount. The time between the day of discount and the day of maturity.

Underwriters. Insurance companies.

MISCELLANEOUS EXERCISES.

Oral.

351. — 1. What makes a fraction larger?

2. What is it that shows how many times the denominator of a fraction is contained in its numerator?

3. In $\frac{28}{32}$ how many 40ths?

4. What is the ratio of $\frac{2}{3}$ to $\frac{5}{6}$?

5. Compare $16\frac{1}{4}$ with $3\frac{1}{4}$.

6. Add $\frac{1}{2}$, $\frac{1}{6}$, and $\frac{2}{3}$.

7. From $7\frac{1}{7}$ take $6\frac{1}{6}$.

8. $16 \times \$8\frac{1}{5} = \x .

9. $18\frac{3}{4}\%$ of $\$14.40 = \x .

10. $\$17$ is $\frac{2}{3}$ of $\$x$.

352. — 1. $\frac{5}{6}$ of 48 is 80% of x .

2. $\frac{2}{3}$ is $\frac{2}{3}$ of x .

3. $7 : 3\frac{1}{2} = 1\frac{2}{3} : x$.

4. $\$6$ is $\frac{3}{4}\%$ of what?

5. $0.06 \times 1.20 = x$.

6. $0.006 \times .0120 = x$.

7. $\sqrt{5\frac{1}{16}} = x^2$.

8. $4.2 \div 0.007 = x$.

9. $0.42 \div 70 = x$.

10. $0.042 \div 0.0007 = x$.

353. — 1. $3\frac{1}{2}\%$ of 500 = x .

2. $\$2$ is what % of $\$800$?

3. $\$6$ is $\frac{2}{3}\%$ of what number?

4. What per cent of a 2 ft. cube are 3 cubic feet?

5. A miller takes 4 quarts, or $x\%$ toll, out of every bushel.

6. Spent $\$27$, or 30% of my money. I had $\$x$ remaining.

7. What rate of income have I on an $\$8000$ house, rented at $\$40$ a month.

8. The tax is 15 mills on a dollar, or $x\%$.

9. $\$160$ is 0.8% of $\$x$.

10. Wages are increased 10% . Men that now receive $\$1.65$ formerly had $\$x$.

354. — 1. $\$22.50$ is what % of $\$750$?

2. $\frac{1}{4}$ of the selling price was gain. The gain per cent?

3. Selling at $\$3.50$ I lost $12\frac{1}{2}\%$. What should I have gained by selling at $\$5$?

4. Sold stock at 120 that cost 125. Loss % = x .

5. 10ϕ is 125% of what?

6. Sold for $\$2$, and gained 400% .

7. A patent medicine costs 16¢ a gallon and sells for \$1 an 8 oz. bottle, or $x\%$ of the cost. (16 fluid ounces = 1 pt.)

8. $\frac{4}{7}$ yd. = x ft. y in.

9. Cost of $\frac{2}{3}$ lb. at \$ $\frac{1}{2}$ an oz.

10. If from $\frac{4}{5}$ of my money I had taken \$2.50, $\frac{2}{3}$ of my money would have remained.

355.—1. $4\frac{1}{2} \times 2\frac{2}{3} \times x = 25$.

2. 0.0375 = what common fraction?

3. Change $\frac{2}{3}$ of a week to hours.

3. $\frac{1}{8}$ mile = how many feet?

4. $\frac{\frac{3}{4} \times \frac{8}{9}}{\frac{2}{3}} + \frac{37\frac{1}{2} \times 2}{25 \times 3} = x$.

5. Divide $\frac{4}{5}$ into three equal parts; 3 such parts make how much?

6. $\frac{2}{7}$ of $\frac{2}{3}$ = what?

7. $2 - \frac{3}{2} - \frac{1}{8} - \frac{3}{4} - \frac{1}{4} - \frac{3}{16} = x$.

8. $2\frac{5}{8}$ pounds = x ounces.

9. $33\frac{1}{3} \div 6\frac{2}{3}$ = what?

10. Add $\frac{7}{10}$, $\frac{5}{12}$, $\frac{2}{15}$.

11. Repeat the table of length measures, giving the value of each unit in feet.

356.—1. Sum of prime numbers between 20 and 30.

2. G. C. D. of 51 and 85.

3. L. C. M. of 6, 8, 16.

4. $33\frac{1}{3}\%$ of $7\frac{1}{2}$ yards.

5. 20% of half a million.

6. $41\frac{1}{4}$ yd. at \$0.12 $\frac{1}{2}$.

7. The square root of 90,000.

8. Surface of a 10-ft. cube.

9. 20 da. interest of \$120.

10. Proceeds of a 60-day note for \$800, 9%, without grace.

357.—1. Longest straight line that can be drawn on a 6 × 8 sheet.

2. Cost \$18. Proceeds of sale \$22. Gain per cent?

3. Sold at $\frac{1}{2}\%$ commission and earned \$40. Sales?

4. \$36,000 insurance at $\frac{3}{4}\%$. Premium?

5. Rate of taxation, \$16 on \$1000. Property tax, \$112. Assessed value of property?

6. $8:24 = 7\frac{1}{2}:x$.

7. Weight of an inch cube, 10 oz. Of a 5-inch cube?

8. Cost of an acre at 10¢ per square foot.

9. Square root of $2\frac{7}{9}$.

10. Cost of 11 at \$2 a dozen.

358.—1. Add 8 to $\frac{2}{3}$ of a number, and we have 32. The number?

2. $6\frac{2}{3}$ yd. cost \$ $5\frac{1}{2}$. 20 yd. ?
3. Divide 50 in the ratio of 7 to 3.
4. How many yards of carpet will be needed for a hall 9 ft. by 15 ft. ?
5. A building lot contains 3,382 sq. ft. and is 100 ft. long. How wide is it ?
6. Cost of $\frac{1}{2}$ gross at 8 for a dime.
7. Divide 0.6 by 0.015.
8. $4 \times \sqrt{256} = x^2$.
9. $\frac{1}{3}$ is what per cent of $\frac{1}{2}$?
10. An arc of 80° is what part of a circle ?

359.—1. A house bought for \$3,200 was sold for \$3,000. What per cent was lost ?

2. \$21 pays for how much insurance at $\frac{3}{4}\%$?

3. $12\frac{1}{2}\%$ of 64 is $33\frac{1}{3}\%$ of x .

4. Find the interest of \$400 for 66 days at 6% .

5. What is an agent's commission at $1\frac{1}{2}\%$ on \$3,000 ?

6. What will pay a note of \$500 that has been running $\frac{2}{3}$ of a year at 9% ?

7. Cost of 5 yards at \$1.50 a yard, $33\frac{1}{3}\%$ discount.

8. Bought 3 for 4ϕ and sold at the rate of 2 for 3ϕ . Did I gain or lose, and how much ?

9. Bought for \$10 and sold for \$2.50. What per cent was lost ?

10. What did I pay as an assessment of 2% on 15 \$100-shares of stock ?

360.—1. What is the perimeter of an equilateral triangle whose side is $3\frac{3}{4}$ ft. ?

2. How many feet in $4\frac{1}{2}$ rd. ?

3. How many degrees does the long hand of a watch move over in 25 minutes ?

4. What is my milk bill for 3 weeks, 3 pints being taken daily at 7ϕ a quart ?

5. Cost of 49 lb. of flour at \$5 per barrel.

6. Cost of $12\frac{3}{4}$ yd. at \$2 a yard.

7. Contents of a piece of cloth containing 240 $\frac{1}{2}$ -inch checks.

8. What will 5 tons of coal cost at 4ϕ a pound ?

9. My weight increased from 150 to 175. What was the per cent of increase ?

10. The side of a rhombus measures $6\frac{2}{3}$ in. Its perimeter is what ?

361.—1. 9 months interest of \$200 at 8%.

2. How long will \$500 be in earning $\frac{1}{3}$ of itself at 5%?

3. $\frac{3}{4}$ yd. is what part of $\frac{7}{8}$ yd.?

4. What is the ratio $\frac{1}{8}$ to $\frac{1}{6}$?

5. $13\frac{1}{2}$ = product of means; one extreme is 27. The other?

6. The junior partner has $\frac{4}{5}$ the capital and \$720 gain. What is the whole gain?

7. What time is it when the hours before noon equal the hours after noon?

8. Eighty cents remains after losing 2 dimes and spending $\frac{1}{5}$ of my money. I had \$ x at first.

9. A can do $\frac{2}{15}$ of his work in 20 minutes. When will he finish if he begins at 10 A.M.?

10. How many yards of 3 in. bandages can be made from 7 yards of yard-wide linen.

362.—1. If it is $\frac{3}{4}$ of a mile to the post-office, how far will you travel in making 7 round trips to it?

2. It is between 10 and 11, and the long hand has moved over 72° since the clock struck. What time is it?

3. How wide is a yard of ribbon that contains 1 sq. ft.?

4. To $\frac{2}{3}$ add its cube.

5. How many acres in a farm $\frac{1}{4}$ mile square?

6. From $\frac{3}{4}$ take its square.

7. $4 : x = x : 9$.

8. The inverse ratio of $\frac{2}{3}$ to $\frac{3}{4}$?

9. The perimeter of a square

10 A. field, or $\frac{1}{100}$ of a sq. mile?

10. What per cent of a day has gone at 2 P.M.?

363.—1. $0.075 \times 0.4 = x$.

2. $\frac{7}{8}$ yd. at \$2, and $1\frac{2}{3}$ yd. at $37\frac{1}{2}\phi$ cost \$ x .

3. What cost $4\frac{2}{3}$ yd. at \$1.25?

4. What shall I pay for $16\frac{2}{3}$ lb. at 18ϕ ; and $87\frac{1}{2}$ lb. at 32ϕ ?

5. How many eggs in a basket if 12 are bad and 96% good?

6. How many 18 in. napkins will cover a table 2 yards square?

7. At \$4 a dozen how many brooms can be bought for \$2.50?

8. \$ $12\frac{1}{2}$ is $8\frac{1}{3}\%$ of what?

9. 25 eggs at 25ϕ a dozen?

10. What will \$62.50 amount to in 200 months at 6%?

364.—1. Sold for \$16 and gained $33\frac{1}{3}\%$. What per cent should I have gained at \$15?

2. Gave \$2 for a bunch of 108 bananas. What per cent was gained by selling 2 for 5 cents?

3. $10\frac{1}{2}$ lb. cost \$2.60; 21 lb.?

4. Is it better to buy for \$4 and sell for \$4.50, or to sell for \$8 what is bought for \$7?

5. A can roof a house in 3 days; B, in 4 days; both in x .

365.—1. Add $12\frac{7}{18}$, $19\frac{5}{24}$, $10\frac{17}{36}$.

2. From $108\frac{9}{16}$ take $96\frac{1}{8}$.

3. Find the cost of $18\frac{3}{4}$ yd. at \$3.37 $\frac{1}{2}$. 16 $\frac{7}{12}$ doz. at \$166 $\frac{2}{3}$.

4. 4650 lb. coal at \$5.40; 37 bbl. cement at 1.58 $\frac{3}{4}$; $\frac{1}{6}$ off, special discount.

5. $(6.25 \div 3\frac{1}{2}) \div (3\frac{7}{8} - 0.275)$.

6. $\frac{4}{7}$ of a lot of land sells for \$1185.50. What is the rest of it worth at that rate?

7. 0.6825 less $\frac{1}{3}\frac{1}{20} = x$.

8. A can do $\frac{1}{3}$ of a piece of work in a day. B can do the work in 5 days. Together they can do it in x days.

9. From $\frac{3}{8}$ mile take $15\frac{3}{8}$ rd.

10. From $\frac{1}{2}\frac{9}{4}$ cu. ft. take 19 cu. in.

11. At \$ $\frac{5}{8}$ a yard, how many yards can be bought for \$17?

12. Find the cost of $\frac{7}{8}$ gal. at 12 $\frac{1}{2}$ ¢ a quart; 25 eggs at 20¢ a dozen; 11 quires at \$2.50 a ream?

13. How many square feet in $\frac{7}{11}$ of an acre?

14. 1728 sq. ft. = what fraction of an acre?

366.—1. My profits of 35% were \$210 one week, and \$180 the next week. My sales for both weeks were \$ x .

2. Sold a house at an advance of 18% for \$3240. What was its cost?

3. Bought goods for \$400 at a discount of 10 and 5%, and sold at the same price discounting 5 and 10%. Did I gain or lose, and what?

4. Received $\frac{1}{3}\%$ for collecting x dollars, and earned \$22.50.

5. Sold to A for \$400; he sold to B for \$500. His gain and mine were at the same rate. What was the cost to me?

6. Bought 2000 lb. of sugar at $4\frac{3}{4}$ ¢ and sold 18 lb. for \$1. What was my gain per cent on a pound? On the whole lot?

7. A dealer sold two bicycles for \$100 each. On one he gained 25%, and on the other he lost 25%. Total gain or loss?

8. Had I better pay \$256 cash or \$260 in 3 months?

9. Sold 60% of a lot of land for what $\frac{2}{3}$ of the whole cost. My gain was $x\%$.

10. 288 is $\frac{9}{20}$ of how many 5ths of 84?

367.—1. Describe one's position after travelling 600° due west.

2. $186,360 \div 5,280 = 186,360 \times x$.

3. $\frac{16\frac{2}{3} \times 66}{12\frac{1}{2}} = x$.

4. $\frac{1}{7} + \frac{2}{49} + \frac{3}{196} - \frac{3}{343} = x$.

5. The times at which successive miles of a bicycle race were completed were:

h.	min.	sec.	h.	min.	sec.
2	2	10	2	10	$12\frac{1}{5}$
2	4	$46\frac{2}{5}$	2	12	$45\frac{3}{5}$
2	7	$26\frac{2}{5}$	2	15	$27\frac{3}{5}$

Find the average time for the five miles.

6. If the wind through a tree pulls at each of 50,000 leaves with a force of 1 oz., what is the strain on the trunk?

7. 1,000 strips of canvas are wanted 38 in. long. In cutting every 20 strips 1 foot is wasted. How many yards must be bought?

8. Six dogs can lift 800 lb., but they can pull 4 sledge loads weighing together 9 times as much. Find by cancellation each dog's share of each load.

9. Capital \$23,000, profits \$3,680 or $x\%$.

10. \$238 at 7% for 178 d.

368.—1. Amount of a \$525 note, 1 yr. 17 d. after date, at 5% .

2. Proceeds of a \$2,750 note discounted 117 days before maturity, at 7% .

3. Cost of putting a 6-inch coating of loam on a garden patch 60 ft., by 36 ft., at 50ϕ a cubic yard.

4. My agent's commission is $2\frac{1}{2}\%$ on a purchase of hides, all expenses being covered by a remittance of \$1132. Freight cost \$25. Required his commission.

5. Recovered \$8,500 insurance on a total loss. What was my actual loss of 34% ?

6. The surface of a hemispherical dome 15 ft. high is x sq. ft.

7. A cubic foot of iron weighs 450 lb., a 15-in. cannon ball, x lb.

8. A cylindrical oil tank $3\frac{1}{2}$ ft. by 22 ft. holds x gallons.

9. The compound interest of \$350 for 2 yr. 3 mo., at 4% , is \$ x .

10. A note for \$500 running 18 mo. has had two semiannual \$100 payments. What is now due?

369.—1. Find the average receipts for a full day at an office which is closed from Saturday noon to Monday morning, and takes in \$72,147.84 in three weeks.

2. What per cent of discount lets a \$225 horse go for \$185 $\frac{1}{2}$?

3. A conical sugar loaf 6 in. at the base and 1 ft. high would make a square cake 1 in. thick, x in. wide.

4. What is the rate of duty on jewelry if \$402.50 is charged on an invoice of \$1,150?

5. After my bill has been reduced by successive discounts of 20% and 10%, I can pay it for \$101.664. What was the gross amount charged?

6. A debt bearing 7% interest increases \$61.52 by standing 2 yr. 1 mo. How large is it at first?

7. In 1885 aluminum sold for \$12 a pound. In 1895 it can be bought for 35 cents. What is the per cent of reduction?

8. China is to pay Japan a war indemnity of \$160,000,000. One half is to be paid at once, and the remainder in 6 years, with simple interest at 6%. What will be the amount of the second payment?

9. The successful candidate received 6,750 votes. His opponent had 3,825. How were every 100 votes divided?

10. What is the tax rate on \$1 where \$73,000 are to be raised on a valuation of \$5,309,090.

11. If the Great Pyramid covers 13 A., how long is one side? How much more stone would be required to make a prism of the same base and altitude?

12. How large a square could be made from the cover of a 3-inch ball?

13. \$1,200 is due with interest and without grace in 1 year. If half is prepaid now, how much will discharge the debt at the end of 6 months?

370.—1. A man, engaged at \$45.00 a month, worked from Dec. 14 to May 9. His wages were x dollars.

2. I bought 1185 lb. of hay, at \$19.50 per ton, for x dollars.

3. The assessed value of an estate is \$55,000. The house is assessed for \$28,200. The property has a frontage of 36 ft. and is 126 ft. deep. The land is assessed for x dollars per foot.

4. Water freezes at 32° above 0 (Fahrenheit) and boils at 212° ; but, according to the Centigrade thermometer, it freezes at 0° and boils at 100° . 68° F. equals x° C.

5. Bought an estate comprising 19,367 sq. ft. for \$27,800. I sold the buildings for \$2685, and paid the interest on a mortgage of \$12,500 for 3 years at 5%, also \$565.00 for taxes. I then sold the land at \$1.45 per sq. ft. I gained or lost x dollars, not counting interest on my money.

6. The last reading of my gas-meter was 67,300 cu. ft. The previous reading was 64,900. At \$1.50 per thousand, with a discount of 15¢ per thousand ft., my gas bill was x dollars.

7. By working every week day in the month of August, which began on Sunday, a boy picked $2\frac{1}{2}$ bu. of berries, which he sold for $6\frac{1}{2}$ ¢ a quart. He earned x cents a day.

8. Mr. Brown is taxed for x dollars. His tax bill is \$110.00, including \$2.00 for poll tax, the rate being 9 mills on a dollar.

9. A man received x dollars for sawing a pile of cord wood, 36 ft. long and 4 ft. high, @ \$1.50 per cord.

10. Two coaches set out on the same journey. The red one completes $\frac{5}{6}$ of it, the yellow one, $\frac{4}{5}$. Which is greater, the distance between them or the distance the yellow one has to go?

11. Two ships sail the same course in opposite directions. One has finished $\frac{3}{4}$, the other $\frac{2}{3}$ of it. What part of the whole distance lies between them?

12. For 1895 a clerk's salary was \$1242. It had been increased 10% in 1893 and 8% in 1894. What was it in 1892?

13. What was the per cent of increase from 1892 to 1895?

371.—1. How far is a man from his starting-point, who travels west 48 miles, then due north 62 miles, and then east 14 miles?

2. Show that 90 days' interest on any sum of money at 4% is found by removing the point two places to the left.

3. 15° difference in longitude makes an hour's difference in the time of sunrise. The sun rises at 4.50 at A; at B, 80° farther west, it is then what o'clock by sun time?

4. Each side of a triangle measures 30 ft. Its area is x .

5. A lot containing 30,000 square feet is $\frac{1}{3}$ as wide as it is long. What will it cost to fence it at $12\frac{1}{2}\phi$ a running foot? (What part of the lot makes a square?)

6. A cubic foot of distilled water weighs 1000 oz. The weight of any substance, as compared with that of an equal bulk of pure water, is called its *specific gravity*. The specific gravity of water is 1; of gold, 19.258; of cork, 0.240; of pure alcohol, 0.794. What is the specific gravity of granite that weighs $165\frac{3}{4}$ lb. to the cubic foot? 7. What does a gallon of alcohol weigh?

8. Bought railroad stock at $114\frac{3}{4}$, and sold at $117\frac{1}{4}$, the purchaser in each case paying $\frac{1}{8}\%$ brokerage. What was my profit on 200 shares?

9. $\frac{3}{4}$ carpeting is used for a room 20 feet square. The waste in matching is 6 inches to a strip. The cost at \$1.75 per yard is \$ x .

\$800—

Chicago, March 19, 1895.

Four months after date I promise to pay to the order of _____ Gordon Bacon _____

_____ Eight Hundred _____ Dollars

at _____ the North Bank.

Value received.

Victor Robinson.

10. This note was discounted April 17, at 5%. Proceeds?

372.—1. If $\frac{2}{3}$ of a day's wages are \$1.40, what shall be paid for $\frac{7}{15}$ of a day?

2. $\frac{7}{32}$ is $\frac{4}{5}\%$ of what number?

3. A merchant gains $7\frac{1}{2}\%$ by selling goods for \$2128.50. What did they cost?

4. 24 carat gold is pure gold. What per cent of alloy in 14 carat gold?

5. Bought 300 gross of files listed at \$14.25, at 20, 10, and 5 off. Required the cost.

6. A dealer obtained \$480 for a piano on the list price of which he had discounted 50%. He still made a profit of 20%. What did it cost him?

7. Bought \$15,000 worth of goods on 4 months, and sold immediately for \$14,900 cash. Money being worth 5%, what did I gain?

8. The diameter of a circle is 20 in. Find the side of an approximately equal square.

9. How many cubic inches will a box contain, 9 by 12 by 15 in., outside measure, if it is made of $\frac{1}{2}$ in. stuff.

10. A was to receive $\frac{2}{3}$, B $\frac{3}{5}$, and C the remainder of a bequest of \$18,000. C died, and his share was equitably divided between A and B. What did each receive?

11. What sum will cancel a 5% note for \$763, dated April 19, 1894, and maturing Aug. 11, 1896?

12. If a speculator saves himself from failure by borrowing \$50,000 for 4 hours at 1% per day, how much interest does he pay?

13! I bought cloth by the meter (39.37 in.), and sold at the same price per yard. What per cent did I gain?

14. The 600 shade trees in the common are increased in five years to 1350. What is the yearly per cent of increase?

15. A 17 in. square of basket work is plaited into 289 small squares. How wide is the material used?

373.—1. What is the semiannual dividend on 15 \$50 shares of 7% railroad stock?

2. 4% government bonds yield an annual income of \$1000. What is their face?

3. How many shares of stock that sell at a premium of $37\frac{1}{2}$ can be bought for \$5,000, no brokerage?

4. \$120 yields \$8 annually. What rate per cent is this?

5. What is the market value of 25 shares of New York Central stock at $46\frac{3}{4}$ premium?

6. Paid \$200 for a share of telephone stock, and received an annual dividend of \$10. This was x per cent on my investment.

7. What does a \$6,500 mortgage yield semiannually at $5\frac{1}{2}\%$?

8. Stone & Co. purchase \$1,500,000 city bonds at $2\frac{3}{4}\%$ premium, one third of which they sell at a premium of $4\frac{3}{4}$, one third at $5\frac{3}{8}$, and the remainder at $4\frac{9}{16}$. What is their profit, the cost of selling being $\frac{3}{4}\%$?

9. Sold a 7% mortgage for \$3,000 at 25% premium, and bought 6% railroad stock with the proceeds. Did I increase or lessen my income, par of stock being \$50?

10. Which is the better investment, 5% stock at 120, or 6% at 150? $\frac{5}{120}$ or $\frac{6}{150}$?

11. \$328 is my annual income from 4% stock. What is it worth at \$125 per share?

12. A charitable organization has the following assets. What is its annual income? \$6,000 in 5% mortgages; \$25,000 in government 4's; 900 shares 4% stock; 2,000 \$10 shares yielding $3\frac{1}{2}\%$ semiannually.

13. A watch appears to be 5 h. slow because of a change of longitude. Has it been taken east or west, and how many degrees?

14. A side of a hexagon is 6 in. Find the distance between its parallel sides. 15. Give the area of the hexagon, and that of the largest circle to be drawn within it.

**374. — Questions
on Principles and
Methods.**

1. Is a fraction a number? 2. Mention four integral units of different kinds and four fractional units of different values.
3. Show how the fundamental process of arithmetic may be reduced to two, — putting numbers together and separating a number into parts.
4. Show the effect of moving the decimal point.
5. What is the difference between a decimal and a common fraction?
6. Show why doubling the denominator halves a fraction.
7. Explain the principle on which the change of a fraction to larger units is based.
8. How is the number of decimal places in the product determined? Explain why this is so.
9. When does a note begin to draw interest? 10. Why are notes sometimes indorsed?
11. Why does multiplying the antecedent multiply the ratio?
12. What is the difference between specific and ad valorem duties?
13. Show why pointing off two places from the right of a principal gives 2 months' interest at 6%.
14. Why do leap years contain 366 days? 15. What are the *natural* divisions of time? 16. Why will your watch be an hour fast if you go 15° west?
17. Why should a 5-inch ball be 125 times as heavy as a 1-inch ball of the same material?
18. Why should a pound of feathers be heavier than a pound of gold?
19. How would you show the ratio of circumference to diameter to be 3.1416?
20. What is meant by saying that the specific gravity of cork is 0.240?
21. Show by a drawing that the "square of the hypotenuse equals the sum of the squares of the two short sides of a right triangle."

375.—1. If $38\frac{3}{4}$ bushels of turnips cost $\$28\frac{3}{5}$, what will $12\frac{1}{2}$ bushels cost?

2. The cost of 50 gallons of molasses is $\$25$. If $\frac{1}{5}$ is lost by leakage, and 20 gallons are sold at $62\frac{1}{2}\phi$ a gallon, at what price per gallon shall the remainder be sold to gain $\$5$?

3. A gentleman paid $\frac{3}{5}$ of his money for a farm; had he paid $\$100$ more, he would have paid $\frac{5}{8}$ his money. The farm cost $\$x$.

4. An agent received $\$32$ as his commission at 4% for buying flour at $\$5$ a barrel. How many barrels were bought?

5. At $2\frac{1}{2}\%$ discount, what shall be paid for 4% stock so that the annual income shall be $\$2400$?

6. A house was sold for $\$1850$ at an advance of 15% on the cost. What would it have brought at a gain of 20%?

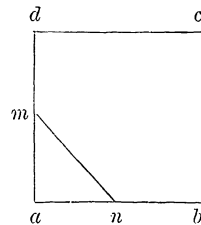
7. Three drovers rent a 9-acre field at $\$5$ an acre. A puts in 6 cows for 2 months; B, 9 cows for 1 month; and C 12 cows for 2 months. What should each pay?

8. 75% of a farm is arable; of the remainder, 80% is pasture, and the remaining 3 A. 20 sq. rd. is waste. What is the area of the farm?

9. How long must a stick of timber be which is $1\frac{1}{2}$ feet square throughout its length and whose volume is 100 cubic feet?

10. A rectangular field is 64.8 rods long and 36.05 rods wide, and a square field is of equal area. At $\$1.10$ a rod, how much more will it cost to fence one than the other?

11. The accompanying figure represents a surface containing 15,376 square feet. Find the length of mn , connecting the middle point of two sides.



12. *Memorandum.* — Note for $\$1824$, dated Chicago, Oct. 10, 1895. Payment of $\$500$ was made April 25, 1896. What was due Jan. 2, 1897, interest at 6%?

376.—1. From a field containing 50 A., I sold a corner 100 rods long and 40 rods wide. What % remained?

2. I paid \$25 for linoleum, at \$1.25 per square yard. The length of my floor was 15 ft. What was its width?

3. What is the rate of interest when \$62.50 is paid for the use of \$800 for 1 yr. 3 mo.?

4. I paid $\frac{3}{4}\%$ brokerage on a sale of wheat at \$1.15 a bushel. The brokerage was \$63.48. How much wheat was sold?

5. Sold my carriage at 30% gain, and with the money bought another, which I sold for \$182 and lost $12\frac{1}{2}\%$. What did each carriage cost?

6. A owns $\frac{4}{9}$ of a manufacturing plant. The plant is valued at \$48,870. A sells a part of his interest for \$10,860. What part of his interest does he sell?

7. What rate of interest do I get for my money when I buy 6% bonds at 108?

8. A horse tied to a stake can graze over $218\frac{1}{8}$ sq. yd. of surface. How far from the stake?

9. How many acres in a street $1\frac{1}{8}$ miles long and 4 rods wide?

10. A man with \$27,000 has his choice of two equally safe investments,—a 5% mortgage, or bonds at 135 paying 7% on the par value. Which would you advise him to take?

11. An irregular piece of land, containing 4 A. $62\frac{1}{4}$ sq. rd., is exchanged for a square piece at the same price per foot. What is the length of a side of the latter?

12. Find the lateral area of an octagonal pyramid whose slant height is 30 in., each edge at base 20 in.

13. How long is a chain of 1600 one-inch links made from $\frac{1}{16}$ -inch wire?

14. A coal-bed 10 ft. thick covers a sq. mile. How many 15 T. car-loads, allowing 45 cu. ft. to a ton?

377.—1. The product of two numbers multiplied by $\frac{1}{2}$ is 0.0005. One of the numbers is 0.05. What is the other?

2. What will it cost to cement a cellar bottom 36 ft. long, 23 ft. 7 in. wide, at 96¢ a square yard?

3. A dry article was exposed to dampness, and absorbed 4 ounces of water. It then weighed 9 pounds. What per cent of this weight is water?

4. Find the value of the liquid in a cylindrical can 11 in. in diameter and 21 in. high, at 20 cents a pint.

5. A piece of cloth, when measured with a yard-stick that is $\frac{2}{3}$ of an inch short, appears to be 18 yards long. What is its true length?

6. A square field, containing $27\frac{1}{2}$ A., has a diagonal path across it x rods in length.

7. Sold $\frac{2}{5}$ of an article for what $\frac{3}{4}$ of it cost. What was the gain per cent?

8. Find the surface of a sphere 25 inches in diameter.

9. A square court, whose side is 42 yards, is paved with 28,224 square tiles. Find the dimensions of each tile.

10. A merchant buys goods for \$1125. He sold $\frac{1}{2}$ at an advance of 25% on the cost, $\frac{2}{3}$ at an advance of $12\frac{1}{2}\%$, and the remainder at one-half their cost. What was his profit?

11. What will \$1350 amount to in 125 days at $4\frac{1}{2}\%$?

12. Sold a horse so that $\frac{4}{5}$ of the gain equalled $\frac{2}{15}$ of the cost. What was the gain per cent?

13. A, B, and C trade in partnership. A furnished $\frac{1}{2}$ the capital, and is to have $\frac{1}{3}$ the gain for extra services. B and C furnished \$3000 each, and the gain is \$5760. How is it divided?

14. $1.92 \div 0.0048 - 0.0048 \div 192 =$ what?

15. Twelve persons hired a boat for a certain sum. Four of them withdrew without paying, and thus the expense of each of the others was increased by \$2. What was the rent of the boat?

378.—1. A merchant bought 3 yards for \$2 and sold 2 yards for \$3. What was his gain per cent?

2. What is the entire surface, in square feet, of a box 18 in. by 16 in. by 14 in.?

3. A coat cost \$8. How shall it be marked to lower the price 20% and still gain 20% on the cost?

4. How much will it cost for steel rails to lay one mile of double-track railway at the rate of \$56.25 for 100 feet of rail?

5. Paid an attorney \$18.16 for collecting \$72.64. How much would he need to collect to earn \$5,000 a year at the same rate?

6. A tree 100 ft. high casts a shadow, on level ground, 75 ft. long. How far from the end of the shadow to the top of the tree?

7. What is the cost of 35 miles of telephone wire at 40 cents a pound, supposing a pound to stretch 20 feet?

8. If 25 men can do a piece of work in 15 days, how many men will be needed to do 4 times as much in $\frac{1}{3}$ the time?

9. At 8 cents a foot, what will be the cost of a board 12 feet long, 10 inches wide at one end, and tapering to a point?

10. I sold two cows at \$45 apiece. On one I gained 20%, and on the other I lost \$17.50. For what should I have sold the two to gain $5\frac{1}{3}\%$?

11. How many square feet are there in a tight board fence, 4 feet 6 inches high, round a circular lot 10 rods in diameter?

12. Leaving $\frac{3}{5}$ of my money at home, I spend 5% of the rest for eggs that cost me 29 cents per dozen. I bought eggs enough to fill 8 baskets, 5 dozen to a basket. How much money had I at first?

13. Into a gallon of water I put a pint of alcohol. I then drew off a quart of the mixture. What per cent of the alcohol did I draw off?

14. Assuming a meridian to be 12,500 miles in length, how many miles is it from one tropic to the other?

379.—1. The list price of a carriage is \$ 260. I am allowed 20% and 10% discounts. What is the net price?

2. The net price of a mowing machine is \$ 158.40, and the trade discounts are 20% and 10%. Find the list price.

3. If by selling an article for \$ 9.50 I lose 5%, for how much should I sell it to gain 5%?

4. A building was insured on $\frac{3}{4}$ its value at $\frac{3}{4}$ %, the premium being \$ 19.50. What was the owner's loss when it burned?

5. Purchased stock at 80, and realized $12\frac{1}{2}$ % from my investment by getting one annual dividend and selling immediately at cost. What was the stock paying?

6. A bookseller buys books from the publishers at 40% off the list price. He sells a set of Thackeray's works which list at \$ 30 at a discount of 20%. What per cent does he make?

7. A man, who had been paying \$ 35 a month rent, borrowed \$ 5,000 at 5% and bought the house. Instead of rent he now pays the interest on his borrowed money, \$ 60 a year taxes, \$ 12 water rate, and \$ 50 for repairs. Find his gain or loss per year.

8. A bought a ship for \$ 8,750; expended upon her for repairs the amount of 10% of her first cost; paid $2\frac{1}{3}$ % for insuring her on $\frac{2}{3}$ of her total cost; and she burned at sea. How much did he pay on her account altogether? How much did he receive?

9. A farmer has a trapezoidal lot whose parallel sides are 35 rd. and 84 rd. The perpendicular distance between the parallel sides is 32 rd. How much will it cost to plow the field at \$ 2.50 an acre?

10. The owner of a wrecked steamboat sells her wrought iron crank shaft, which is 18 ft. long and averages 8 in. in diameter, @ 7 cents per lb. How much does he realize if there are 483.4 lb. in a cu. ft.?

11. I have \$ 1,000, with which I buy as many watches as possible at \$ 43.75 each. How much more money do I need to buy one watch more?

380.—1. If 32 men can lay city railroad tracks a distance of 600 rods in 15 days, in what time can 40 men lay 840 rods of tracks?

2. A house rents for \$40 a month, the annual expenses on it are, —taxes \$92.50, water rate \$20, and repairs \$60. The landlord has five per cent clear profit. What did he pay for the house?

3. On goods invoiced at \$8,000 the duty was 15%; on others invoiced at \$17,000 the duty was 25%; the total cost of another invoice, including the duty of 30%, was \$12,090; what was the whole amount of duties?

4. After buying some goods, a merchant lost 20% of them by fire. He sold the remainder at a gain of $33\frac{1}{3}\%$, receiving \$250.75 more than he paid for the whole. What did the goods cost?

5. The perimeter of one square field is 500 feet, and of another 360 feet; what would be the perimeter of a square field that is equal in area to both?

6. A cylindrical tub is $3\frac{1}{2}$ feet in diameter; the recent heavy rain filled it with water to the depth of six inches, making how many gallons, 231 cu. in. to a gallon?

7. How many board feet in a stick of timber 27 ft. long, 11 in. wide at one end, 8 in. wide at the other, and 12 in. thick?

8. I sold a flouring mill, receiving 45 per cent of the price in cash, and invested 75 per cent of the sum received in a city lot worth \$2,160. For how much did I sell the mill?

9. In three successive years a manufacturer invests in his business sums equal to 15 per cent, 10 per cent, and 20 per cent, respectively of his capital at the time of the investment, and in the fourth year he loses 30 per cent of his capital. At the end of the fourth year his capital is \$23,695.98. How much was his original capital?

10. I ordered my agent to buy flour which I afterward sold at 20 per cent profit and gained \$1.56 per barrel. If my agent's rate of commission was 4 per cent and his total commission \$23.40, how many barrels did he buy?

11. 4,000 copies of a book, containing 420 pages, were printed from 650 reams of paper; how many reams of paper would have been required to print 7,000 copies, containing 528 pages of the same size?

381. — 1. If a man can dig $\frac{1}{7}$ of a ditch in $2\frac{2}{3}$ days, how long will he be in finishing it?

2. At \$ $3\frac{1}{4}$ a cord, a pile of 4-ft. wood 32 ft. long cost \$ $17\frac{7}{8}$. How high was the pile?

3. At \$ 60 an acre, what is the value of land whose two parallel sides are 25 rds. and 75 rd., the distance between them being 55 rd.?

4. Every person breathes on an average 28 cu. ft. of air an hour. How many hours will the air in a room 16 ft. by 12 ft. by $9\frac{1}{2}$ ft. last 12 men?

5. A note for \$ 550, dated Jan. 1, 1896, due in 4 months, was discounted Jan. 17, at 7%. Proceeds?

6. Find the square root of $(4\frac{1}{2} - 4.08\frac{1}{3} + 8) \div 5000$.

7. Bought 150 shares of stock at $\frac{3}{4}\%$ premium, and sold at $1\frac{1}{2}\%$ discount. Brokerage in each case $\frac{1}{8}$. Loss?

8. How many square inches are left of a sheet of paper 14 in. by 21 in. after the largest possible circle is cut out of it?

9. A grocer bought 75 lb. of soap at $6\frac{1}{4}$ cents a pound. While on hand it dried away one-fourth in weight. He sold it at $8\frac{1}{3}$ cents a pound. What was his gain or loss per cent?

10. A broker bought 60 shares of stock, par value \$ 50, at 94, and after receiving a dividend of 3% sold them at $100\frac{1}{2}$. What did he gain?

11. Foster & Rich began business Jan. 1, 1894, with a capital of \$ 7,500, of which Rich furnished \$ 5,000. Foster was to have \$ 800 a year salary from the profits. How did they divide \$ 2,000 of profit Dec. 31, 1894?

12. The wheels of a bicycle are 30 in. in diameter; the gearing is such that each wheel makes two revolutions to every turn of the pedals. How many times will each pedal turn in a ride of one mile?

13. The end of a square prism 25 ft. long contains 625 sq. in. What is the area of one side?

382.—1. The 15 minutes' recess is 5% of the time devoted to study and recitation in a certain school. How many hours are so employed?

2. Mr. James owned $\frac{2}{3}$ of a mill and sold 18% of his share for \$1,296, which was $12\frac{1}{2}\%$ more than it cost him. What was the value of the mill at this rate?

3. A man bought a pair of horses for \$400, which was 20% less than their real value, and sold them for 25% above their real value. What was the selling price?

4. A merchant having a debt due him of \$6,424, compromises for 80%. What will he receive if he pays his agent $2\frac{1}{4}\%$ for collecting?

5. A has in bank \$8,000, and loans $\frac{5}{16}$ of it to B for 2 years 3 months 6 days, at 9%. How much does B owe him at the expiration of the time?

6. I bought a house for \$2,500, and sold it so that 20% of the selling price was profit. What did I receive for it?

7. \$800.

CLEVELAND, June 1, 1894.

Six months after date I promise to pay to James Hutton, or order, Eight Hundred Dollars, for value received, interest 6%. Discounted Sept. 15, at 4%. Find proceeds and bank discount.

8. When a post 11.5 ft. high casts a shadow 17.4 ft. long, a neighboring steeple casts a shadow 63.7 yards long. How high is the steeple, supposing the ground to be level?

9. What will it cost to dig a well 5 ft. in diameter and 30 ft. deep, at 55 cents a cubic yard, estimating to the nearest cubic yard?

10. A box 6 ft. long, 4 ft. wide, and 3 ft. deep, is full of oats. What is the value of the oats at 30 cents a bushel?

11. A man, owning $\frac{7}{8}$ of a farm, sold 12 per cent of his share to B, and then the remainder to C for \$20,020. What was the value of the whole farm?

383.—1. If $4\frac{1}{2}$ tons of coal cost $\$30\frac{2}{3}$, how many tons may be bought for $\$100$?

2. A, B, and C bought a horse for $\$100$, and sold it for $\$150$, by which A gained $\$18$, and B $\$19$. How much had A, B, and C each paid toward the purchase?

3. What ought I to pay a broker for two $\$500$ 6's at $106\frac{1}{8}$, three $\$100$ $4\frac{1}{2}$'s at $104\frac{5}{8}$, and five $\$500$ 4's at $99\frac{1}{2}$, with his brokerage of $\frac{1}{4}\%$ in addition to the prices named?

4. A dealer bought 100 bushels of potatoes at 40 cents a bushel. If he lost 30% of them, at what price per bushel must he sell the remainder to gain 20% on his investment?

5. In what time will the interest of $\$884.60$ at 5% equal one-twelfth of the principal?

6. A man bought cloth costing $\$245\frac{1}{4}$. The number of yards and the number of dollars per yard were the same. Find the number.

7. Find the cost of 40 planks at $\$50$ per M, board measure, each plank 12 ft. long, 2 in. thick, 20 in. wide at one end, 16 in. at the other, and tapering regularly.

8. How many gallons of water, each 231 cubic in., in a circular cistern, 4 ft. 4 in. in diameter, the water 16 feet deep?

9. Find the result of $1.76 \times 49.647 \div 0.0088$.

10. A bin is 10 ft. square at the bottom. How deep must it be to contain 1,000 bushels of grain, allowing 1.244 cubic ft. to a bushel?

11. Find the ratio of lighting surface to floor surface in a room 30 by 35 ft., with 4 windows, each 3 ft. by 8 ft. 9 in.

12. Find the cost of boards to make two contiguous rectangular rooms, each of whose dimensions is 10 ft., at $\$18$ per M, allowing for a single partition between the rooms, no floors, and 25% waste in the lumber purchased.

13. Each side of a pentagon measures 5 ft.; the perpendicular distance from the centre to one of the sides is $4\frac{1}{2}$ ft. What is the area of the pentagon?

384. — 1. B endowed a professorship with a salary of \$ 2,000 per annum. What sum must he invest at 6% to provide this salary?

2. A man has \$ 8,000 which he wishes to loan for \$ 600 per year for his support. At what per cent must he loan it?

3. On a note for \$ 425, at 8%, dated March 25, 1898, are the following indorsements: June 1, 1898, \$ 75; Dec. 30, 1898, \$ 120. What is due Sept. 1, 1899?

4. A railway train runs $\frac{3}{4}$ of a mile in $\frac{1}{4}$ of a minute. Find its rate per hour.

5. A pupil who attended school 68 days during a term was marked 85% for attendance. How many days was he absent?

6. The spire of a church is an octagonal pyramid. Suppose the slant height to be 80 ft. and each side 6 ft. at the base. Find the cost of covering it at \$ 6 a square.

7. A's farm is 240 rods wide; he sells 18 acres off one end. How much shorter is his farm than it was before?

8. A wagon weighed 638 lb.; when loaded with wheat it weighed 4,313 lb.; the box was 9 ft. 4 in. long, 3 ft. 6 in. wide. How deep was it?

9. If a cubic foot of iron weighs 500 lb., what will a cannon ball 6 in. in diameter weigh?

10. What will it cost to gild the cannon ball at 3¢ a square inch.

11. Which will be the greater, 6% simple interest on \$ 3,000 for 3 yr. 6 mo., or compound interest at 5% on the same sum for the same time?

12. A certain house was built by 40 workmen in 48 days, but, being burned, it is required to rebuild it in 30 days. How many men must be employed?

13. A garrison of 600 men has bread enough to allow 16 ounces a day to each man for 15 days; but, the garrison being reinforced by 200 men, how many ounces a day may each man have in order that the bread may last 20 days?

385.—1. A miller keeps $\frac{1}{3}$ of the wheat for grinding it. How many bushels, pecks, and quarts must a man take to mill in order to carry back the flour from exactly 13 bushels?

2. At what price must I mark a set of furniture costing \$ 31.20, so that I may take 4% from the marked price, and still make 16%?

3. A grain merchant buys through his agent 4,500 bushels of wheat at $62\frac{1}{2}\text{¢}$ per bushel, commission 2%. He insures it for \$ 2,800 at $1\frac{1}{4}\%$. He had borrowed the money to buy the wheat Mar. 1, 1892. He receives his insurance money Mar. 1, 1895, and pays his note, which has drawn interest at 6%. Find his total loss.

4. A man bought Pacific R. R. bonds at 107, sufficient to give an annual income of \$ 252 at 6%. What did he pay for them, brokerage $\frac{1}{4}\%$?

5. Write a note in which Benj. Simpson agrees to pay Sam'l Hill, or order, \$ 240, with interest at 6%. Date Sept. 1, 1895, to run a year. What will Mr. Hill get at the bank for the note May 10, 1896?

6. How many cubic feet of iron in a gas pipe 20 ft. long, 18 in. in circumference, with 3 in. bore?

7. What per cent should I receive on my investment if I should buy, at 10% discount, stock which pays an annual dividend of $4\frac{1}{2}\%$?

8. Find the cost of papering the walls of a hall 36 ft. long, 24 ft. wide, and 18 ft. high, with paper $1\frac{1}{2}$ ft. wide at \$ 2.50 a roll of 16 yd., allowing 64 sq. yd. for doors and windows.

9. Three men, A, B, and C, engage in partnership; A puts in \$ 1200, B \$ 1500, and C \$ 1900. They gain \$ 350. What is the share of each in the profits?

10. The owner of $\frac{3}{11}$ of a mine sold $\frac{9}{10}$ of his share for \$ 40,500. What should he who owns $\frac{2}{3}$ of the mine get for $\frac{5}{7}$ of his share?

11. What is the amount of \$ 897.25, at 6% simple interest, from Sept. 19, 1891, to March 13, 1894?

12. A bookseller buys a book whose catalogue price is \$ 3.50, at a discount of 20% and 5%, and sells it at 10% above the catalogue price. What per cent profit does he make?

386. — 1. An estate sold for \$45,000, which was $37\frac{1}{2}\%$ below the appraised value. What was the appraised value?

2. When chairs are sold at \$4.80 per dozen, with a discount of 5% for cash, what is the cash value of 200 chairs?

3. A owned $\frac{5}{6}$ of a factory, and sold $\frac{3}{4}$ of his share to B, who sold $\frac{1}{2}$ of his share to C, who sold $\frac{2}{3}$ of what he bought to D. What part of the factory did each then own?

4. The gross amount of a bill of goods is \$750.35, and the rates of discount are 10%, 10%, and 5%. What is the net cash to the purchaser?

5. When the duty on a quantity of lace at 30% ad valorem was \$115.80, what was the cost of the lace, and the duty in francs at \$0.193 per franc?

6. A man has settled on his wife \$1,200 income a year. What sum must he invest in government 4 per cent bonds at $107\frac{1}{8}$ to produce the required amount of income?

7. A yard is 84 feet long and 80 feet wide. What is the length of a clothes line that will reach from one corner to the corner diagonally opposite?

8. A note of \$1,050, dated Feb. 13, 1895, due six months after date, and drawing interest at 6 per cent, was discounted at a bank at 8 per cent, May 13, 1895. Find the proceeds.

9. I bought two houses for \$11,700, paying 25 per cent more for one than for the other. I sold the cheaper house at a profit of 20 per cent, and the higher priced house at a profit of $16\frac{2}{3}$ per cent. What was my total gain?

10. A and B offer the same quality of goods at the same list price. A offers a discount of 15 per cent and 5 per cent, and B offers a single discount of 20 per cent. Of whom will it be the more advantageous to buy, and how much would be saved on a bill, the list price being \$185.50?

11. The ice on a circular pond is two feet thick. If the pond is 1,000 feet in circumference, how many cubic feet of ice does it contain?

**387. — From Civil
Service Tests.**

1. If $\frac{6}{7}$ of a pound of butter cost $\frac{36}{100}$ of a dollar, what will $3\frac{2}{3}$ lb. cost?
2. If $\frac{2}{5}$ of a ton of coal cost \$ 2.56, how much will $1\frac{1}{2}$ tons cost, the latter being 25% cheaper per ton than the former?
3. A house rents for \$ 30 a month, and the owner pays \$ 75 a year for taxes and repairs. What is the value of the house if his net profit is 5% per annum?
4. What number must be added to the sum of $\frac{3}{7}$, $\frac{7}{9}$, $\frac{19}{21}$, to make $8\frac{1}{3}$?
5. An army officer, in preparing for a march of 6 weeks, buys oats for 32 horses at $57\frac{2}{3}$ cents a bushel. Each horse will eat $\frac{3}{8}$ of a bushel a day. How many bushels does he buy and what is the total cost?
6. Change $\frac{19}{32}$ to the form of a decimal and multiply it by .035.
7. The steamer "City of Paris" made the run from Queenstown to Sandy Hook, 2,788 miles, in 5 da. 19 hr. and 18 min. What was the average rate of speed per hour?
8. How many tons of coal can be put into a bin 12 ft. square and 6 ft. high, allowing 55 lb. of coal to a cubic foot, and 2,240 lb. to the ton?
9. What is the cost of 8 pieces of paper, each $13\frac{1}{2}$ yards, at \$ 1.75 per piece of 11 yards?
10. If a merchant's gain on \$ 15,000 worth of sales is \$ 937.50, what amount must he sell to gain \$ 5,060?
11. The cost of insuring a warehouse at $1\frac{1}{2}\%$ is \$ 72 a year, and the cost of insuring its contents at $2\frac{1}{4}\%$ is \$ 129.42. What is the whole amount insured?
12. In 1888 a railway company paid dividends on its stock as follows: 3 mo. at the rate of 7% a year, 6 mo. at the rate of 6% a year, 3 mo. at the rate of 2% a year. What did the dividends amount to that year on 28 shares of stock?

388. — 1. Three gross of lead pencils are divided equally among the clerks in a post-office, giving to each clerk 11 and leaving a remainder of 14 pencils. How many clerks are there in the office?

2. A physician whose charges are \$2 a visit, made on an average 5 visits per day in a year of 365 days. He collected 55% of his charges and saved \$2 out of every \$5 collected. At this rate how much did he save in 2 yr. and 6 mo.?

3. In an office employing 35 carriers, each carrier loses 20 minutes a day in idle talk. Suppose the average salary of each to be \$2.50 for 10 hours' work, what is the cost to the government of the lost time each day, and what will it amount to in a year of 313 working days?

4. A merchant imported 120 tons of English iron, costing $1\frac{1}{2}$ pence per pound, on which he paid a duty of 20%. The freight was 5 shillings sterling per ton. What was the total cost in United States currency? (The ton equals 2,240 lbs. The pound sterling equals \$4.8665.)

5. Owing to a deficiency in the appropriation bill, the salaries of the clerks in a bureau were reduced 18% for the last quarter of the fiscal year. How much did a clerk who was paid \$287 for the last quarter receive during the whole fiscal year?

6. A grocer sells goods to a customer for \$352 by weights averaging $15\frac{1}{4}$ ounces to the pound, and afterwards sells goods for \$320 by weights averaging $16\frac{1}{2}$ ounces to the pound. How much does the grocer make or lose by the false weights?

7. A merchant buys 42 gallons of whiskey at \$2.50 per gallon, and keeps it for 3 years. He then finds that he has lost 7 gallons by leakage and evaporation. Estimating the value of money at 6%, how much per gallon must he charge in order that he may realize the full amount of the cost, including the estimated interest?

8. If the consular fees collected by the United States consul at Liverpool in a year amount to £4,000, and his salary and expenses are \$7,591.74, what percentage of the fees can be paid to the United States after deducting the salary and expenses, the estimated value of the pound sterling being \$4.8665?

APPENDIX I.

I. — A rule in arithmetic gives directions for performing the operations necessary to obtain a desired result.

A clear understanding of subjects and principles will make rules unnecessary. The following, which refer to the more difficult processes, are given for reference. Quantities described concretely must often be considered as abstract.

RULES FOR REFERENCE.

2. — FACTORING.

Greatest Common Divisor. I. (1) *From the given numbers reject all multiples of any of the given numbers.* (2) *Separate each of the remaining numbers into its prime factors.* (3) *Find the product of all the common prime factors. This will be the g. c. d. required.* Or, —

II. *Divide the greater number by the less and the preceding divisor by the last remainder till nothing remains. The last divisor is the g. c. d. If there are more than two numbers, find the g. c. d. of any two of them, and then of this g. c. d. and a third number, and so on.*

Least Common Multiple. I. (1) *Reject from the given numbers all that are divisors of any of the rest.* (2) *Separate each of the remaining numbers into its prime factors.* (3) *Multiply the largest of the given numbers by all the prime factors of the other numbers that are not found among its own. This product will be the l. c. m. required.* Or, —

II. (1) *Strike out any of the given numbers that are factors of any of the others, and divide the remaining numbers by any prime factor common to two or more of them.* (2) *Strike out from the resulting quotients and undivided numbers all that are factors of any of the rest, and divide as before.* (3) *Thus proceed until no two of the remaining numbers have a common factor. The product of the divisors and remaining numbers will be the l. c. m. required.*

3. — FRACTIONS.

To Lowest Terms. *Divide both terms of the fraction by their g. c. d.*

To a Required Denominator. (1) *Divide the required denominator by the given denominator.* (2) *Multiply each term of the given fraction by the quotient.*

To Mixed Numbers. *Divide the numerator by the denominator.*

Mixed Numbers to Improper Fractions. *Multiply the integer by the denominator, and to the product add the numerator. Write the sum over the denominator.*

To a Common Denominator. (1) *Find the l. c. m. of the denominators of the given fractions.* (2) *Divide this by the denominator of each fraction and multiply both terms by the quotient.*

Addition. (1) *Change the fractions to like fractions with a common denominator.* (2) *Find the sum of the numerators.* (3) *Simplify the result.*
Add integers and fractions separately in adding mixed numbers.

Subtraction. *Proceed as in addition of fractions, excepting that the difference of the numerators is to be taken instead of their sum.*

Multiplication. I. When one factor is an integer. *Either (a) multiply the numerator by the integer, or (b) divide the denominator by it.*

II. When one factor is an integer and the other a mixed number. (1) *Multiply the integer first by the fractional part, then by the integral part of the multiplier.* (2) *Add the partial products.*

III. When the factors are mixed numbers or fractions. (1) *Change integers and mixed numbers to improper fractions.* (2) *Write the product of the numerators over that of the denominators.* (3) *Simplify the result.* (The process may often be shortened by cancellation.)

Division. I. A fraction by an integer. *Either (a) divide the numerator or (b) multiply the denominator by the integer.*

In general. II. (1) *Change dividend and divisor to fractional form, and then (2) to a common denominator.* (3) *Write the quotient of the numerators over the common denominator; or, III. Multiply the dividend by the divisor inverted.*

4. — DECIMALS.

To Common Fractions. *Express the denominator and change the fraction to smallest terms.*

Common Fractions to Decimals. *Annex decimal ciphers to the numerator and divide by the denominator.*

Addition and Subtraction. *As with integers.*

Multiplication. (1) *As with integers.* (2) *Give the product as many decimal places as there are in all its factors.*

Division. (1) *Make the divisor an integer by removing the decimal point.* (2) *Move the decimal point in the dividend an equal number of places in the same direction.* (3) *Divide as with integers.* (4) *Give the quotient as many decimal places as have been used in the dividend.*

MEASUREMENTS.

NOTE. — In the following rules dimensions are spoken of as abstract numbers.

5. — OF LINES.

To find —

The Perimeter of a Polygon. *Take the sum of its bounding lines.*

One Dimension of a Parallelogram. *Divide its area by the given dimension.*

One Dimension of a Rectangular Prism. *Divide its volume by the product of the two given dimensions.*

The Side of a Square. *Take the square root of its area.*

The Diagonal of a Square. *Take the square root of twice its area.*

The Diagonal of a Rectangular Prism. *Take the square root of the sum of the squares of its three dimensions.*

The Circumference of a Circle or Sphere. *Multiply the diameter by 3.1416.*

The Diameter of a Circle or Sphere. *Multiply the circumference by 0.31831, or divide by 3.1416.*

An Hypotenuse of a Right Triangle. *Take the square root of the sum of the squares of the short sides of the triangle.*

One of the Short Sides of a Right Triangle. *Take the square root of the difference of the squares of the given sides.*

6. — OF SURFACES.

NOTE. — Before multiplying, dimensions must be changed to like numbers.

To find the Area of —

A Parallelogram. *Find the product of base and altitude.*

A Triangle. *Find half the product of base and altitude.*

A Trapezoid. *Multiply the altitude by half the sum of its parallel sides.*

Other Polygons. *Divide into triangles and find the sum of their areas.*

A Sector. *Find half the product of arc and radius.*

A Circle. *Multiply the square of the diameter by 0.7854.*

The Lateral Surface of a Prism or Cylinder. *Multiply the perimeter of the base by the altitude.*

The Lateral Surface of a Pyramid or Cone. *Multiply the perimeter of the base by half the slant height.*

A Sphere. *Multiply circumference by diameter; or, multiply the square of the diameter by 3.1416.*

7.—OF SOLIDS.

To find the Volume of a—

Rectangular Solid. *Find the product of its three dimensions.*

Prism or Cylinder. *Multiply the area of the base by the altitude.*

Pyramid or Cone. *Multiply the area of the base by one-third the altitude.*

Sphere. *Multiply the cube of the diameter by 0.5236.*

A Frustum of a Pyramid or Cone is what remains after cutting off the top in a plane parallel to the base. To find its volume: (1) *Find the area of each of the two bases.* (2) *Add to their sum the square root of their product.* (3) *Multiply this result by one-third the altitude.*

To find—

The Number of Board Feet in a Piece of Lumber. *Multiply the product of its length and width in feet by its thickness in inches. (Disregard thickness when it is one inch or less.)*

8.—PERCENTAGE.

To find the—

Percentage. *Multiply the base by the rate per cent.*

Base. *Divide the percentage by the rate per cent.*

Rate per cent. *Divide the percentage by the base.*

What constitutes base, percentage, rate%, etc., in business operations, is shown in the following table:—

APPENDIX I.

5

	BASE.	PERCENTAGE.	RATE PER CENT.	AMOUNT.	DIFFERENCE.
Profit and Loss.	Cost.	Gain or loss.	Rate of Gain or Loss.	Cost plus Gain; or Selling Price.	Cost less Loss; or Selling Price.
Insurance.	Sum insured.	Premium.	Rate of Insurance.	—	—
Commission.	Amount of Purchase or Proceeds of Sale.	The Agent's Commission.	Rate of Commission.	Investment plus Commis- sion; or Remittance.	Proceeds of Sale less Commission.
Trade Discount.	Face of Bill or List Price.	The Discount.	Rate of Discount.	—	Base less the Discount.
Taxes.	Valuation of all Property.	Total tax less poll-tax.	Shown by tax on \$ 1.	—	—
Duties.	Cost as per Invoice.	Ad Valorem Duty.	Rate of Tariff.	Cost of Importation.	—

9. — INTEREST.

General Method. Find the product of the principal, rate per cent, and time in years. (The time may be taken in months or in days, provided the product is divided by 12 or 360, as the case may be.) Use cancellation.

Principal and interest added will give the amount.

One Dollar Method. To find the interest of one dollar at 6%: Reckon 6 cents for each year, 5 mills for each month, and $\frac{1}{16}$ of a mill for each day.

To find the interest of any sum: Multiply one dollar's interest by the principal.

For any rate except 6: Add to or subtract from the interest at 6% a proportional part of itself.

Bankers' Method. To find interest at 6%: Find 20 months' interest by taking $\frac{1}{10}$ of the principal, or 60 days' interest by taking $\frac{1}{100}$ of it. Then take such parts or multiples of these as the given time may require.

Exact Interest. For any part of a year: *Find the common interest for the exact number of days in the given part of a year; then lessen it by $\frac{1}{3}$ of itself.*

To find the Principal. *Divide the interest by the product of the rate and the time in years.*

To find the Rate. *Divide the interest by the product of the principal and the time in years.*

To find the Time. *Divide the interest by the product of the principal and the rate per cent. The quotient is the time in years.*

To find the Amount due on a Note on which Partial Payments have been made. United States rule: *Find the amount of the principal to the time when a payment or the sum of several payments shall equal or exceed the interest due at the time. Subtract such payment or sum of payments from the amount, and with the remainder as a new principal, proceed as before to the time of settlement.*

Compound Interest. *Find the amount of the principal for the first period of time. Treat this amount as a new principal, and find its amount for the second period, and so on for the entire time. The last amount less the given principal will be the compound interest.*

Present Worth. *Divide the given debt by the amount of \$1 for the time to elapse before the debt is due. The debt less the present worth is the true discount.*

10. — DISCOUNT.

Bank Discount. *Compute bank discount as if it were simple interest on the face of the note for the term of discount. The face of the note less the bank discount will be the proceeds.*

True Discount. *See Present Worth, § 9.*

11. — PROPORTION.

Rule of Three. *Make that number in the problem which is of the same kind as the desired result the third term of a proportion.*

If, from the conditions of the question, the result is to be larger than the third term, use the two like numbers in making the first ratio of the proportion less than 1; but if the result is to be smaller, make the first ratio greater than 1.

Divide the product of the means by the given extreme, and the quotient will be the fourth term of the proportion, or the result desired.

Partnership. Give each partner such part of the whole gain or loss as his capital for any time is part of the whole capital for the same time.

12. — ROOTS.

Extracting the Square Root.

I. Beginning at the decimal point, separate the given number into groups of two figures each.

II. Find the greatest square in the left group and place its root at the right; subtract the square of this root from the left group, and to the remainder annex the next group for a dividend.

III. Divide this dividend, omitting the last figure, by double the root already found, and annex the quotient to the root, and also to the divisor.

IV. Multiply the divisor as it now stands by the last root figure and subtract the product from the dividend.

V. If there are more groups to be brought down, proceed in the same manner as before.

Extracting the Cube Root.

I. Beginning at the decimal point, separate the given power into groups of three figures each.

II. Find the greatest cube in the left group and place its root at the right. Subtract the cube of this root from the left group, and to the remainder annex the next group for a dividend.

III. Annex a cipher to the root already found and take three times its square for a trial divisor. Divide the dividend by this trial divisor and place the quotient as the next root figure.

IV. Multiply the number last squared by the last root figure and add three times this product and the square of the last root figure to the trial divisor for a complete divisor.

V. Multiply the complete divisor by the last root figure, subtract the product from the dividend, and to the remainder annex a new group.

VI. Form a second trial divisor, using two figures of the root with a cipher annexed, and proceed as before until all the groups have been used.

13. — Roman Notation.

Seven capital letters were used by the Romans to represent numbers. They are of almost no use in computations.

I	V	X	L	C	D	M
1	5	10	50	100	500	1000

To represent other numbers, these letters are combined according to the following principles :

- I. Repeating I, X, C, or M, repeats its value.
- II. When I is used before V or X, X before C or L, and C before M, the difference of the values is to be taken.
- III. When any numeral follows one of greater value, a sum of values is to be taken.
- IV. A dash (—) over any numeral but I increases its value 1000 times.

Show how these principles are illustrated in CC; IX; LX; CM; MM; \overline{C} ; MDCCCXCV. Mention four uses of Roman numerals.

14.—Least Common Multiple.

The least common multiple of two or more numbers contains only such prime factors as are needed to produce each number.

The following method of finding the l. c. m. is a useful one, though not different in principle from that given on page 61.

To find the l. c. m. of 24, 40, 72, 108.

2	24	40	72	108
2	20	36	54	
2	10	18	27	
3	5	9	27	
3	5	3	9	
	5	1	3	

$$2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 3 = 1080, \text{ l. c. m.}$$

EXPLANATION.—We discard 24, for any multiple of 72 is a multiple of 24. We then divide the remaining numbers by any prime factor common to any two of them, until quotients are obtained that are prime to each other. The product of the divisors and the remaining quotients is the desired least common multiple.

Select from the process shown above the prime factors of each number.

15.—Leap Years.

A True or Solar Year is the exact time in which the earth revolves once around the sun. Its length is 365 d. 5 h. 48 min. 49.7 sec., or about $11\frac{1}{2}$ min. less than $365\frac{1}{4}$ days.

To avoid the confusion and inaccuracy of the methods of reckoning time then in use, the Roman dictator, Julius Cæsar, 46 B.C., reformed the calendar by establishing what is now known as the *Julian year*, of $365\frac{1}{4}$ days. To avoid the inconvenience of counting the fractional part of a day every year, he decreed that three successive years should consist of 365 days and the fourth year of 366 days, the extra day being added to the month of February. The year containing the extra day is called *bissexile* or *leap year*.

But this arrangement of the calendar made the civil year too long by about $11\frac{1}{2}$ minutes, an error that amounted to 1 day in about 130 years. To correct this and other errors, Pope Gregory XIII. struck out ten days from the calen-

dar, calling Oct. 5, 1582, Oct. 15; and ordaining that thereafter only those centennial years should be leap years whose numbers are divisible by 400.

The *Gregorian year* is now the *civil* or *legal* year in nearly all civilized countries but Russia and Greece, where the Julian calendar is still in use, and the dates 12 days behind ours.

The Gregorian calendar was not adopted in Great Britain till 1752. The error had then amounted to 11 days, and hence the third of September was called the fourteenth. *Old style* dates are according to the Julian calendar, *new style* dates conform to the Gregorian calendar.

When the number of a year is divisible by 4, it is a leap year; but centennial years whose number is not divisible by 400 are exceptions.

16.—Land Measurements.

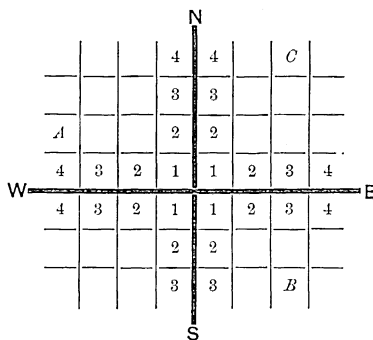
Government Lands are divided by parallels and meridians into *townships* six miles square, containing 36 sections or square miles. Each section is divided into *half sections* and *quarter sections*.

A township is designated by its number north or south of a *base line* running east and west, and east or west of a *principal meridian* running north and south.

Thus, C is *Township 4 N., Range 3 E.* What is A? B?

The 36 sections into which a township is divided are numbered as in the accompanying figure. Point out section 15.

Half and quarter sections are designated as W. or N. half sections, etc.; and S. W. or N. E. quarter sections, etc.



6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

Township.

N. $\frac{1}{2}$ Sect.

S. W. | S. E.

Section 15.

Surveyors generally use, in measuring land, a steel chain 100 ft. long, divided into foot links, or a steel tape line of the same length graduated to feet and tenths. Sometimes a *Gunter's Chain* is used. It contains 100 links, each 7.92 in. long. The chain is 4 rods, or 66 ft., or 792 in., in length. 80 chains, or 320 rods, measure a mile.

17.—The Metric System of Weights and Measures.

The **Metric System** of weights and measures is named from the **Meter**, from which all the units of length, surface, volume, capacity, and weight are derived.

The **Meter** is approximately one ten-millionth of the distance from equator to pole on the earth's surface.

NOTE.—The Metric System is in general use by nearly all civilized nations except Great Britain and the United States. It is used by some departments of the United States government, and in the sciences.

The Metric System is a *decimal system*, ten units of one denomination making one of the next higher.

Decimal parts of the standard units are denoted by Latin prefixes; multiples of the standard, by Greek prefixes.

<i>Milli</i> means 1000th	<i>Myria</i> means 10000
<i>Centi</i> means 100th	<i>Kilo</i> means 1000
<i>Deci</i> means 10th	<i>Hekto</i> means 100
	<i>Deka</i> means 10

In the tables units in common use are in *italics*.

Length Measures. Standard unit, the *Meter*.

	<i>Table.</i>	<i>Equivalents.</i>
10 <i>millimeters</i> (^{mm})	= 1 <i>centimeter</i> (^{cm})	= 0.3937079 inch
10 centimeters	= 1 <i>decimeter</i> (^{dm})	
10 decimeters	= 1 <i>meter</i> (^m)	= 39.37079 inches
10 meters	= 1 <i>dekameter</i> (^{Dm})	
10 dekameters	= 1 <i>hektometer</i> (^{Hm})	
10 hektometers	= 1 <i>kilometer</i> (^{Km})	= { 3280.9 feet 0.621382 miles
10 kilometers	= 1 <i>myriameter</i> (^{Mm})	

Surface Measures. Principal unit, the *Square Meter*.

NOTE.—As the units of surfaces are squares whose dimensions are the corresponding linear units, it takes 10² or 100 units of one denomination to make one of the next higher.

	<i>Table.</i>	<i>Equivalents.</i>
100 sq. millimeters (^{sq mm})	= 1 <i>sq. centimeter</i> (^{sq cm})	= 0.155 sq. inch
100 sq. centimeters	= 1 <i>sq. decimeter</i> (^{sq dm})	
100 sq. decimeters	= 1 <i>sq. meter</i> (^{sq m})	= 10.764 sq. feet
100 sq. meters	= 1 <i>sq. dekameter</i> (^{sq Dm})	
100 sq. dekameters	= 1 <i>sq. hektometer</i> (^{sq Hm})	
100 sq. hektometers	= 1 <i>sq. kilometer</i> (^{sq Km})	= 247.114 acres

NOTE.—When used in measuring land the square meter is called a *centare* (^{ca}), the square dekameter an *are* (^a), and the square hektometer a *hektare* (^{Ha}).

Volume Measures. Principal unit, the *Cubic Meter*.

NOTE. — As the units of volume are cubes whose edges are the corresponding linear units, it takes 10^3 or 1000 units of one denomination to make one of the next higher.

<i>Table.</i>	<i>Equivalents.</i>
1000 cu. millimeters (cu mm)	= 1 cu. centimeter (cu cm) = 0.06103 cu. inch
1000 cu. centimeters	= 1 cu. decimeter (cu dm)
1000 cu. decimeters	= 1 cu. meter (cu m) = 35.314 cu. feet

NOTE. — In measuring wood the cubic meter is called a *Stere* ($1^{\text{st}} = 0.2759 \text{ cu. ft.}$); a *decistere* ($1^{\text{st}} = 0.1 \text{ stere}$) is one tenth of a stere.

Measures of Capacity. Principal unit, the *Liter* = a cu. decimeter.

<i>Table.</i>	<i>Equivalents.</i>
10 milliliters (ml)	= 1 centiliter (cl) = 0.6102 cu. inch
10 centiliters	= 1 deciliter (dl)
10 deciliters	= 1 liter (l) = { 1.0567 liquid quarts 0.908 dry quart
10 liters	= 1 dekaliter (Dl)
10 dekaliters	= 1 hektoliter (Hl) = { 26.417 gallons 2.8375 bushels
10 hektoliters	= 1 kiloliter (Kl)

NOTE. — The *liter* is used in measuring liquids and small fruits, the *hektoliter* in measuring grain, vegetables, and liquids in larger quantities.

Measures of Weight. Principal unit, the *Gram*.

<i>Table.</i>	<i>Equivalents.</i>
10 milligrams (mg)	= 1 centigram (cg) = 0.15432 grain
10 centigrams	= 1 decigram (dg)
10 decigrams	= 1 gram (g) = 15.432 grains
10 grams	= 1 dekagram (Dg)
10 dekagrams	= 1 hektogram (Hg)
10 hektograms	= 1 kilogram (Kg) = 2.20462 pounds
10 Kilograms	= 1 myriagram (Mg)
10 Myriagrams	= 1 quintal (Q)
10 Quintals	= 1 metric ton (T) = 2204.621 pounds

NOTE. — The *gram* is the weight of a cubic centimeter, the *kilogram* of a cubic decimeter, and the *metric ton* of a cubic meter of distilled water at its greatest density.

The *gram* is used in mixing medicines, and in weighing jewels, precious metals, letters, etc. Ordinary articles are weighed by the *kilogram* (commonly called *kilo*), and heavy articles by the *metric ton*.

18.—TABLE OF EQUIVALENTS.

<i>Common.</i>	<i>Metric.</i>	<i>Common.</i>	<i>Metric.</i>
1 inch	= 2.54 ^{cm}	1 cu. foot	= 28.317 ^{cu dm}
1 foot	= 30.48 ^{cm}	1 cu. yard	= 0.7645 ^{cu m}
1 yard	= 0.9144 ^m	1 cord	= 3.624 st
1 rod	= 5.029 ^m	1 liquid quart	= 0.9463 ^l
1 mile	= 1.6093 ^{Km}	1 gallon	= 3.785 ^l
1 sq. inch	= 6.452 ^{sq cm}	1 dry quart	= 1.101 ^l
1 sq. foot	= 9.2903 ^{sq dm}	1 bushel	= 0.3524 ^m
1 sq. yard	= 0.8361 ^{sq m}	1 grain	= 0.0648 ^g
1 sq. rod	= 0.2529 ^a	1 ton	= 0.9072 ^{met ton}
1 sq. mile	= 2.59 ^{sq Km}	1 troy ounce	= 31.1035 ^g
1 Acre	= 0.4047 ^{Ha}	1 av. ounce	= 28.35 ^g
1 cu. inch	= 16.387 ^{cu cm}	1 av. pound	= 0.4536 ^{Kg}

Approximate Equivalents.

1 decimeter	= 4 inches	1 liter	= 1.06 liq. qt. or $\frac{9}{16}$ dry qt.
1 meter	= 3 ft. 3 $\frac{3}{8}$ in.	1 dekaliter	= 1 $\frac{1}{8}$ pecks
1 dekameter	= 2 rods	1 hektoliter	= 2 $\frac{5}{8}$ bushels
1 kilometer	= $\frac{5}{8}$ mile	1 gram	= 15 $\frac{1}{2}$ grains
1 are	= 4 sq. rds.	1 kilogram	= 2 $\frac{1}{2}$ av. pounds
1 hektare	= 2 $\frac{1}{2}$ acres	1 metric ton	= 2200 pounds
1 stere	= $\frac{1}{4}$ cord		

19.—Metric System. The units of the Metric System form a decimal system. Hence the following principles apply :—

I. *Excepting in square and cubic measures, any metric number may be changed from one denomination to the next smaller or the next larger by moving the decimal point one place to the right or left, as the case may be.*

II. *In square or surface measures this reduction is effected by moving the point two places, and in cubic or volume measures three places, instead of one.*

III. *Any denomination may be taken as the unit, the number at the right of the point being read as its decimal.*

Explain the following changes or reductions :—

- 3247.28^m = 324728^{cm} = 32.4728^{dm} = 3.24728^{Km} = 3247280^{mm}.
- 67317.96^{sq cm} = 673.1796^{sq dm} = 6.731796^{sq m} = 0.06731796^{sq Km}.

3. $8.3724^{\text{Ha}} = 837.24^{\text{a}} = 83724^{\text{ca}} = 83724^{\text{sq m}}$.
4. $47.234^{\text{cu m}} = 47.234^{\text{st}} = 47234^{\text{cu dm}} = 47234000^{\text{cu cm}}$.
5. $247.831^{\text{l}} = 2.47831^{\text{hl}} = 24783.1^{\text{cl}}$.
6. $1346.982^{\text{g}} = 1.346982^{\text{Kg}} = 134698.2^{\text{cg}}$.
7. In 847.2^{K} , how many grams? How many pounds?
8. Change 75 bushels to hektoliters.
9. How many square meters in a rectangle 18 ft. by 10 ft.?
10. An importer pays duty on 1,200 meters of cloth. How many yards?
11. How many square rods in a square hektometer?
12. How many liters in a cubic meter?
13. An importer buys 250^{l} of liquor at \$0.75 a liter. He sells it for \$3 a gallon. What does he gain or lose?
14. A rectangular stone is 1^{m} long, 5^{dm} wide, and 24^{cm} thick. How many kilograms does it weigh, being eight times heavier than water?
15. How many kilograms of flour in a barrel?
16. Add 18.32^{Km} , 648^{m} , 94.8^{Hm} , 38.4^{dm} .
17. What will a stere of wood cost at \$12 a cord?
18. How many hectares in a field 14^{Hm} long and 400^{m} wide? How many acres?
19. How many gallons in a cubic meter of water?
20. How many times is 16^{dm} contained in 1.28^{Km} ?
21. If goods are bought at \$2.35 per yard, at what price per meter must they be sold to gain 25%? (1 meter = 39.37 inches.)
22. A hektoliter of fruit weighs 63 kilograms, and 32 liters of syrup can be obtained from it. How many kilograms of fruit will it take to make a hektoliter of syrup?
23. The distance between two places on a map is 12.5 centimeters. What is the actual distance between the places if the scale of the map is 1 to 60,000?
24. If a certain stone is 2.83 times as heavy as water, what is the weight of a piece of this stone which is 5.36^{m} long, 17.36^{dm} wide, and 52.6^{cm} thick?

20.—COMPOUND INTEREST TABLE.

Yr.	2 per cent.	2½ per cent.	3 per cent.	3½ per cent.	4 per cent.	5 per cent.	6 per cent.
1	1.020000	1.025000	1.030000	1.035000	1.040000	1.050000	1.060000
2	1.040400	1.050625	1.060900	1.071225	1.081600	1.102500	1.123600
3	1.061208	1.076891	1.092727	1.108718	1.124864	1.157625	1.191016
4	1.082432	1.103813	1.125509	1.147523	1.169859	1.215506	1.262477
5	1.104081	1.131408	1.159274	1.187686	1.216653	1.276282	1.338226
6	1.126162	1.159693	1.194052	1.229255	1.265319	1.340096	1.418519
7	1.148686	1.188686	1.229874	1.272279	1.315932	1.407100	1.503630
8	1.171660	1.218403	1.266770	1.316800	1.368569	1.477455	1.593848
9	1.195093	1.248836	1.304773	1.362897	1.423312	1.551328	1.689479
10	1.218994	1.280085	1.343916	1.410599	1.480244	1.628895	1.790848
11	1.243374	1.312087	1.384234	1.459970	1.539454	1.710339	1.898299
12	1.268242	1.344889	1.425761	1.511069	1.601032	1.795856	2.012197
13	1.293607	1.378511	1.468534	1.563956	1.665074	1.885649	2.132928
14	1.319479	1.412974	1.512590	1.618695	1.731676	1.979932	2.260904
15	1.345868	1.448298	1.557967	1.675349	1.800944	2.078928	2.396558
16	1.372786	1.484506	1.604706	1.733986	1.872981	2.182875	2.540352
17	1.400241	1.521618	1.652848	1.794676	1.947901	2.292018	2.692773
18	1.428246	1.559659	1.702433	1.857489	2.025817	2.406619	2.854339
19	1.456811	1.598650	1.753506	1.922501	2.106849	2.526950	3.025600
20	1.485947	1.638616	1.806111	1.989789	2.191123	2.653298	3.207136

21.—Annual Interest. 1. I borrow \$800, agreeing to pay 6% interest at the end of each year. I do not, however, pay any interest until the end of 4 years 4 months, when I pay principal and interest. What should I then pay?

In cases where there is an agreement to pay interest annually the custom is to charge simple interest on the principal and on the overdue interest for the time each interest payment is overdue. This is called **annual interest**.

Process.

Each payment of annual interest should have been \$48.

The 1st annual interest payment of \$48 has been due 3 yr. 4 mo.

The 2d annual interest payment of \$48 has been due 2 yr. 4 mo.

The 3d annual interest payment of \$48 has been due 1 yr. 4 mo.

In all, interest on \$48 has been due for . . . 7 yr.

APPENDIX I.

15

Interest of \$ 800 for 4 yrs. 4 mo.	\$208
7 yr. interest of \$ 48	20.16
Total interest due	\$228.16
Principal due	800
Amount due at settlement	\$1028.16

Find the annual interest of —

- | | |
|-----------------------------------|-------------------------------------|
| 2. \$ 1,200 for 4 yr. 6 mo. at 6% | 6. \$ 900 for 9 yr. 6 mo. at 7% |
| 3. 1,800 for 5 yr. 8 mo. at 5% | 7. 720 for 3 yr. 8 mo. 12 d. at 4½% |
| 4. 4,200 for 3 yr. 2 mo. at 4% | 8. 618 for 4 yr. 5 mo. 17 d. at 3% |
| 5. 640 for 5 yr. 7 mo. at 5% | 9. 427 for 6 yr. 8 mo. 2 d. at 12% |

22.—TABLE SHOWING RATE OF INTEREST ALLOWED IN THE STATES AND TERRITORIES.

The legal rate is given in the first column. The rate allowed by contract is in the second column. * Any rate. † Any rate on call loans of \$5000 and upwards on collateral security.

States.	Rate per cent.		States.	Rate per cent.		States.	Rate per cent.	
Alabama.....	8	8	Louisiana.....	5	8	Ohio.....	6	8
Arizona.....	7	*	Maine.....	6	*	Oklahoma.....	7	12
Arkansas.....	6	10	Maryland.....	6	6	Oregon.....	8	10
California.....	7	*	Massachusetts.....	6	*	Pennsylvania.....	6	6
Colorado.....	8	*	Michigan.....	6	8	Rhode Island.....	6	*
Connecticut.....	6		Minnesota.....	7	10	South Carolina.....	7	8
Delaware.....	6	6	Mississippi.....	6	10	South Dakota.....	7	12
Dist. of Columbia.....	6	10	Missouri.....	6	8	Tennessee.....	6	6
Florida.....	8	10	Montana.....	10	*	Texas.....	6	10
Georgia.....	7	8	Nebraska.....	7	10	Utah.....	8	*
Idaho.....	7	18	Nevada.....	7	*	Vermont.....	6	6
Illinois.....	5	7	New Hampshire.....	6	6	Virginia.....	6	6
Indiana.....	6	8	New Jersey.....	6	6	Washington.....	8	*
Iowa.....	6	8	New Mexico.....	6	12	West Virginia.....	6	6
Kansas.....	6	10	New York.....	6	6†	Wisconsin.....	7	10
Kentucky.....	6	6	North Carolina.....	6	8	Wyoming.....	12	*
			North Dakota.....	7	12			

23. — Partial Payments of Promissory Notes. Merchants' Method.

When a note is settled *within a year from its date*, it is the custom of business men to use the following method: —

Find the amount of the face of the note from its date to the time of settlement.

Find the amount of each payment from its date to the time of settlement. Add their amounts and take the sum from the amount of the note. The remainder is what is due.

1. A note for \$1200 dated May 7, 1895, is settled April 25, 1896. What is due at settlement if it draws 6% interest and has the following indorsements?

Aug. 19, 1895, \$400. Oct. 25, 1895, \$300. Jan. 25, 1896, \$450.

Process.

Amount of \$1200 from 5/7, '95 to 4/25, '96, 11 mo. 18 d.	\$1269.60
Amt. of \$400 from 8/19, '95 to 4/25, '96, 8 mo. 6 d.	\$416.40
“ “ 300 10/25, '95 to 4/25, '96, 4 mo.	306.
“ “ 450 1/25, '96 to 4/25, '96, 3 mo.	456.75
Total amount of payments	1179.15
Amount due at settlement	\$90.45

2. An 8% note for \$1500 dated Aug. 4, 1896, is settled June 28, 1897. \$460 was paid Oct. 4, 1896; \$500, Jan. 16, 1897. What is due at settlement?

3. Face of note \$6000; its date May 15, 1897; rate 4%. \$2000 paid Aug. 20, 1897, and \$3000 May 1, 1898. What is due at settlement, May 9, 1898?

24. — Concerning Promissory Notes, etc. 1. A *Joint Note* is one which two or more persons *jointly* promise to pay, each one of whom must put his signature to the note and is held liable for his share and no more.

2. A *Joint and Several Note* is one which two or more persons *jointly* and *severally* promise to pay, each of whom must put his signature to the note and is held liable for the whole amount provided others fail to pay their share.

3. Notes written in pencil are valid, but should be written in ink.

4. When the words and figures expressing the face of a note or check, or draft, differ, the words govern the amount to be paid.

5. Unless otherwise stated a note is payable at the maker's place of business or residence.

6. Notes cannot be collected before maturity.

7. Demand notes are due and payable immediately on presentation. They bear legal interest in all cases after demand for payment is made.

8. In most States a note becomes outlawed or worthless at the expiration of six years from its maturity, or from the date of the last payment of either principal or interest.

9. Notes are void if given on Sunday, or by a minor, or without consideration, or if obtained by force or illegally.

10. Grace is not generally allowed on demand notes nor on sight drafts.

11. Bankers throughout the country are seeking to secure the abolishment of days of grace on all commercial paper by action of State legislatures. Hence changes are likely to occur in States now allowing grace.

12. Business men generally date their notes so that they shall not fall due on Sunday or on a legal holiday. A note falling due on Sunday, or on a legal holiday, is in most States payable on the business day next preceding. In the following named States and Territories it is payable on the next business day succeeding:—

Alabama	Idaho	Nebraska	Pennsylvania
Arizona	Louisiana	New Jersey	Rhode Island
California	Massachusetts	New Mexico	South Dakota
Connecticut	Michigan	New York	Vermont
Dist. of Columbia	Mississippi	North Dakota	Wisconsin
Florida	Missouri *	Oregon	

25. — Equation of Payments.

1. Show that the interest of \$200 for 4 mo. is the same as the interest of \$800 for 1 mo. or of \$1 for 800 months.

2. May 10, a debtor owes his creditor \$200, due without interest in 3 months, \$600 in 4 months, and \$500 in 6 months. When may the \$1300 of indebtedness be paid at one time without loss of interest to either debtor or creditor?

Process.

The debtor is entitled to the use of $\left\{ \begin{array}{l} \$200 \text{ for } 3 \text{ mo., or of } \$1 \text{ for } 600 \text{ months.} \\ 600 \text{ for } 4 \text{ mo., or of } 1 \text{ for } 2400 \text{ months.} \\ 500 \text{ for } 6 \text{ mo., or of } 1 \text{ for } 3000 \text{ months.} \end{array} \right.$

The debtor is entitled to the use of \$1 for a total of 6000 months.

This is the same as the use of \$1300 for $\frac{1}{1300}$ of 6000 mo., or for $4\frac{8}{13}$ months, or 4 mo. 18 d.

4 mo. 18 d. after May 10 is Sept. 28, the date on which a single payment of \$1300 may be equitably made. This date is the *average or equated time of payment*, and the process of finding it is the *Equation of Payments*.

* Unless a Sunday or holiday follows.

3. Find the equated time of payment of \$100 due in 2 mo., \$200 due in 3 mo., and \$300 in 4 mo.

4. When may these debts be equitably paid: \$350 due in 60 days, \$400 due in 90 days, \$700 due in 30 days?

5. Aug. 10, \$320 is due; Sept. 15, \$424.75; Dec. 4, \$612.40. What is the equated date of payment?

Process.

Aug. 10, \$320 is due.

Sept. 15, or 36 days after Aug. 10, \$425* is due.

Dec. 4, or 116 days after Aug. 10, \$612* is due.

$$320 \times 0 \text{ days} = 0 \text{ days}$$

$$425 \times 36 \text{ days} = 15,300 \text{ days}$$

$$612 \times 116 \text{ days} = 70,992 \text{ days}$$

$$\underline{1357}$$

$$\underline{86,292 \text{ days}} \text{ (64 d.}$$

64 days after Aug. 10 is Oct. 13, the equated time.

6. \$500 is due March 31, \$600 May 1, and \$400 June 1. On what date may a single payment of \$1,500 be made?

7. Brown & Fisher owe Chandler & Co., \$350, which is to be paid July 10, \$62 due Aug. 1, \$108.75 due Sept. 10. What is the equated date of payment?

Find the equated time of payment for these obligations:—

8. \$800 due May 15; \$650 due June 11; \$300 due Aug. 4.

9. \$397.25 due Nov. 1, 1895; \$64.90 due Dec. 10, 1895; \$480 due Feb. 1, 1896.

10. I hold three non-interest-bearing notes: one for \$86.40 maturing June 7, one for \$348.96 due Aug. 5, and a third payable Sept. 16 for \$408. I exchange them for a single note maturing at the equated time of payment. Required its face, time to run, and maturity, without grace.

NOTE.—If there is a common term of credit, we may disregard it in finding the equated time, and add it to that time when found.

11. There is a common term of credit of 2 months on the following: \$600 bought Oct. 5, and \$500 bought Dec. 9. When may both be paid at once?

12. May 1, Samuel Rich bought a bill of \$375 worth of merchandise, July 10 he bought \$480 worth, and on Aug. 22, \$218.75. On each purchase he was to have 60 days' time. What is the equated time of settlement?

* Reckoning to the nearest dollar.

26. — Average of Accounts.

1. The following is a statement from the books of Bainbridge & Co., of their account with William Henderson. We wish to find the average date or equated time for the settlement of the account without loss of credit or interest to either party.

WILLIAM HENDERSON.					
Dr.			Cr.		
1896.			1896.		
Mar. 20	Mdse. 30 d.	\$400	Apr. 7	Mdse. 2 mo.	\$300
Apr. 5	" 2 mo.	500	May 5	Note 4 mo.	400
June 12	" 60 d.	800			

Debits.						Credits.					
Date.	Term of Credit.	Due.	Debt.	Days.	Product.	Date.	Term of Credit.	Due.	Credit.	Days.	Product.
Mar. 20	30 d.	Apr. 19	\$ 400	19	7600	Apr. 7	2 mo.	June 7	\$ 300	68	20400
Apr. 5	2 mo.	June 5	500	66	33000	May 7	4 mo.	Sept. 5	400	158	63200
June 12	60 d.	Aug. 11	800	133	106400				\$ 700		\$3600
			\$ 1700		147000						
			700		83600						
			\$ 1000		63400						

(63 days after Mar. 31 = June 2.

We find when each item of the debits and credits becomes due by adding its term of credit to its date.

We assume March 31, the last day of the month preceding the earliest date at which any item falls due on either side of the account, as the *focal date*.

We then multiply each item by the number of days between this *focal date* and the date of its maturity, and find the sum of the items and the sum of the products on each side of the account.

The aggregate time credit of the debit items is \$1 for 147,000 days. That of the credit items is \$1 for 83,600.

This is an excess of \$1 for 63,400 days in time credits on the debit side of the account. But the balance of the account is \$1000, and \$1 for 63,400 days = \$1000 for 63 days, which is the average term of credit for the balance due, and is to be reckoned forward from March 31, the focal date.

This gives June 2 as the date when the balance is equitably due.

NOTE. — When the balance of items and the balance of credits are on different sides of the account, the average term of credit comes before the focal date.

Find the equated time for paying the balance of these accounts: —

	Dr.		Cr.
2. Jan. 4, Mdse 3 mo.	\$ 600	Mar. 1, Mdse 2 mo.	\$ 500
Feb. 19, " 4 mo.	700	Apr. 1, " 1 mo.	200
3. Mar. 17, Mdse 60 d.	\$ 425	Jan. 24, Mdse 2 mo.	\$ 900
Apr. 7, " 90 d.	600		
May 11, " 3 mo.	783		
4. Aug. 15, Mdse 4 mo.	\$ 725	May 11, Mdse 3 mo.	\$ 611.84
		July 15, " 60 d.	500.
		Sept. 9, " 30 d.	317.16
5. Sept. 15, Mdse 60 d.	\$ 215	Aug. 19, Mdse 90d.	\$ 131.75
Oct. 19, " 60 d.	641	Nov. 10, " 3 mo.	250
6. July 17, Mdse 3 mo.	\$ 1500	July 24, Cash	\$ 500
Aug. 17, " 3 mo.	800	Aug. 19, Mdse 90 d.	400
Sept. 24, " 2 mo.	1000	Sept. 28, Note 30 d.	1500
Sept. 24, " 3 mo.	1500	Oct. 29, Cash	200

**27. — Extraction of
the Cube Root.**

1. Find the value of 3^3 ; 30^3 ; 300^3 ; 9^3 ; 90^3 ;
 $\sqrt[3]{27}$; $\sqrt[3]{27'000}$; $\sqrt[3]{27'000'000}$; 5^3 ; $\sqrt[3]{125}$; 50^3 ;
 $\sqrt[3]{125'000}$; 600^3 ; $\sqrt[3]{216'000'000}$.

2. Compare the number of places in the 3d power or cube with the number in the 3d or cube root.

3. If the cube of a number contains three times as many places as the number, or one or two less, how many places in the cube of 253? 67? 8000?

4. How many figures in the cube root of 729? 512,000? 8,000,000?

5. Cube 0.08. How many decimal places in the cube? Why? 6. How many decimal places in the cube of 0.716? Of any number? 7. Why must the number of decimal places in the cube of a decimal be a multiple of 3?

The square of a number of two digits consists of three parts [§ 333]. So its cube is made up of four parts. Let us analyze the process of cubing a number to see what these four parts are.

To find the cube of 64.

$$64^3 = 64 \times 64 \times 64; \text{ or, as } 64 = 60 + 4,$$

$$64^3 = (60 + 4) \times (60 + 4) \times (60 + 4) = (t + o)(t + o)(t + o).$$

$$\begin{array}{r}
 60 + 4 = t + o \\
 60 + 4 = t + o \\
 \hline
 60 \times 4 + 4^2 = to + o^2 \\
 60^2 + 60 \times 4 = t^2 + to \\
 \hline
 60^2 + 2(60 \times 4) + 4^2 = t^2 + 2to + o^2 \\
 60 + 4 = t + o \\
 \hline
 60^2 \times 4 + 2(60 \times 4^2) + 4^3 = t^2o + 2to^2 + o^3 \\
 60^3 + 2(60^2 \times 4) + 60 \times 4^2 = t^3 + 2t^2o + to^2 \\
 \hline
 60^3 + 3(60^2 \times 4) + 3(60 \times 4^2) + 4^3 = t^3 + 3t^2o + 3to^2 + o^3 \\
 216,000 + 43,200 + 2880 + 64 = 262,144
 \end{array}$$

The four parts of which the cube of 64 is composed, are

- | | |
|---|---|
| I. The cube of the tens. | III. 3 times the tens times the ones ² . |
| II. 3 times the tens ² times the ones. | IV. The cube of the ones. |

Let us try to find the factors of these four parts while finding the cube root of 262,144.

Process.

$$\begin{array}{rcl}
 t^3 + 3t^2o + 3to^2 + o^3 & = & 262'144 (60 + 4. \\
 t^3 & = & 216\ 000 \\
 3t^2o & = & 10,800 \\
 3to & = & 720 \\
 o^2 & = & 16 \\
 3t^2 + 3to + o^2 & = & 11,536
 \end{array}
 \begin{array}{l}
 46\ 144 = 3t^2o + 3to^2 + o^3 = (3t^2 + 3to + o^2)o. \\
 \hline
 46\ 144
 \end{array}$$

EXPLANATION.—262,144 comes between the cubes 216,000 and 343,000; its cube root then comes between 60 and 70, and

Part I. the t^3 must be 216,000. Taking this out of the cube, what remains?

Part II., or $3t^2o$ is much the largest of the three remaining parts of the cube. It is made up of two factors, $3t^2$ and o . The remainder, 46,144, is approximately the product of these two factors. $3t^2 = 3 \times 60^2 = 10,800$. If we divide the approximate product, 46,144, by one of its factors, 10,800, we get the other factor, 4 or o .

Part III., or $3to^2 = 3to \times o$; $3to = 3 \times 60 \times 4 = 720$. Part IV., or $o^3 = o^2 \times o$; $o^2 = 16$; uniting $3t^2 + 3to + o^2$, $10,800 + 720 + 16$, we have 11,536; multiplying this number by o or 4, we have $3to^2 + 3to^2 + o^3 = 46,144$. Hence we conclude that 64 is the cube root required.

28.—Exercises in

Cube Root.

Written.

7. 175,616.

8. 238,328.

As shown above find one of the three equal factors of—

1. 12,167.

2. 32,768.

3. 46,656.

4. 74,088.

5. 103,823.

6. 157,464.

9. 373,248.

10. 474,552.

11. 614,125.

12. 884,736.

13. What is the edge of a cubic block containing 421,875 cubic inches ?
 14. How large a cube will weigh as much as a rectangular prism 24 in. long, 16 in. wide, and 9 in. thick ?
 15. Extract the cube root of 320.013504.

Process.

		320.013504(6.84
		216
$3 \times 60^2 =$	10,800	104. 013
$3 \times 60 \times 8 =$	1,440	
$8^2 =$	64	
	12,304	98. 432
$3 \times 680^2 =$	1,387,200	5. 581,504
$3 \times 680 \times 4 =$	8,160	
$4^2 =$	16	
	1,395,376	5. 581,504

1. We begin at the decimal point to separate the power into groups of three figures. How many figures will then be in the root? How many integral?

2. Having found two figures of the root, we consider the tens to be 68, and proceed as before.

3. If the number is an imperfect power, we may annex decimal ciphers and approximate the root.

16. $\sqrt[3]{3796416}$

18. $\sqrt[3]{37.259704}$

20. $\sqrt[3]{0.087640}$

17. $\sqrt[3]{12977875}$

19. $\sqrt[3]{0.1001728}$

21. $\sqrt[3]{25}$

In extracting the cube root of fractions or mixed numbers, follow the directions given under square root (page 221).

22. $\sqrt[3]{\frac{1728}{2197}}$

24. $\sqrt[3]{3\frac{3}{8}}$

26. $\sqrt[3]{72\frac{1}{8}}$

23. $\sqrt[3]{\frac{19683}{24389}}$

25. $\sqrt[3]{4\frac{12}{125}}$

27. $\sqrt[3]{405\frac{28}{125}}$

28. Find the entire surface of a cube whose volume is 37 cu. ft. 64 cu. in.
 29. If a cubical pile of wood contains 108 cords, how long is it ?

29.—Foreign Moneys. The director of the United States Mint publishes from time to time a list of nearly forty foreign countries with the value of their money standards in United States gold coin. Many of these values change from year to year, especially in the case of countries which have a silver standard, and the rates of exchange vary accordingly.

APPENDIX II.

INTRODUCTION TO ALGEBRA.

1.—Likened to In arithmetic we use (1) numbers expressed
Arithmetic. in figures, (2) signs that indicate some process
or shorten an expression, and we may use
(3) letters that represent a number when we cannot or do not wish
to give the size of it in figures.

In algebra we use the same numbers, signs, and processes, and
we constantly use letters to represent quantities both known and
unknown.

Adding: $5 + 5 =$ how many 5's? $a + a?$

Subtracting: $10 - 5?$ Two 5's $- 5?$ $2a - a?$

Multiplying: $4 \times 5 =$ how many 5's? $4 \times a?$

Factoring: $20 = ?$ $4 \times 5 = ?$ $2 \cdot 2a?$ $4a?$

Dividing: $20 \div 4?$ $20 \div 5?$ $\frac{4a}{4}?$ $\frac{4a}{a}?$

Squaring: $5^2 = \sim$ or $\sim \times \sim$; $a^2 = ?$ $\sqrt{25} = ?$

2.—Contrasted with The most important differences are
Arithmetic. these:—

I. In algebra, before solving a problem,
it must usually be stated in one or more *equations* to show the rela-
tion between the known quantities and the unknown.

The chief thing to learn is how to find from *equations* the value
of the unknown quantities which they contain.

II. All algebraic problems involve the use of literal quantities.

[§§ 76-79 of the arithmetic may be reviewed here.]

Though arithmetic serves for ordinary computations, there is many a problem that cannot be easily solved without the equations in which it may be stated.

3. — Letters and Signs.

The first letters of the alphabet, a, b, c, d , etc., are used as abbreviations or convenient substitutes for quantities whose value is given or supposed to be known.

The last letters, x, y, z , etc., are used for quantities of unknown value.

Signs have the same force as in arithmetic.

4. — Reading and Interpreting.

Read the following expressions and say what each means.

Thus, in $\frac{3}{c-d}$ the literal quantity d is to be subtracted from c , and the numerical quantity 3 is to be divided by the remainder.

- | | | | |
|---------------|------------------------|--------------------|-----------------------|
| 1. $7 + b$ | 4. $g \div 7$ | 7. $\frac{n}{c}$ | 9. $\frac{3}{c-d}$ |
| 2. $a \div b$ | 5. $f + 8 - d$ | 8. $\frac{a+b}{4}$ | 10. $\frac{a-b}{c+d}$ |
| 3. $3 - c$ | 6. $d \times a \div 2$ | | |

11. Which of the preceding expressions have the fractional form? Read them as if the sign \div had been used.

Notice that $2abc$ is read "Two a, b, c "; that it means the same as $2 \cdot a \cdot b \cdot c$ or $2 \times a \times b \times c$, and is the common way of indicating the product of such factors, the numerical factor coming first.

Read and interpret the following. Notice which are the unknown quantities.

Thus: $5ax$ indicates the product of the numerical factor 5 and of the literal factors a and x .

- | | | |
|------------------------|--------------------|---------------------------|
| 12. $6def$ | 15. $25mn$ | 18. $4ab + 3bc + 5cd - 4$ |
| 13. $2a - 4x$ | 16. $ay - 3z + 10$ | 19. $\frac{a+b}{cd}$ |
| 14. $5abm + 2cd$ | 17. $7admaxy$ | 21. $\frac{a+x+3my}{bcn}$ |
| 20. $3a - \frac{d}{z}$ | | |

- 5.—Problems Suggested by Equations. 1. Make a problem of which the proper statement is $3 \times \$12 = x$. What is meant by x ? Its value is ~.

Try to make a problem for each of these statements:—

2. $x = 13 - 4$ 4. $\$17 - x = \10 6. $4 \text{ wk.} = 32 \text{ wk.} - x$
 3. $x - 12 \text{ h.} = 8 \text{ h.}$ 5. $5 \times x = \frac{10\phi}{2}$ 7. $\frac{7 \text{ bu.}}{x} = 3\frac{1}{2} \text{ bu.}$
 8. $\$1 - x\phi = \$\frac{3}{4} + \$\frac{1}{10}$ 9. $\$3 + \$4 = \frac{\$21}{x}$

- 6.—Writing Equations. The *first member* of an equation, the part at the left of the sign $=$, must always equal the *second member*, the part at the right.

Put into the form of equations, and test each one:—

1. 10 added to 35 is the same as 9 times 5.
 2. 3 more than half of 7 is like a quarter of 26.
 3. $15 =$ twice 10, less 2 more than 3.
 4. What quantity $=$ 19 less 5 times the double of $1\frac{1}{2}$?
 5. $3 =$ a half as much as 1 more than 10 from 15.
 6. a from a leaves 1 less than a fifth of five.
 7. $5 =$ as much more than 4 as 3 exceeds 2.
 8. A quarter of a is as much as the whole of b . $a = 44$; $b = 11$.
 9. 4 less than a and $b = 1$ more than $\frac{1}{2}$ a hundred.
 10. a less 11 comes within 3 less than 70 of being 100.

- 7.—Expression of Unknown Quantities. 1. One chimney has x flues; another twice as many, or $2x$; a third has as many as the first two, or ~. In all how many?

2. y is a certain number. One farm employs $3y$ workmen, another 4 times as many, or ~. The two together ~.

3. Compare the values of x and $\frac{x}{2}$; of $\frac{x}{2}$ and $\frac{1}{2}x$; of $3\frac{1}{2}x$ and

$$3x + \frac{x}{2}$$

Express given statement

4. One kind of pillow costs 3 times a certain sum, another 8 times as much as the first, or $\sim x$.

5. One ladder contains x rounds; another $\frac{1}{2}$ as many, or \sim ; another 3 more than the first or \sim ; another contains 2 more than the second ladder. All four contain \sim .

6. If $16x$ will pay for 4 tons, express the cost of $\frac{1}{8}$ of a ton as more or less than x .

7. Compare $\frac{x}{1\frac{1}{2}}$ of x with $2x$; with $\frac{1}{3}$ of $2x$; with $\frac{2x}{3}$.

8. Explain the difference between $y - 5 + 7$ and $y - (5 + 7)$. Each expression is how much more or less than y ? [\S 75.]

9. How many x 's added will equal $x \times (5 - 2)$ or $(5 - 2)x$.

10. One shelf in a case holds within 1 of a quarter of all the books, or $\frac{x}{4} - \sim$; another holds a quarter as much as 4 more than the

- whole, or $\frac{x + \sim}{4}$. 11. There are $\frac{3}{11}$ of x or $\frac{3x}{11}$ on the third shelf, and on the fourth $\frac{5}{6}$ as many, or \sim of $x = \frac{5x}{6}$.

12. x = the value of an apple; 6 peaches cost $12x$, 4 plums $6x$. How many plums have the value of 9 peaches?

Explain
8. — Statement of Problems.

I. Tell what unknown quantity is to be found.

II. Make an equation for each case.

III. Then give x or y its numerical value.

1. A bag contained x coins. Each of 3 persons took out 7, and 4 remained.

2. When six marbles had been lost from a bag containing y , each of 4 persons took out 8 and 3, leaving none.

3. The present yield of apples (x) is 10 barrels more than last year's, when the crop brought \$160 at \$2.50 a barrel.

4. A certain bicycle cost \$ y . One half that amount is the same as $\frac{2}{3}$ the cost of a \$75 bicycle.

5. If you take $\frac{1}{4}$ of x and add 3 to it, the result is $\frac{1}{4}$ of 20. Is $\frac{x}{4}$ more or less than 5?

6. x is the cost of a house. A quarter and a half of it was $\frac{3x}{4}$, which was the cost of two other buildings. The cost of all three was $x + \frac{3}{4}x$, or \$1750.

7. After x persons were seated in church 4 times as many more came in, or $\sim x$. When they went out, it was found that there were 25 men and 3 times as many women.

8. The long cars in a freight train are to the short cars as 2 to 3. There were 30 or $5x$ in all. The long cars are $\sim x$.

9. One of three brothers is a year old, another is 7, the third is half way between. $x = \frac{1}{2}$ of what?

10. An iceberg moves x miles a day, a ship 10 times as fast. When they approach each other, they are $\sim \times x$ or 110 miles nearer in a day. When the berg follows the ship, they are $\sim \times x$ miles further apart.

**9.—Analysis and
Statement of
Problems.**

I. Without solving, decide upon some quantity whose value is lacking, but may be found from what is given. Say—"let x = the distance from," etc.

II. Show what operations must be performed upon the known quantities to find the value of the unknown. Thus, $x = \frac{7+4}{11}$.

1. It is 90 m. from New York to Philadelphia. Two men set out on bicycles to meet each other. One has gone 30 m., the other 35 m.

2. Twenty rows; eight in a row; every other tree dug up and sold.

3. 100 young apple trees; all but 25 bearing; 150 bu. the yield.

4. Two boys working together earn \$1.50 a day; the first gets $\frac{1}{2}$ as much as the second.
5. If you will bring 2 more like those I have, there will be enough for 7 of us.
6. I had to double my money 4 times before I could buy a \$160 horse.
7. There were 400 grown persons at the fair yesterday, as shown by the receipts, which were \$200. The same amount was received on children's day for half tickets.
8. There are 144 pegs in a shoe. costing 1¢ a gross. At one bench 24 gross are used in a day.

10.—Problems
Stated in General
Terms.

If x = the sum of 5 and 6, or of any two numbers, it can be found from the equation $x = a + b$, a and b representing the two numbers, whatever they are.

1. What kind of problem is to be stated as $x = \frac{a}{b}$? $x = a - b$?
2. What do 3 yd. cost at \$0.80 a yard? Substitute a and b for the known quantities, and give the equation for all such problems.
3. Burning a cords of one kind of wood, and b cords of another kind, what is burned in m years?
4. If the cost for a year is d , the cost for each of n parts of a year would be what part of d ?
5. One person is b inches tall, another $b + 1$, a third $b + 2$. Express briefly the sum of their heights as equal to y .
6. Compare $b + b + 1 + b + 2$; $3b + 3$; $3 \times (b + 1)$. [$b = 5$.] If $3 \times b = 15$, why does $3 \times (b + 1) = 18$, and $3 \times b + 1 = 16$?
7. A pole x feet long is cut into 3 fractional parts, $\frac{1}{a}$, $\frac{1}{b}$, $\frac{1}{c}$. If you take $\frac{1}{b}$ from x , what remains of the pole?
8. A man paves a gutter in x days. What part of it does he pave in 1 day? In m days he paves m times as much, or \dots

9. I walk how many miles in b hours if I go n miles an hour?
 10. I go at the rate of how many miles an hour if in c hours I walk d miles; *i.e.*, in one hour I walk $\frac{d}{c}$ miles.

11. — Terms.

An expression that may be separated into two or more parts connected by $+$ or $-$ contains two or more **terms**. If it cannot be so separated, it consists of one term.

As $ab + 5 - \frac{c}{d}$ contains three terms, it is a *trinomial*.

As $5c - \frac{ad}{b}$ contains two terms, it is a *binomial*.

As $3axy$ and $\frac{a+b}{c}$ contain each one term, each is a *monomial*.

An expression of more than three parts or terms so connected is a *polynomial*.

NOTE. — In the fraction $\frac{a+b}{c}$, the plus sign connects merely the terms of the numerator. The expression as a whole is a single term, a monomial.

Remember that where there is a parenthesis marked by *curves* $()$, *brackets* $[\]$, or the *vinculum* $\frac{\quad}{\quad}$, the included quantities are to be taken as one.

Apply the names "*monomial*," etc., to each of the following expressions, and explain the process indicated: —

- | | | |
|---------------------------|----------------|--------------------------------|
| 1. $3 - (4 - z)$ | 4. $3(c + d)$ | 7. $7ax - \overline{2x - 4af}$ |
| 2. $a - \overline{b - c}$ | 5. $3c + d$ | 8. $3(a + 4) - 4(b - 3)$ |
| 3. $(2 - x) - (y - 4)$ | 6. $x(mn - 1)$ | 9. $(4a[b - 2])(c + d)$ |

12. — Literal Powers

and
 Numerical Coefficients
 as Factors.

1. Define a *power* of any number. 2. What does the *exponent* show?

1st powers: 5, a , xy , $(c - g)$.

2d powers, or squares:

$$5^2, a \times a, a^2, x^2y^2, x \times y \times x \times y, (c - g)^2.$$

Read, and give all the factors of—

- | | | | | |
|----------------------|-------------------|---------------------|--------------|------------------|
| 3. b^3 | 5. a^2b | 7. ab^2 | 9. $(ab)^2$ | 11. $(a+b)^2$ |
| 4. $\overline{ac^2}$ | 6. $a \times b^2$ | 8. $a^2 \times b^2$ | 10. a^2b^2 | 12. $a^4b^3xy^2$ |

13. How much is 3×5 ? 5^3 ? Explain the difference.

14. $3a = a + a + a$; the factors are 3 and a . $3a^2 = ?$

The coefficient of a literal quantity consists of the factors preceding it in the same term, usually the numerical factor, which comes first. Thus, 3 is the coefficient of $3xyz$, or $3xy$ is coefficient of z . When no figure is written, 1 is understood: $a = 1 \times a$, or $1a$.

Give the numerical coefficient, and factor each term, thus:

$$3m = m + m + m, \text{ or } 3 \times m; m^3 = m \times m \times m.$$

- | | | | |
|-----------|---------------|-----------------------|---|
| 15. n | 18. $2a^2m$ | 21. $\frac{abcde}{x}$ | 24. $b^2(a+b) - c(d-e^2)$ |
| 16. $4a$ | 19. $2x^2y^2$ | 22. $3ab - b^3$ | 25. $3a(b+c) + 2y(f-g)$ |
| 17. $3ab$ | 20. $3am^2$ | 23. $2c^2d + 5z$ | 26. $ab(x-y) - 4\left(\frac{3}{x}\right)$ |

Similar terms must contain the same powers of the same literal quantities. The numerical coefficients and the preceding signs may differ.

m^2b and bm^2 are similar, but in each the b should be written before the m .

27. *Select similar terms among the following:—*

$7m^2b$	$-2a^2n^2$	$5an$	$-2xy$	x^3y^2
an	$3b^2m^2$	$-2bm^2$	$2x^3y^2$	$-axy$
$-4x^3y^2$	$-5bm^2$	$4x^2y^3$	$2bn$	$-9an$

13.—The Value of the Unknown Quantity found by Subtraction.

I. If the same quantity be taken from equal quantities, equal quantities remain.

Apply this *axiom* or self-evident truth in finding the value of x in these equations.

1. $x + 4 = 10$ Subtracting 4 from each member, (1) $x = 10 - 4$
Performing the process indicated, (2) $x = 6$

2. $4x = 3x + 17$

Taking $3x$ from each member, we have (1) $4x - 3x = 17$
Performing the subtraction, we find (2) $x = 17$

3. $x + 24 = 32$ 5. $31 + x = 64$ 7. $x + \$19 = \82

4. $5x = 4x + 20$ 6. $27 = 15 + x$ 8. $x + \$2\frac{5}{8} = \20

9. \$5 added to my money will give me \$28. How much have I?
10. The price of corn has risen 13¢ and is now 63¢. What was it at first?

Let $x =$ my money now.

Then $x + 5 =$ { my money after \$5
is added.

Then $x + 5 = 28$, etc.

11. The watch and chain cost

\$125. The chain cost \$30.

12. Show that subtracting a quantity from each member of an equation is the same as *transposing* it to the other side with a minus sign.

14.—The Value of the Unknown Quantity found by Addition.

II. If the same quantity be added to equal quantities, the sums will be equal.

Apply this second axiom in finding the value of x .

1. $x - 7 = 12$

Adding 7 to each member, we have (1) $x - 7 + 7 = 12 + 7$
Performing the processes indicated, (2) $x = 19$

What is the effect of taking away 7 and then adding 7 to any number?

2. $x - \$12 = \19 3. $x - \frac{3}{4} = 2$ 4. $x - \$24.75 = \82.75

5. The cost was \$18; the discount was $\$4\frac{1}{2}$. What was the list price?

Let x = the list price.

(1) $x - \$4\frac{1}{2}$ = the cost.

(2) $x - \$4\frac{1}{2}$ = \$18, etc.

8. Show that in an equation like $x - 3 = 10$, adding 3 to each member is the same as transposing -3 to the other side, with its sign changed.

6. He was in school 26 half days, and absent 12. How long had school kept?

7. 327 miles had been travelled. The journey was x miles, and 487 yet remained.

15.—The Value of the Unknown Quantity found by Division.

III. *If equal quantities be divided by the same quantity, the quotients will be equal.*

Apply this third axiom in finding the value of x .

1. $3x = 24$

Dividing each member by 3, we have

(1) $x = 8$

If $3x = 24$, $1x$ or $x = \frac{1}{3}$ of 24 or 8.

2. $12x = 36$

4. $2\frac{1}{2}x = 25$

6. $0.8x = 5.12$

3. $19y = 57$

5. $13x = \$52.13$

7. $\frac{2}{3}x = 90$

8. I sold my house for \$6,000. If this was three times the cost, what did I gain?

9. Seven times a number less 5 equals 51.

10. Eight times my money and \$40 is 12 times my money.

16.—The Value of the Unknown Quantity found by Multiplication.

IV. *If equal quantities be multiplied by the same quantity, the products will be equal.*

Apply this fourth axiom in finding the value of x .

1. $\frac{x}{3} = 7$

Multiplying each member by 3, we have

(1) $x = 21$

If $\frac{1}{3}$ of $x = 7$, the whole of $x = 3 \times 7$ or 21.

$$2. \frac{x}{12} = 6 \quad 3. \frac{3}{4}x = 9 \quad 4. \frac{5x}{6} = 20 \quad 5. \frac{7x}{10} = \$210$$

6. Lost $\frac{3}{8}$ of my money, but \$35 still remained. How much had I?

7. $\frac{5}{12}$ of the whole distance and 21 miles cover the journey.

8. 18 of the 30 miles of the yacht's triangular course were sailed in 72 minutes. The full time was what?

9. A third and a fourth of my money made \$18.60. How much had I?

(1) $\frac{x}{3} + \frac{x}{4} = \18.60 . Multiply both members by the l. c. m. of 3 and 4.

10. $\frac{3}{7}$ of what I received was silver, $\frac{2}{7}$ was gold, and the remaining \$24 was in bank bills.

17.—Reduction of Equations.

The value of x in any simple equation may be found by applying one or more axioms; that is, by increasing, diminishing, multiplying, or dividing both members of the equation by the same quantity.

The steps of the process should be taken in this order:—

- I. *Combine similar terms; reduce to lowest terms.*
- II. *Clear the equation of denominators by multiplying both members of it by their l. c. m.*
- III. *Transpose unknown quantities to the first member, and known quantities to the second, by addition or subtraction.*
- IV. *Combine similar terms.*
- V. *Divide both members of the equation by the coefficient of x .*

1. Given $\frac{2x}{3} + \frac{3x}{4} + 9 = \frac{5x}{6} + 16$, to find the value of x .

Multiplying by 12, we have (1) $8x + 9x + 108 = 10x + 192$

Subtracting 108 and $10x$, (2) $8x + 9x - 10x = 192 - 108$

Combining similar terms, (3) $7x = 84$

Dividing by 7, (4) $x = 12$

2. Given $\frac{x}{2} + \frac{4x}{5} + 6 = \frac{x}{5} + 17$, to find the value of x .

To *verify* an equation is to prove that its members are equal by substituting numbers for the letters that represent them.

Thus, in the equation in Example 1, $\frac{2x}{3} + \frac{3x}{4} + 9 = \frac{5x}{6} + 16$, we found that $x = 12$. Using 12 in the equation wherever x occurs, we have

$$\frac{2 \times 12}{3} + \frac{3 \times 12}{4} + 9 = \frac{5 \times 12}{6} + 16,$$

or
$$\frac{24}{3} + \frac{36}{4} + 9 = \frac{60}{6} + 16,$$

or
$$8 + 9 + 9 = 10 + 16,$$

or
$$26 = 26.$$

Find the value of x in the following equations, and verify each:

3. $13x + 7 - 15 = 5x + 32$

8. $\frac{3x}{4} - 7 = \frac{2x}{3} + 8$

4. $12x - 13 + x = 35 + 7x$

9. $2x + \frac{5x}{6} - 17 = 0$

5. $108 + 20x + 14 = 12x + 19x$

10. $\frac{x}{4} + \frac{x}{5} + 3 = \frac{x}{10} + 10$

6. $13x - 7 + 16 = 5x + 18 - x$

11. $\frac{x}{10} + \frac{x}{6} + \frac{x}{5} - 3 = 41 - x$

7. $x + \frac{x}{2} + \frac{x}{3} = \frac{x}{6} + 30$

12. $\frac{x-4}{3} + \frac{x-6}{4} = 5\frac{2}{3}$

13. $\frac{2x}{3} + \frac{x}{4} - 5 = \frac{x}{12} - \frac{x}{6} + 7$

14. $\frac{2x-2}{3} + \frac{x}{2} - 1 = \frac{5x-20}{6} + 3$

15. $\frac{5x+2}{3} - \frac{3}{4} = \frac{x+5}{2} + 3\frac{1}{4}$

18.—Problems.

1. A horse and wagon cost \$280. The horse cost 3 times as much as the wagon.
2. Together the girls picked 45 quarts of berries. One picked $\frac{4}{5}$ as much as the other.
3. Divide 23 into two parts, one 5 less than the other.

4. The sum of two numbers is 87, and their difference is 9. What are the numbers?

5. I bought 6 lb. of coffee and 5 lb. of tea for \$6.40. The tea cost twice as much as the coffee. Find the price of each per pound.

6. Of three candidates at an election, A had 3 times as many votes as B, lacking 220, and C had $\frac{2}{3}$ as many as B. If there were 3300 votes cast, how many did each candidate receive?

7. Four times a certain number divided by 3 added to $\frac{3}{4}$ the number equals the difference between the number and 518.

8. A certain number diminished by 40 is the same as 40 diminished by $\frac{1}{3}$ of the number.

9. Take from a number $\frac{2}{3}$ of itself, $\frac{1}{3}$ of itself, and 40, and nothing remains.

10. A man bought an equal number of horses, cows, and sheep. For each horse he paid \$125; for each cow, \$50; and for each sheep, \$12. How many did he buy for \$1,309?

19. — Problems.

1. A's capital in business is twice B's. A loses \$5,000, while B gains \$3,000. They then have \$13,000 together. What did they have originally?

2. A farmer sold $\frac{1}{2}$ his wood at \$3 a cord, $\frac{1}{3}$ of it at \$4, and $\frac{1}{6}$ of it at \$5. He received \$44 for the lot. How many cords did he sell?

3. A certain sum of money was divided among F, G, and H. F and H received \$150; F and G, \$216; and G and H, \$178. How much did each receive?

4. Two tanks contain an equal quantity of water. But after 75 gallons have been taken from one, and 50 gallons added to the other, one contains twice as much as the other.

5. Divide 30 into two such parts that 4 times the greater shall equal 6 times the lesser.

6. Take twice a number from 19, divide the remainder by 3, and add the number. The sum will be the same as if half the sum of the number and 10 were taken. Find the number.

7. The sum of three consecutive odd numbers is 39. What are they?

8. The sum of three consecutive multiples of 7 is 273. What are they?

9. A grocer sells 80 pounds of tea at 50 cents a pound. But this tea is a mixture of poor tea at 45 cents with a better quality worth 65 cents. How many pounds of each kind in the mixture?

10. A has \$250, and B has \$75. How much must one give the other, that they each may have the same sum?

20. — Addition: the As in arithmetic only like numbers can be

Signs + and —. combined in one sum, so in algebra only **similar terms** can be added. To add $3a$, $2b$, and $1c$ is in a sense like adding 3 acres + 2 bushels + 1 cent; we can only indicate the addition of the different units, thus: $3a + 2b + 1c$.

1. How many n 's are $4n$, n , $2n$, $13n$?

2. Arrange similar terms in columns; then add —

$2ab$, $5c$, $7ax$, $4bx$, c , $\frac{1}{2}ab$, bx , $3ax$, $7c$, $\frac{1}{2}ab$, ax , bx .

Indicate the sum of the four amounts just found.

Suppose a man begins the week with no money, and in his book puts a + before each amount that he gains and a — before each

Gain \$ 3	amount that he loses. Instead of	+ 3d
Gain \$ 14	entering each transaction as shown	+ 14d
Loss \$ 2	at the left, he might keep the	— 2d
Gain \$ 3	account as at the right. He would	+ 3d
Loss \$ 4	first add the + sums which in-	— 4d
	crease his property, then the —	
\$ 19 gain less \$ 7	sums which diminish it, and then	+ 19d and —
loss = \$ 12 gain	put the two together. If it were	7d = + 12d

+ 19d and + 7d that he put together, the sum would be + 26d, but adding — 7d to + 19d has an opposite effect. The sum of his gains and losses is + 12d.

Consider that the signs $+$ and $-$ belong to the quantities that follow them, showing their character as gains or losses, and we may write $(+3d) + (+14d) + (-2d) + (+3d) + (-4d) = (+19d) + (-7d) = (+12d)$. Adding -7 is like taking away $+7$, and as all quantities are $+$ unless marked $-$, we may write $19d - 7d = 12d$.

3. Add a loss of \$10 to a gain of \$15; what results?

Add $+$ and $-$ quantities separately.

4. $-4a$	5. $+2x^2l$	6. $8(b+c)$	7. $-2x$
$+3a$	$-1x^2l$	$-2(b+c)$	$-3x$
$+a$	$+4x^2l$	$-3(b+c)$	$-x$
$-5a$	$-2x^2l$	$3(b+c)$	$-5x$
<u>$+5a$</u>	<u>$+7x^2l$</u>	<u>$(b+c)$</u>	<u>$-x$</u>

21.—Problems.

1. A yacht goes 10 m off shore and 3 m back. Give these distances opposite signs, and add.
2. Again she sails from port $+7m$, $-6m$, $+6m$, $+4m$, $-10m$. How far from port is she at the finish?
3. A man has no money, but there are due to him \$10, \$3, \$4.50, \$2.50. He owes \$2, \$6, \$0.75, and \$1.25. If all these sums should be paid, would you mark the balance $+$ or $-$? In adding the eight sums given, which four should be marked $-$?
4. A bin is kept for corn and oats mixed. One bagful $= (c + o)$. The changes in a month are 10 $(c + o)$ received, 4 $(c + o)$ sold, 25 $(c + o)$ received, 2 $(c + o)$ received, 9 $(c + o)$ sold, 1 $(c + o)$ sold. Arrange in one column with the proper signs, and add.
5. Suppose a man's debts are \$100 greater than the money that he has together with what is due to him. Which will represent the amount of his property, $+$ \$100 or $-$ \$100?
6. If the sum of the $-$ or *negative* terms is greater than the sum of the $+$ or *positive* terms, will the result be $+$ or $-$?
7. If you go forward 100 miles $(+100m)$ and backward 150 miles $(-150m)$, is your final position in reference to the starting point $+$ or $-$?

22.— Positive and Negative Quantities.

Any quantity may be taken in two opposite senses, **positive** and **negative**, which we indicate by the signs + and -.

	+	-
When plus means	positive	negative
	above	below
	forward	back
	before	after
	west	east
	north	south
	excess	deficiency

1. What is the difference between 10° above zero and 5° below zero?

2. If we take away a man's gains (+ quantities), is his property diminished or increased? If we lessen the amount of his losses (- quantities), will he have more dollars or fewer? Taking away a positive quantity makes the minuend smaller.

Taking away nothing leaves the minuend the same.

Taking away a negative quantity makes the minuend larger.

$$10 - (+3) = 7 \qquad 10 - 0 = 10 \qquad 10 - (-3) = 13$$

Let us compare these four cases:—

To add				To subtract			
Like signs.		Unlike signs.		Like signs.		Unlike signs.	
+ 6	- 6	+ 6	- 6	+ 6	- 6	+ 6	- 6
+ 4	- 4	- 4	+ 4	+ 4	- 4	- 4	+ 4
+ 10	- 10	+ 2	- 2	+ 2	- 2	+ 10	- 10
Here the first quantity is increased in the same direction by adding the second quantity.		Here we combine opposing quantities. A part of the larger is counteracted by the smaller. If the negative is larger, the result is -.		Here each upper quantity is lessened by taking away 4 of the same kind, leaving 2 of each kind.		If taking (+4) from (+6) leaves the (+6) smaller, the (-4) from (+6) leaves it larger. If (-6) denotes a deficiency, taking away (+4) will increase the deficiency to (-10).	

23. — Subtraction.

1. 2 from 1 = \sim

10 from 0 = \sim

2. -2 and -1 ?

-2 from -1 ?

Subtracting a positive quantity is adding a negative,
and subtracting a negative quantity is adding a positive.

3. From	5	7	9	-3
subtract	2	0	-2	1

4. From	-9	11	11	-11
subtract	-6	-8	8	8

*To subtract a quantity,
change its sign and add.*

5. Read each of the preceding quantities with its sign, thus:
 $+5$ less $+2$.

In such operations do not confuse the sign of subtraction with the sign which marks a quantity as negative.

$2 - 1$ may equal $(+2) - (+1)$ or $(+2) + (-1)$.

What number must be added to the subtrahend to produce the minuend in the following cases?

6. From	$a - a$	$b - a$
subtract	b	$b - a - b$

7. $2a$	$5b - 7x$	$5xy$
$a - 3b - 6x$	$-10xy$	

8. If $a = 5$ and $b = 3$, compare in value $-a + b$ and $b - a$.

By changing signs and adding, —

9. Subtract	19	-8	-7	4
from	17	-6	4	-7

10. x	$3y - x$	$4z$
$2x$	$2y - x - 8z$	

	11.	12.	13.	14.
Min.	$2a + 3b$	$a - 3d$	$4m + 5n - z$	In 13 is $5n$ a part
Subt.	<u>$a + b$</u>	<u>$2a - d$</u>	<u>$3m - 4z$</u>	of what remains?

Shorten the expressions:

**24. — Terms in
Parenthesis.**

1. $a + b + c - b$

4. $2x - 3mx - x + mx$

2. $a + 2b - c + 3a$

5. $3ac + z - ac - z$

[Review § 73.]

3. $a^2 - 2ab - a^2$

6. $4x^2 - 2y^2 - 3x^2 + y$

7. Explain the difference in value between

$6 - (3 + 2)$ and $6 - 3 + 2$

If $+3+2$ is to be taken from 6, we change both signs and write $6-3-2$. If the quantities in curves were to be added to 6, there would be no change of sign, for $6+3-2=6+(3-2)$.

Remove the curves, etc., without changing the value:—

$$8. 7-(3+3); 4+7-(4-2) \quad 9. a-m-(b+c); a+(n-c)-(m+c)$$

Remember that an expression in parenthesis is a single term; thus, $8-[4-(2+1)]$ becomes first $8-4+(2+1)$, $(2+1)$ having its sign changed as a single term.

Remove parentheses, making change for one pair at a time.

Combine similar terms in the result.

$$10. 9-(8-3) \quad 12. 9-[4+(2-1)] \quad 14. 2ax-(3ax-[y-ax])$$

$$11. c-\overline{b+d} \quad 13. x-[a-(x-y)] \quad 15. (2bc+n)-(n+2bc)$$

Without changing values, enclose in parentheses the second and third terms of:—

$$16. 5-z-1+2 \quad 17. a+b-c-e \quad 18. x-y+z+a$$

$$25.—\text{Multiplication: } 2 \times 3 \text{ ft.} = 6 \text{ ft.}$$

$$\text{Product of Coefficients; } 2 \times 3a = 6a$$

$$\text{Sum of Exponents. } 2a \times 3b = 6ab$$

$$3c \times 5x = 15cx$$

$$4ab \times 12y^2 = 48aby^2$$

$$7m^2 \times 9n^3 = 63m^2n^3$$

In multiplying algebraic quantities, *the product of the coefficients is taken first.*

$$1. 5a \times 2bc$$

$$3. 9a \times 4xy$$

$$5. 19ax \times 12by$$

$$2. 6ab \times 3x$$

$$4. 12b \times 4cy$$

$$6. 3\frac{1}{2}bc \times 18dz$$

7. What does an exponent show?

$$a \times a = a^2$$

$$b^3 \times b^3 = b^6$$

$$3a^2b \times 3ab^2 = 9a^3b^3$$

$$a \times a \times a = a^3$$

$$2x^2 \times x = 2x^3$$

$$x^7 \times x^2 = x^9$$

$$a \times a \times a \times a \times a = a^2 \times a^3 = a^5$$

$$3m^4 \times 2m^2 = 6m^6$$

8. Show how the exponents of each product (above) were obtained.

In multiplying algebraic quantities the exponent of a power is found by *adding the exponents of its factors*.

NOTE.—As in arithmetic, so in algebra, the order of the factors does not affect the product. The alphabetical order is generally followed.

9. Let $a = 2$, $b = 3$, $c = 4$, and show that $abc = cba = bac = cab$.

Shorten and rearrange the expressions:—

$$10. \quad xa^2cb \ 2 \ cbx$$

$$11. \quad x^2amc^2xa^34$$

Put all the factors of each group into one shorter term:—

$$12. \quad 2 \ ab \times 4 \ ab$$

$$15. \quad (m+n)(m+n)$$

$$18. \quad 6(a-b) \times (a-b)$$

$$13. \quad 3 \ bc \cdot axy \cdot 2 \ cx$$

$$16. \quad a^2x \times xa^2 \times bac$$

$$19. \quad (m+n)(m+n)^2$$

$$14. \quad a^2, a^3, a^4$$

$$17. \quad 3(a-b) \times 2$$

$$20. \quad 3(x+y) \times 4(x+y)^2$$

Explain:—

$$21. \quad \begin{array}{r} 3 + 2 + 4 \\ \hline 5 \\ 15 + 10 + 20 \end{array}$$

$$22. \quad \begin{array}{r} x + y + z \\ \hline a \\ ax + ay + az \end{array}$$

$$23. \quad \begin{array}{r} 7ab + 3bc + 4cd \\ \hline 2ac \\ 14a^2bc + 6bc^2 + 8ac^2d \end{array}$$

Multiply:—

$$24. \quad \begin{array}{r} x^2 + y^2 + z^2 \\ \hline 3x \end{array}$$

$$25. \quad \begin{array}{r} x^2 + y + z^2 \\ \hline xyz \end{array}$$

$$26. \quad \begin{array}{r} (m-n) + m + n \\ \hline 3a \end{array}$$

$$27. \quad \begin{array}{r} 2x^2y + 3xy^2 + 4xyz \\ \hline 5aa^2y \end{array}$$

28. In finding the product of $2a + 3b + c$ and $3 + 4$, how many times is the multiplicand to be repeated? Find the two partial products and add them.

Similar terms in the partial products must be written under one another and combined in the result.

Find and add the partial products:—

$$29. \quad \begin{array}{r} a^2 + 4ab + b^2 \\ \hline a + b \\ a^3 + 4a^2b \quad \text{etc.} \\ + a^2b \quad \text{etc.} \end{array}$$

$$30. \quad \begin{array}{r} 2x + 3ay + 4bz \\ \hline 2ax + 2by \end{array}$$

$$31. \quad \begin{array}{r} x^3 + 2m + y^3 + m \\ \hline x^2 + m + y^2 \end{array}$$

26. — Multiplication: If $3 \times (+5) = +15$, $3 \times (-5) = -15$.
Like Signs give + ; Un- If $3 \times a = a + a + a$, $3 \times (-a) = -a - a - a$,
like, —. or $-3a$.

If multiplying x by 3 means adding x repeated 3 times, then multiplying x by -3 means subtracting x 3 times, or $-x - x - x = -3x$.

When only one of the factors is negative, the product is negative.

Read the products :—

1. $\begin{array}{r} 7 \\ -7 \end{array}$	$\begin{array}{r} -4 \\ 12 \end{array}$	$\begin{array}{r} -ab \\ 3 \end{array}$	$\begin{array}{r} 2xy \\ -4 \end{array}$	$\begin{array}{r} a^2b \\ - \end{array}$	$\begin{array}{r} -xy^2 \\ xy \end{array}$	$\begin{array}{r} 3a^2b^2 \\ -4a^3x \end{array}$
2. $\begin{array}{r} a - b \\ 3a \end{array}$	3. $\begin{array}{r} 2a^2 + 3b \\ -3 \end{array}$	4. $\begin{array}{r} 3am - 4m^2 + mn \\ m + 1 \end{array}$	5. $\begin{array}{r} 2x^2y + y^3 + ab \\ x - y \end{array}$			

If we indicate the product of 4 multiplied by -2 as 4 to be *subtracted* twice, or -8 , we may write the product of -4 multiplied by -2 as 4 to be *added* twice, or $+8$. -4 times $-2 = +4 \times +2$.

6. $\begin{array}{r} -3ab^4 - 2a^2x \\ -ax \end{array}$	7. $\begin{array}{r} 4mn^3 - 4n^4 + 7 \\ -2mn \end{array}$	8. $\begin{array}{r} m^2y - 3y - n \\ -ay \end{array}$
---	--	--

$$9. -(x^2 - 3xy - y^2) \times (-x^2 + xy - y^2) = \dots$$

27. — Division: 1. Division is the inverse or opposite of what process?
Quotient of
Coefficients, Difference 2. In $3a^2 \times 4a^3 = 12a^5$ show how coefficient and exponent in the product are obtained?
of Exponents. 3. What terms in division and multiplication correspond to each other?

4. Explain: $\frac{12a^5}{3a^2} = 4a^3$; $\frac{12a^5}{4a^3} = 3a^2$; $\frac{3 \times 4aaaaa}{3aa} = 4a^3$.

5. Show how the coefficients and the exponents of the quotients were obtained.

Give all the factors of the dividend, and show which remain after taking out those found in the divisor:—

$$6. \frac{3xy}{x} \quad 7. \frac{15a^2b^3}{5ab} \quad 8. \frac{8x^2y^2z}{2z} \quad 9. \frac{4(a+b)(a-b)}{a-b} \quad 10. \frac{x^8}{x^5}$$

Divide by giving the quotients of coefficients and the difference of exponents of the same letters:—

$$11. \frac{12a^2b^3}{6ab^2} \quad 13. \frac{40ab^2x}{8b^2x} \quad 15. \frac{12a^3+16a^2d}{4a^2} \quad 17. \frac{13a(x+y)}{13(x+y)}$$

$$12. \frac{18a^4m^5}{9a^2m^2} \quad 14. \frac{14a^5x^4}{2a^3x} \quad 16. \frac{20m^4+24m^2}{4m^2} \quad 18. 90a^5 \div 45a^4$$

19. What is the law of signs in multiplication? 20. When the product (dividend) is negative, what signs have the factors (divisor and quotient)? 21. Show that the law of signs in division is the same as in multiplication, —

Like signs in division give +; unlike signs, —.

22. The dividend is 6. Will divisor and quotient have like signs or unlike? 23. Dividend is 6; divisor, — 3. Why should quotient be — 2 rather than + 2?

24. Dividend is — 2a, divisor 2, quotient?

First give sign of quotient; then its value:—

Give product of quotient and divisor:—

DIVISOR.	DIVIDEND.
25. $a + b$	$-(a + b)$
26. $-ab$	$-a^3b^4$
27. $-mn$	$3mn^2$
28. $-6(x-y)$	$-18(x-y)^2$
29. $-7(a+2)^2$	$-21(a+2)^3$

$$30. \frac{2xy)6x^2y - 4xy^2 + 20xy}{3x - 2y + 10}$$

$$31. \frac{-3y^2z)-3y^2z^3-9y^2z^2+12y^2z}{z^2+3z-4}$$

$$32. \frac{ax)4a^4x-3a^3x^2+2a^2x^3}{}$$

$$33. \frac{-8y^3)-8b^2y^3-8by^3+16y^4}{}$$

$$34. \frac{6b^2y)6b^6y-12b^5y^3-12b^3y^2}{}$$

28. — Factoring at Sight.

1. What quantity will divide every term of the trinomial $ax + bx + cx$? What is the remaining factor?
2. Find the numerical and literal factors of $3am + 6an - 9ay$. How is the third factor found?

In the same way find two or more factors of —

3. $6x - 3b^2y$
4. $3a^4 - 6a$
5. $6ax - 12a^2y + 24ay^2$
6. Find the two prime factors of $4a + 8ab - 12b$
7. Of $3x^2y - xy^2 + 2xy$
8. a^3, b^2 , and 12 are factors of what monomial?

Give two factors : one a trinomial, one the largest possible monomial.

9. $2ab - 6a^2b + 4ab^2$
11. $9ab^2 + 45ab^3 - 27b$
10. $3x^2y + 12x^3y^2 - 15x^2y^4$
12. $-12mx - 16m^2y - 6mxy$
13. What quantity will produce $2a$ when multiplied by 1? By -1 ?
14. 1 is a divisor of $2a + b$; what is the quotient?
15. If -1 is one factor of $-a - b$, what is the other?
16. Give the square of 2; of -2 .

29. — Fractions to Smallest Terms.

All operations with fractions in algebra are based upon the same principles that apply in arithmetic.

Reference may be made if need be to their development on pp. 54-72.

1. Explain: $\frac{2a}{2b} = \frac{a}{b}$ $\frac{ax}{bx} = \frac{a}{b}$ $\frac{3m^2y}{6my} = \frac{m}{2}$

To change a fraction to smallest terms, —

Strike out all factors common to numerator and denominator : —

2. $\frac{4x^2y^2}{2x^2y^3}$
5. $\frac{12a^4d^3e}{15a^3d^2}$
8. $\frac{28mnx^2}{35m^2nx}$
3. $\frac{4a^4x}{16a^2x^4}$
6. $\frac{27a^3x^3y^3}{36ax^2y}$
9. $\frac{21a(a-b^2)}{33b(a-b^2)}$
4. $\frac{ab-cb}{abc}$
7. $\frac{24a^2bx^2}{30abm^2}$
10. $\frac{51abcd(x^2-3)}{68a^2x(x^2-3)}$

30.—Fractions to a Common Denominator.

1. What is the effect of multiplying both terms of a fraction by the same quantity?
2. Change $\frac{x}{4bc}$ and $\frac{y}{2cd}$ to a common denominator. Explain the three steps of the process as shown below.

I.

$$2cd = 2 \times c \times d$$

$$4bc = 4 \times b \times c$$

$$4bc \times d = 4bcd = \text{l. c. m.}$$

II.

$$\frac{4bcd}{4bc} = d$$

$$\frac{4bcd}{2cd} = 2b$$

III.

$$\frac{x}{4bc} \times d = \frac{dx}{4bcd}$$

$$\frac{y}{2cd} \times 2b = \frac{2by}{4bcd}$$

To change fractions to a common denominator:—

- I. Find a common multiple of the denominators.
- II. Divide this multiple by the denominator of each fraction.
- III. Multiply both terms of each fraction by the quotient of its denominator into the common multiple.

Change to common denominators:—

$$3. \frac{3y}{2ab}, \frac{x}{10a^2}$$

$$4. \frac{1}{2xy}, \frac{am}{3x^2y}$$

$$5. \frac{x}{3abm^2}, \frac{y}{6b^2m}, \frac{z}{a4^2}$$

31.—Fractions Added and Subtracted.

To add or subtract fractions:—

- I. Change to a common denominator.
- II. Place the sum or the difference of numerators over the common denominator.
- III. Change results to lowest terms.

$$1. \frac{4a}{xy} + \frac{2b^2}{xy}$$

$$2. \frac{a}{x} + \frac{a}{y} + \frac{a}{z}$$

$$3. \frac{2x}{m} - \frac{3a}{n}$$

$$4. \frac{2}{3xy} + \frac{4}{x^2y^2} + \frac{5}{z^2}$$

$$5. \frac{4}{cd^2y} - \frac{2ad}{c^2y}$$

$$6. \frac{3(a-b)}{x} + \frac{3(a+d)}{y}$$

$$7. \frac{a}{b} + \frac{c}{d} - \frac{m}{n}$$

$$8. \frac{mx}{cd^3y^2} + \frac{nx}{c^2dy^3}$$

$$9. x^2 - \frac{y}{a} + b^2$$

32. — Multiplication of Fractions.

1. Explain: $\frac{a}{bc} \times \frac{a^2}{b^2} \times \frac{ad}{c^2} = \frac{a^4d}{b^3c^3}$

To multiply fractions together, —

Write the product of the numerators over that of the denominators. Cancel when possible.

NOTE. — Integral quantities may be given a fractional form by writing 1 as a denominator.

2. $\frac{4ax}{3by} \times \frac{2b}{ax} \times \frac{3c}{4}$

5. $\frac{a(x+y)}{b^2cd^4} \times \frac{2abc^2}{b^2(x+y)}$

3. $\frac{3a^2b}{16y^2} \times \frac{4y}{6a^4b} \times 2ab$

6. $\frac{(a-b+c)x}{32b^2cd} \times \frac{16abx}{ax-bx+cx}$

4. $\frac{9m^2n^2x}{25d^2a^3y} \times \frac{50d^4ay^2}{27mnz}$

7. $\frac{(a+b)^2}{(x+y)^3} \times \frac{(x+y)^4}{a+b}$

33. — Division of Fractions.

1. Explain: $\frac{a}{b} \div \frac{x}{y} = \frac{ay}{by} \div \frac{bx}{by} = \frac{ay}{bx}$

or

$\frac{a}{b} \div \frac{x}{y} = \frac{a}{b} \times \frac{y}{x} = \frac{ay}{bx}$

In division of fractions, —

Multiply the dividend by the divisor inverted. Cancel when possible.

NOTE. — Integers may be given a fractional form as in multiplication.

2. $\frac{2em}{xy} \div \frac{m}{y}$

5. $\frac{a^2b}{x} \div 5z$

8. $2x^2y \div \frac{3abx^2}{cf}$

3. $\frac{2dz}{5m} \div \frac{10dm}{5z}$

6. $\frac{2a+b^2}{3x^2y^4} \div \frac{5(2a+b)}{6xy^2}$

9. $\left(\frac{x}{y} \times \frac{c}{d}\right) \div \frac{3d^2c^2}{y+d}$

4. $\frac{a+b}{x} \div \frac{(a+b)x}{y}$

7. $\frac{abcd}{yz^2} \div \frac{2ab^2}{7y}$

10. $\frac{-2ad^2}{4b^2c} \div \frac{-4a^2d}{16c^2}$

34. — Equations with Two Unknown Quan- tities.

(1) $x + y = 12$

(2) $x - y = 2$

These equations are *independent*; that is, neither one is made from the other. They are also *simultaneous*; that is, the same unknown quantities have the same values in each equation.

1. $\begin{cases} (1) & x + y = 12 \\ (2) & 2x + 2y = 24 \end{cases}$ Are these equations independent? (2) is made by doubling (1). Are they simultaneous?

2. $\begin{cases} (1) & x + y = 12 \\ (2) & x + y = 17 \end{cases}$ Are these equations simultaneous? Can the sum of the same two numbers be both 12 and 17?

3. Let $x = 3$ and $y = 2$, and make two equations that are both independent and simultaneous.

4. $\begin{cases} (1) & 2x + y = 13 \\ (2) & 3x - y = 7 \\ (3) & 5x = 20 \end{cases}$ If we combine two or more independent simultaneous equations in such a way as to cause one of the unknown quantities to disappear, we *eliminate* it, and obtain an equation with *one* unknown quantity only, which is readily solved.

$$\text{Whence } x = 4$$

$$\text{and } 2x = 8$$

$$(1) \quad 8 + y = 13$$

$$y = 5$$

In Example 4 we have added the equations. As $+y$ and $-y$ neutralize each other, we obtain equation (3), $5x = 20$ and $x = 4$. Substituting 8, the value of $2x$, for $2x$ in equation (1), we obtain 5, the value of y .

5. Verify the equations in the preceding example by substituting their values for x and y .

6. $\begin{cases} (1) & x + 7y = 31 \\ (2) & x - 3y = 1 \end{cases}$ In Example 6 we eliminate x by subtracting (2) from (1), and find values as before.

$$(3) \quad 10y = 30$$

$$y = 3$$

$$x + 21 = 31$$

$$x = 10$$

It will be observed that we eliminate by *addition* when the quantities to be eliminated have unlike signs, and by *subtraction* when they have like signs.

$$7. \begin{cases} 4x - y = 9 \\ x + y = 16 \end{cases}$$

$$8. \begin{cases} 7x + y = 59 \\ 9x + y = 75 \end{cases}$$

9. The sum of two numbers is 19; their difference is 11. Find the numbers.

10. Three times Charles's money plus Henry's = \$325; but if Henry's be taken from four times Charles's, \$200 remains.

35.—Elimination
by Addition or Sub-
traction.

Process.

$$\begin{array}{rcl} (1) & 4x - 2y & = 28 \\ (2) & 5x + 3y & = 57 \\ (3) & 12x - 6y & = 84 \\ (4) & 10x + 6y & = 114 \\ (5) & 22x & = 198 \end{array}$$

$$x = 9$$

$$5x = 45$$

$$\begin{array}{rcl} (2) & 45 + 3y & = 57 \\ & 3y & = 12 \\ & y & = 4 \end{array}$$

$$2. \begin{cases} x + y = 13 \\ 3x - 2y = 14 \end{cases}$$

$$3. \begin{cases} x - y = 4 \\ 3x + 4y = 61 \end{cases}$$

$$4. \begin{cases} 2x + y = 26 \\ 3x + 4y = 44 \end{cases}$$

$$5. \begin{cases} 5x - 3y = 27 \\ 3x - 2y = 16 \end{cases}$$

Find the values of x and y in

$$1. \begin{cases} 4x - 2y = 28 \\ 5x + 3y = 57 \end{cases}$$

I. We choose to eliminate y because its coefficients are the smaller.

II. We make the coefficients of y alike by multiplying equation (1) by 3, and equation (2) by 2.

III. We eliminate by addition because the signs of the y -terms differ.

IV. We find the value of x by division, and of y by substituting the value of $5x$ in (2).

$$6. \begin{cases} 6x - 10y = 24 \\ 2x - 4y = 6 \end{cases}$$

$$7. \begin{cases} 3x - 5y = 11 \\ 10x + 6y = 150 \end{cases}$$

$$8. \begin{cases} 4x - 2y = 28 \\ 3x + 5y = 47 \end{cases}$$

$$9. \begin{cases} 5x - 4y = 7 \\ 7x + 3y = 70 \end{cases}$$

$$10. \begin{cases} x + \frac{1}{2}y = 13 \\ \frac{1}{2}x + y = 8 \end{cases}$$

$$11. \begin{cases} \frac{1}{3}x - \frac{1}{2}y = 0 \\ 3x - 2y = 25 \end{cases}$$

$$12. \begin{cases} 3x = 43 + y \\ 7x = 67 + 9y \end{cases}$$

$$13. \begin{cases} 5y - 4z = 33 \\ 6y + 5z = 118 \end{cases}$$

NOTE. — First clear of fractions after reducing them to lowest terms.

$$14. \begin{cases} \frac{x}{3} - \frac{y}{4} = 4 \\ \frac{x}{5} - \frac{y}{6} = 2 \end{cases}$$

$$15. \begin{cases} \frac{5x}{6} + \frac{y}{2} = 21 \\ \frac{2x}{3} - \frac{3y}{4} = 3 \end{cases}$$

$$16. \begin{cases} \frac{4x}{5} + 2y = 40 \\ 2x + \frac{3y}{4} = 49 \end{cases}$$

$$17. \begin{cases} \frac{6x-8}{4} + 17 = 15 + \frac{12y+9}{7} \\ \frac{3x+27+18}{6} = \frac{5x+4y}{8} + 2\frac{1}{4} \end{cases}$$

$$18. \begin{cases} \frac{3x+4}{2} + \frac{5y+3}{3} = 25 \\ \frac{4x-2}{3} + \frac{3y-8}{5} = 12 \end{cases}$$

$$19. \begin{cases} \frac{x}{2} + \frac{y}{3} = \frac{x}{4} + \frac{y}{6} + 8 \\ \frac{x}{8} + \frac{7y}{12} + 4 = \frac{2x}{3} - \frac{5y}{6} \end{cases}$$

36. — Equations with Two Unknown Quantities. Problems. 1. Three times A's money added to 4 times B's money is \$310; but 4 times A's put with 3 times B's makes \$320. How much has each?

2. A and B together have \$850. A spends \$150, and B earns \$100, and then they have equal amounts. How much had each originally?

3. Divide 100 into two such parts that $\frac{3}{8}$ of the greater shall be equal to $\frac{2}{3}$ of the smaller part.

4. Half the sum of two numbers is 32, and 5 times their difference is 80. What are the numbers?

5. Six pounds of tea and 4 pounds of coffee cost \$6.10, and 3 pounds of tea and 8 pounds of coffee cost \$5.45. Required the price of each.

6. Two men together own 100 acres. If A sells $\frac{1}{2}$ of his share, and B gives away $\frac{1}{3}$ of his, they will then own 55 acres. How many acres had each at first?

7. 30 is the quotient when the sum of two numbers is divided by 3, and $\frac{1}{3}$ of their difference is 4. What are the numbers?

8. Of two numbers, 3 times the larger less 56 equals $\frac{1}{3}$ the smaller, and $\frac{2}{3}$ the larger added to $\frac{1}{4}$ the smaller equals 11. Find the numbers.

9. $\frac{2}{3}$ of A's property is equal to $\frac{3}{4}$ of B's, and the difference between A's and B's is \$18. What has each man?

10. Should 3 be added to the denominator of a certain fraction, it would become $\frac{1}{3}$; but should the numerator be increased by 2, it would become $\frac{2}{3}$. What is the value of the fraction?

ANSWERS.

Art. 21.

1. 80.
2. 90.
3. 100.
4. 118.
5. 107.
6. 93.
7. 90.
8. 131.
9. 160.
10. 250.
11. 1040.
12. 800.
13. 999.
14. 999.
15. 855.
16. 1100.
17. 800.
18. 1099.
19. 6798.
20. 3475.
21. 11587.
22. 9385.
23. $\frac{1}{13}$.
24. 0.48.
25. 63%.
26. 1.00.
27. \$15.87.

Art. 23.

1. \$45.40.
2. 300.49.
3. 3915.24.
4. 35751.37.
5. 48500.80.
6. 11465.13.
7. 13729.55.
8. 15850.62.
9. 16197.45.
10. 14681.69.
11. 16678.86.

Art. 34.

1. \$16654.13.
2. 4881.11.
3. 4194.20.
4. 7198.54.
5. 12726.65.
6. 15138.25.
7. 6668.01.
8. 3960.21.
9. 9668.11.
10. 14113.23.
11. 17595.28.
12. 25258.24.
13. 21218.38.
14. 16970.48.
15. 16985.86.
16. 15912.48.

Art. 24.

A.

1. \$6410.05.
2. 6740.79.
3. 6613.78.
4. 6591.27.
5. 5988.19.
6. 7614.95.
7. 8611.69.
8. 9851.73.
9. 6641.52.
10. 8302.91.
11. 9628.47.
12. 10524.13.
13. 12211.41.
14. 13238.12.
15. 14691.68.

B.

1. \$6378.14.
2. 6712.69.
3. 6419.09.
4. 5764.79.
5. 5257.36.
6. 6598.78.
7. 8106.43.
8. 9539.25.
9. 6164.75.
10. 6875.43.
11. 8700.77.
12. 10266.72.
13. 11669.24.
14. 12921.62.
15. 14200.07.

Art. 36.

1. \$473.23.
2. \$30447.10.
3. \$140.53.
4. 309.52.
5. \$127.73 loss.
6. 256506.
7. 869758.
8. 859 feet.
9. 226 d.
10. \$491.61.

Art. 38.

1. \$10.18.
2. 509.78.
3. 8458.25.
4. 15999.60.
5. 7156.52.

Art. 43.

2. 4753.
3. 6516.
4. \$1.75.

5. \$1548.

6. \$180.

7. \$240.

8. \$6.21.

9. 50 h.

10. \$104.

Art. 45.

1. 28578.
2. 39448.
3. 61101 lb.
4. 41909.
5. 6,797,200.
6. \$96312.
7. 253,138,200.
8. 60,195,000.
9. 62000 in.
10. 3,060,000,000.

Art. 48.

1. 453,456.
2. 2,373,672.
3. 15984¢.
4. 35441.
5. 114,163.
6. \$3,019,365.
7. 62464 oz.
8. 69223.
9. 7,928,712.
10. \$3096.

Art. 49.

3. \$5748.
4. \$131,436.
5. \$128,945.
6. \$12592.
7. 5551¢.
8. \$55.51.
9. 137425¢ or \$1374.25.
10. 198186¢ or \$1981.86.

11. 1,170,000,000¢

or \$11,700,000.

12. 13500¢ or \$135.

Art. 52.

1. \$316.80.
2. 2010.
3. 65.16.
4. 4862.97.
5. 1850.
6. 717.60.
7. 9372.80.
8. 6420.

Art. 55.

1. £28.
2. 1237.
3. 275.
4. 310.
5. 653.
6. 57.
7. 132.
8. 1501.
9. 26.
10. 7200.

Art. 59.

1. 71 $\frac{1}{2}$.
2. 1900.
3. 75 yds.
4. 70 $\frac{3}{4}$.
5. 800 bales.
6. 25.
7. 197 or \$461.
8. 200 times; 22 will remain.
9. \$160 $\frac{3}{4}$.
10. \$71 $\frac{1}{2}$.

Art. 62.

1. a. 1 $\frac{8}{15}$ 1 $\frac{7}{15}$ 1 $\frac{6}{15}$ 1 $\frac{5}{15}$ 1 $\frac{4}{15}$
 b. 2 $\frac{6}{15}$ 2 $\frac{4}{15}$ 2 $\frac{3}{15}$ 2 1 $\frac{1}{2}$
 c. 3 $\frac{4}{15}$ 3 $\frac{1}{15}$ 2 $\frac{13}{15}$ 2 $\frac{11}{15}$ 2 $\frac{9}{15}$
 d. 4 $\frac{2}{15}$ 3 $\frac{14}{15}$ 3 $\frac{9}{15}$ 3 $\frac{6}{15}$ 3 $\frac{3}{15}$
 e. 5 4 $\frac{9}{15}$ 4 $\frac{6}{15}$ 4 $\frac{3}{15}$ 3 $\frac{14}{15}$
 f. 5 $\frac{11}{15}$ 5 $\frac{6}{15}$ 5 $\frac{3}{15}$ 4 $\frac{12}{15}$ 4 $\frac{8}{15}$
 g. 6 $\frac{9}{15}$ 6 $\frac{3}{15}$ 5 $\frac{12}{15}$ 5 $\frac{7}{15}$ 5 $\frac{2}{15}$
 h. 7 $\frac{7}{15}$ 7 6 $\frac{8}{15}$ 6 $\frac{2}{15}$ 5 $\frac{13}{15}$
 i. 6 $\frac{11}{15}$ 6 $\frac{5}{15}$ 5 $\frac{14}{15}$ 5 $\frac{9}{15}$ 5 $\frac{4}{15}$
 j. 6 5 $\frac{3}{15}$ 5 $\frac{3}{15}$ 4 $\frac{14}{15}$ 4 $\frac{10}{15}$
2. a. 1 $\frac{3}{18}$ 1 $\frac{2}{18}$ 1 $\frac{1}{18}$ 1 2 $\frac{1}{2}$
 b. 1 $\frac{1}{18}$ 1 $\frac{1}{18}$ 1 $\frac{1}{18}$ 1 $\frac{1}{18}$ 1 $\frac{1}{18}$
 c. 2 $\frac{7}{18}$ 2 $\frac{5}{18}$ 2 $\frac{3}{18}$ 2 $\frac{1}{18}$ 1 $\frac{17}{18}$
 d. 3 2 $\frac{16}{18}$ 2 $\frac{14}{18}$ 2 $\frac{12}{18}$ 2 $\frac{10}{18}$
 e. 3 $\frac{11}{18}$ 3 $\frac{8}{18}$ 3 $\frac{5}{18}$ 3 $\frac{2}{18}$ 2 $\frac{17}{18}$
 f. 4 $\frac{1}{18}$ 4 3 $\frac{16}{18}$ 3 $\frac{14}{18}$ 3 $\frac{12}{18}$
 g. 4 $\frac{15}{18}$ 4 $\frac{11}{18}$ 4 $\frac{7}{18}$ 4 $\frac{4}{18}$ 3 $\frac{17}{18}$
 h. 5 $\frac{8}{18}$ 5 $\frac{3}{18}$ 4 $\frac{18}{18}$ 4 $\frac{14}{18}$ 4 $\frac{10}{18}$
 i. 4 $\frac{17}{18}$ 4 $\frac{13}{18}$ 4 $\frac{9}{18}$ 4 $\frac{5}{18}$ 4 $\frac{1}{18}$
 j. 4 $\frac{15}{18}$ 4 $\frac{11}{18}$ 3 $\frac{15}{18}$ 3 $\frac{11}{18}$ 3 $\frac{7}{18}$
3. a. 4 $\frac{54}{99}$ 4 $\frac{58}{99}$ 4 $\frac{62}{99}$ 4 $\frac{66}{99}$ 4 $\frac{70}{99}$
 b. 5 $\frac{25}{99}$ 5 $\frac{30}{99}$ 5 $\frac{35}{99}$ 5 $\frac{40}{99}$ 5 $\frac{45}{99}$
 c. 6 $\frac{42}{99}$ 6 $\frac{48}{99}$ 6 $\frac{54}{99}$ 6 $\frac{60}{99}$ 6 $\frac{66}{99}$
 d. 7 $\frac{31}{99}$ 7 $\frac{38}{99}$ 7 $\frac{45}{99}$ 7 $\frac{52}{99}$ 7 $\frac{59}{99}$
 e. 8 $\frac{45}{99}$ 8 $\frac{53}{99}$ 8 $\frac{61}{99}$ 8 $\frac{69}{99}$ 8 $\frac{77}{99}$
 f. 9 $\frac{73}{99}$ 9 $\frac{82}{99}$ 9 $\frac{91}{99}$ 10 $\frac{4}{99}$ 10 $\frac{14}{99}$
 g. 5 $\frac{28}{99}$ 5 $\frac{33}{99}$ 5 $\frac{38}{99}$ 5 $\frac{43}{99}$ 5 $\frac{48}{99}$
 h. 6 $\frac{55}{99}$ 6 $\frac{61}{99}$ 6 $\frac{67}{99}$ 6 $\frac{73}{99}$ 6 $\frac{79}{99}$
 i. 7 $\frac{39}{99}$ 7 $\frac{46}{99}$ 7 $\frac{53}{99}$ 7 $\frac{60}{99}$ 7 $\frac{67}{99}$
 j. 8 $\frac{15}{99}$ 8 $\frac{23}{99}$ 8 $\frac{31}{99}$ 8 $\frac{39}{99}$ 8 $\frac{47}{99}$
4. a. 2840 $\frac{100}{100}$.
 b. 7149 $\frac{75}{125}$.
 c. 3671 $\frac{75}{175}$.
 d. 5821 $\frac{100}{125}$.
 e. 2356 $\frac{200}{275}$.
 f. 2760 $\frac{325}{325}$.
 g. 1050 $\frac{100}{100}$.
- h. 1265 $\frac{300}{300}$.
 i. 907 $\frac{350}{350}$.
 j. 1355 $\frac{225}{225}$.
5. a. 772; 294 rem.
 b. 1794; 96 rem.
 c. 1325; 507 rem.

- d. 829; 202 rem.
e. 414; 773 rem.
f. 485; 854 rem.
g. 1395; 246 rem.
h. 734; 618 rem.
i. 402; 501 rem.
j. 371; 644 rem.

6. a. \$ $72\frac{5576}{33932}$.
b. $12\frac{45}{741}$.
c. $672\frac{250}{94925}$.
d. $943\frac{548}{84428}$.
e. $759\frac{309}{84113}$.
f. $174\frac{7928}{50006}$.
g. $30\frac{9850}{15435}$.
h. $201\frac{3120}{33414}$.
i. $13\frac{13996}{70208}$.
j. $132\frac{3490}{73570}$.

7. a. $367\frac{494}{9430}$.
b. $56\frac{1136}{10040}$.
c. $302\frac{267}{7348}$.
d. $151\frac{3938}{39098}$.
e. $620\frac{861}{54405}$.
f. $628\frac{810}{73264}$.
g. $16\frac{1815}{45501}$.
h. $781\frac{770}{63602}$.
i. $434\frac{173}{65148}$.
j. $448\frac{257}{62946}$.

Art. 65.

1. \$921.42.
2. \$144.99.
3. 1254.35.
4. \$478.04.
5. \$1632.59.
6. \$53.66.
7. \$2165.19.
8. \$230.55.
9. 5346.
10. \$225.63.

Art. 67.

2. $56\frac{3}{4}$ lb.
3. \$350.
4. \$525.
5. \$0.75.
6. 28 m.
7. \$2500., each man's gain;
\$80000., cost of land; \$95000., proceeds of sale; \$15000., whole gain.
8. \$15.75.
9. 3¢ cheaper to send by mail.
10. 22 qt.

Art. 68.

8. 2640.
9. 17741.
10. 15924.
11. 15522.
12. 1,206,060.
13. 32616.
14. 3690.
15. 11466' ft.
16. 10710 lb.

Art. 72.

2. $x = 128.08$,
 $y = 148.92$,
 $z = 209.39$.
Sum of $A =$
1180.11.
Sum of $B =$
693.72.
Sum of $A -$ sum
of $B =$ sum of C
or 486.39.
3. $1,553,424\frac{240}{365}$ m.
4. \$1225.

5. $165\frac{2}{3}$ lb.
6. \$109.85.
7. \$1050.
8. \$1.50.
9. Hans.
10. \$4.

Art. 77.

1. \$16.
2. \$192.
3. 4608 in.
4. 84480 ft.
5. 5185.
6. 30 d.
7. $341\frac{48}{144}$.
8. 243.
9. $12\frac{8}{10}$.

Art. 83.

1. \$16.32.
2. 152.38.
3. 210.11.
4. 655.13.
5. 1107.76.
6. 533.80.
7. 21.46.
8. 25.66.
9. 1197.17.
10. 39.36.

Art. 85.

2. \$1097.10.
3. \$124.20.
4. $13\frac{54}{133}$ yr.
5. \$8100.
6. $1151\frac{250}{1728}$.
7. 27200.
8. 110.
9. 6288 lb.
10. 9504 lb.
11. 334 lb.

Art. 88.

1. $83\frac{2}{3}$ d.
2. 1750 rm.
3. $312\frac{2}{3}$ bu.
4. $694\frac{64}{144}$ sq. ft.
5. $390\frac{80}{128}$ c.
6. 5625 sq. m.
7. $1151\frac{280}{1728}$ cu. ft.
8. $28\frac{160}{5280}$ m.
9. 2894 boxes.
10. $161\frac{28}{376}$ T.

Art. 89.

1. 26000 pkg's.
2. 18944 pt.
3. 5824 pt.
4. \$5,366,760.
5. 2500 quires or
60000 sheets.
6. 86400.
7. 443520.
8. 12375 ft.
9. 4840 sq. yd.,
43560 sq. ft.,
6,272,640 sq. in.
10. 3,317,760 cu. in.

Art. 90.

1. \$38.40.
2. \$11424.89.
3. \$6198.43.
4. \$5255.57.
5. \$604.
6. \$51.60.
7. \$15.03 saved
by buying of
second firm.
8. \$17280.
9. \$19430.65.
10. \$22076.28.
11. 145.8 ft.

12. 7 passengers
fewer than
when \$110.55
was received
from 33 trips.
13. \$0.02.
14. $8333\frac{1}{3}$ in. or
 $694\frac{1}{3}$ ft.

Art. 96.

1. 51,973,650
coins.
2. \$35,506,987.50
3. \$14,989,278.60
4. 2640 ft.;
1320 ft.;
660 ft.;
4620 ft.
5. 4320 sq. rd.;
5200 sq. rd.
6. 23328 cu in.;
315 ft. 2 in.
7. 253,800.
8. 6696; 58590;
334,800.
9. 3180 lb. iron
for 1 lb. silver;
716 lb. iron for
1 lb. nickel;
8 lb. iron for 1
lb. lead.
10. $37\frac{8\frac{1}{2}}{17\frac{1}{2}}$ cu. ft.
11. \$91.25.

Art. 99.

1. $\frac{30}{9}$.
2. $\frac{1}{8}$ bbl. = $24\frac{1}{8}$ lb.;
\$0.35 gain on
1 bbl.
3. \$100 =
£ $20\frac{5\frac{1}{2}}{97\frac{1}{2}}$;

- \$100 =
 $518\frac{26}{193}$ fr.;
\$100 =
 $420\frac{49}{23}$ M.
4. \$0.674.
5. $2\frac{4}{13}$;
52 wk. 2 d.
6. 49 gal. = 11319
cu. in.;
3 bu. = 6451.26
cu. in.
7. No profit.
8. $33\frac{20}{3}$ bu.;
\$19.833 $\frac{1}{3}$.
9. 100 bbl.
10. 63 bbl.

Art. 101.

1. $2\frac{1}{2}$.
2. 420.
3. 12.
4. 2112 m.
5. 42 periods.
6. \$9216.
7. $1\frac{1}{2}$ m.
8. \$270.
9. 30 pieces.
10. 4.53 8-hour
watches.

Art. 102.

1. 38492 deposi-
tors and a re-
mainder of
\$181.04.
2. \$1075.57;
\$185.43.
3. $888\frac{24}{1427}$ ft.
4. \$610 loss.
5. Sun's diame-
ter = $109\frac{3338}{7918}$

× earth's di-
ameter.

6. 234 min.
7. $416\frac{4}{9}$ lb.
8. 2,280,960.
9. 92,795,826 m.
10. $69.2 + m$.

Art. 106.

1. $\frac{355}{9}$.
2. $37\frac{8}{15}$.
3. $\frac{5975}{12}$.
4. $45\frac{3}{25}$.
5. $\frac{10239}{20}$.
6. $416\frac{8}{13}$.
7. $\frac{40514}{101}$.
8. $9\frac{68}{10}$.
9. $\frac{46522}{100}$.
10. $20\frac{9}{10}$.
11. $86\frac{34}{31}$ gal.
12. 2363 eggs.
13. 5678.
14. \$914 $\frac{3}{4}$.
15. $\frac{1429}{14}$.
16. 4.00.
17. 400 $\frac{1}{2}$.
18. 14 $\frac{3}{5}$.
19. $37\frac{1}{2}$.
20. $60\frac{17}{25}$.

Art. 112.

1. $\frac{19}{27}$.
2. $\frac{8}{11}$.
3. $\frac{21}{25}$.
4. $\frac{23}{57}$.
5. $\frac{9}{10}$.
6. $\frac{11}{15}$.
7. $\frac{17}{5}$.
8. $\frac{9}{13}$.
9. $\frac{3}{8}$.
10. $\frac{9}{25}$.

11. $\frac{1261}{1856}$.
12. $\frac{31}{31}$.

Art. 115.

1. $1\frac{1}{5}$.
2. $1\frac{7}{16}$.
3. 0.
4. 1.
5. $1\frac{5}{12}$.
6. $\frac{1}{24}$.
7. 0.
8. $1\frac{1}{8}$.
9. $1\frac{1}{2}$.
10. $\frac{31}{2}$.
11. $1\frac{2}{3}$.
12. $2\frac{5}{8}$.
13. 1.
14. $1\frac{1}{3}$.
15. $1\frac{1}{3}$.
16. 5.
17. $1\frac{5}{2}$.
18. $\frac{1}{20}$.
19. $1\frac{1}{10}$.
20. $1\frac{1}{4}$ d.

Art. 117.

4. 420.
5. 1080.
6. 5; 5.
8. 4620.
9. 1800.
11. 315.
12. 432.
13. 400.
14. 288.
15. 1020.
16. 12000.

Art. 119.

3. $10\frac{333}{791}$;
 $30\frac{10}{33}$;

- 87½;
 $6\frac{251}{1000}$.
 4. $\$2\frac{1}{2}$;
 40½ doz;
 27 $\frac{71}{128}$ cd.
 5. $13\frac{25}{100}$.
 6. $4\frac{59}{100}$.
 7. $50\frac{9}{100}$.
 8. $2\frac{43}{729}$; $7\frac{16}{29}$.
 9. 864.
 10. $\frac{1}{4}$.
 11. $1\frac{9}{11}$.
 12. $141\frac{1}{2}$.
 13. $29\frac{24}{15}$ is $\frac{1}{15}$ more
 than $196\frac{1}{3}$.
 14. $\frac{1}{3}$.
 15. $2\frac{1}{5}$.

Art. 121.

1. $2\frac{3}{33}$.
 2. $11\frac{75}{198}$.
 3. $1\frac{3}{56}$.
 4. $1\frac{19}{40}$.
 5. $11\frac{7}{40}$.
 6. $\frac{7}{40}$.
 7. $\frac{7}{40}$.
 8. $11\frac{1}{28}$.
 9. $27\frac{11}{280}$.
 10. $7\frac{17}{80}$.
 11. $2\frac{1}{16}$.
 12. $1\frac{31}{288}$.
 13. $\frac{7}{16}$ is $3\frac{1}{400}$ greater
 than $\frac{9}{25}$.
 14. $\frac{7}{8}$.
 15. 155.
 16. $5\frac{31}{1000}$.
 17. $50\frac{2}{3}$.

Art. 123.

1. $58\frac{3}{50}$.
 2. $75\frac{3}{4}$.

3. $56\frac{113}{120}$.
 4. $13\frac{125}{144}$.
 5. $71\frac{7}{20}$.
 6. $2\frac{9}{40}$.
 7. $165\frac{3}{5}$.
 8. $20\frac{5}{6}\%$.
 9. $11\frac{5}{14}$.
 10. $81\frac{73}{200}$.

Art. 124.

1. 84, the un-
 known num-
 ber is larger.
 2. $\frac{1}{60}$.
 3. $\$20\frac{1}{4}$.
 4. 75%.
 5. 68 courses.
 6. $1\frac{1}{2}$ cd.
 7. $\frac{1}{6}$ more.
 8. $\frac{1}{5}$.
 9. $\frac{1}{65}$.
 10. $\$15$.

Art. 128.

5. $19\frac{1}{2}$.
 6. 35.
 7. 45.
 8. 10.
 9. $95\frac{5}{8}$ T.
 10. $396\frac{2}{3}$ yd.

Art. 129.

1. $\$1\frac{5}{16}$.
 2. $833\frac{1}{3}$ sec.
 3. 6560.
 4. $5106\frac{3}{4}$.
 5. $\$71.03\frac{3}{4}$.
 6. $\$160.78\frac{1}{8}$.
 7. $681\frac{5}{8}$ lb.
 8. $281\frac{1}{4}$ lb.
 9. $\$173.95\frac{3}{4}$.

10. $389\frac{7}{12}$ m.
 11. $\$201\frac{3}{4}$.
 12. $5534\frac{7}{12}$ m.
 13. $\$294\frac{3}{8}$.

Art. 130.

1. 6 yd.; $\$0.51\frac{3}{4}$.
 2. $\$2.69$.
 3. $\$149.50$.
 4. $49\frac{43}{73}$ ft.
 5. $80\frac{7}{16}$ cd.
 6. 102 sec.
 7. $43\frac{1}{3}$ m.
 8. $\$38.99$.
 9. $\$0.09$.
 10. $1161\frac{1}{4}\frac{1}{2}$ bbl.

Art. 133.

1. $3\frac{25}{34}$.
 2. 5.
 3. $56\frac{1}{4}$.
 5. $19\frac{1}{2}$.
 6. 75.
 7. $\frac{3}{4}$.
 8. 84.
 9. $9\frac{2}{3}$.
 10. $122\frac{5}{8}$.
 13. $1108\frac{1}{5}$.
 14. $1142\frac{1}{3}$.
 15. $1806\frac{15}{16}$.
 16. $1329\frac{23}{30}$.
 17. $21085\frac{1}{6}$.
 18. $54440\frac{23}{30}$.
 19. $10524\frac{1}{35}$.

Art. 134.

1. $\$43.30$.
 2. $\$2.85$.
 3. $\$71.14$.
 4. $\$3.62\frac{1}{2}$.

5. $\frac{1}{2}$.
 6. $\$0.57\frac{3}{4}$.
 7. $\$1.60$ is 20¢ a
 gross better.

Art. 140.

5. $1\frac{19}{24}$.
 6. $1\frac{5}{22}$.
 7. $1\frac{5}{72}$.
 8. $3\frac{35}{48}$.
 9. $\frac{9}{10}$.
 10. $\frac{5}{12}$.
 11. $3\frac{1}{3}$.
 12. $\frac{5}{8}$.
 13. $2\frac{2}{5}$.
 14. $1\frac{1}{5}$.

Art. 141.

1. $3\frac{78}{115}$.
 2. $1\frac{187}{288}$.
 3. $1\frac{295}{1029}$.
 4. $16\frac{7}{19}$.
 5. $21\frac{78}{43}$.
 6. 15.
 7. $1\frac{1}{3}$.
 8. $\frac{5}{6}$.
 9. $\frac{1}{6}$.
 10. $\frac{3}{16}$.
 11. $\frac{2}{17}$.
 12. $2\frac{2}{5}$.

Art. 142.

1. $182\frac{6}{7}$.
 2. $434\frac{29}{48}$.
 3. $610\frac{49}{60}$.
 4. $1023\frac{55}{72}$.
 5. $79\frac{2}{3}$.
 6. $15\frac{27}{20}$.
 8. $21\frac{73}{104}$.
 9. $443\frac{11}{15}$.
 10. $32\frac{21}{296}$.

11. $11\frac{359}{1200}$ lb.

12. $16\frac{1}{2}$ ft.

Art. 143.

5. $58\frac{1}{3}$ times.

6. $6\frac{42}{121}$ sq. rd.

7. 106; 50 rem.

8. 85 spoons;

$\S 0.41\frac{1}{4}$ re-
maining.

9. 1075 m.

Art. 144.

1. (1) $1\frac{1}{3}$.

(2) $1\frac{5}{12}$.

(3) $1\frac{1}{2}$.

(4) $1\frac{13}{24}$.

(5) $1\frac{1}{5}$.

(6) $1\frac{5}{9}$.

(7) $1\frac{11}{30}$.

(8) $1\frac{1}{12}$.

(9) $1\frac{2}{3}$.

(10) $1\frac{7}{15}$.

2. (1) $1\frac{11}{36}$.

(2) $1\frac{1}{28}$.

(3) $1\frac{3}{8}$.

(4) $5\frac{3}{60}$.

(5) $1\frac{29}{140}$.

(6) $1\frac{7}{20}$.

(7) $1\frac{11}{100}$.

(8) $1\frac{3}{16}$.

(9) $1\frac{1}{12}$.

(10) $1\frac{13}{100}$.

3. (1) $1\frac{9}{14}$.

(2) $1\frac{1}{4}$.

(3) $12\frac{3}{2}$.

(4) $1\frac{1}{6}$.

(5) $1\frac{5}{6}$.

(6) $1\frac{29}{120}$.

(7) $1\frac{1}{5}$.

(8) $11\frac{1}{2}$.

(9) $7\frac{91}{40}$.

(10) $1\frac{31}{26}$.

4. (1) $1\frac{2}{9}$.

(2) $1\frac{1}{8}$.

(3) $1\frac{11}{24}$.

(4) $1\frac{1}{120}$.

(5) $3\frac{5}{6}$.

(6) $1\frac{17}{40}$.

(7) $1\frac{3}{20}$.

(8) $1\frac{5}{8}$.

(9) $1\frac{2}{11}$.

(10) $4\frac{7}{30}$.

5. (1) $1\frac{17}{30}$.

(2) $3\frac{5}{6}$.

(3) $1\frac{5}{12}$.

(4) $1\frac{7}{40}$.

(5) $2\frac{7}{10}$.

(6) $2\frac{3}{4}$.

(7) $1\frac{37}{200}$.

(8) $1\frac{67}{144}$.

(9) $2\frac{5}{6}$.

(10) $2\frac{29}{300}$.

6. (1) $2\frac{5}{36}$.

(2) $1\frac{3}{8}$.

(3) $16\frac{5}{72}$.

(4) $1\frac{3}{10}$.

(5) $1\frac{3}{70}$.

(6) $1\frac{5}{9}$.

(7) $1\frac{77}{200}$.

(8) $1\frac{59}{144}$.

(9) $1\frac{5}{8}$.

(10) $1\frac{97}{300}$.

7. (1) $8\frac{29}{40}$.

(2) $25\frac{13}{5}$.

(3) $14\frac{23}{60}$.

(4) $17\frac{2}{15}$.

(5) $25\frac{11}{11}$.

(6) $30\frac{17}{24}$.

(7) $14\frac{73}{120}$.

(8) $22\frac{9}{10}$.

(9) $28\frac{9}{4}$.

(10) $24\frac{53}{60}$.

8. (1) $22\frac{5}{24}$.

(2) $46\frac{8}{15}$.

(3) $30\frac{11}{24}$.

(4) $74\frac{47}{90}$.

(5) $101\frac{2}{12}$.

(6) $112\frac{11}{15}$.

(7) $26\frac{3}{8}$.

(8) $84\frac{31}{60}$.

(9) $55\frac{13}{28}$.

(10) $46\frac{11}{15}$.

9. (1) $24\frac{27}{20}$.

(2) $54\frac{1}{5}$.

(3) $35\frac{9}{20}$.

(4) $81\frac{16}{45}$.

(5) $111\frac{23}{84}$.

(6) $121\frac{73}{120}$.

(7) $31\frac{13}{20}$.

(8) $94\frac{19}{10}$.

(9) $65\frac{24}{4}$.

(10) $50\frac{41}{60}$.

10. (1) $341\frac{37}{20}$.

(2) $547\frac{13}{3}$.

(3) $677\frac{61}{120}$.

(4) $909\frac{7}{45}$.

(5) $1048\frac{255}{198}$.

(6) $1585\frac{31}{40}$.

(7) $2556\frac{5}{24}$.

(8) $4863\frac{17}{20}$.

(9) $8527\frac{1}{12}$.

(10) $7601\frac{23}{60}$.

11. (1) $17\frac{2}{5}$.

(2) $12\frac{1}{6}$.

(3) $14\frac{7}{60}$.

(4) $13\frac{1}{6}$.

(5) $10\frac{1}{2}$.

(6) $11\frac{1}{8}$.

(7) $15\frac{4}{15}$.

(8) $10\frac{1}{4}$.

(9) $9\frac{1}{4}$.

(10) $16\frac{1}{20}$.

12. (1) $2\frac{3}{40}$.

(2) $12\frac{2}{5}$.

(3) $3\frac{17}{60}$.

(4) $4\frac{1}{2}$.

(5) $9\frac{13}{30}$.

(6) $16\frac{1}{30}$.

(7) $4\frac{3}{10}$.

(8) $6\frac{7}{15}$.

(9) $12\frac{13}{20}$.

(10) $15\frac{2}{13}$.

13. (1) $\frac{1}{9}$.

(2) $\frac{13}{8}$.

(3) $2\frac{5}{4}$.

(4) $\frac{89}{120}$.

(5) $\frac{35}{36}$.

(6) $\frac{53}{90}$.

(7) $\frac{17}{50}$.

(8) $\frac{35}{48}$.

(9) $\frac{8}{24}$.

(10) $\frac{9}{50}$.

14. (1) $\frac{13}{6}$.

(2) $\frac{5}{30}$.

(3) $\frac{13}{72}$.

(4) $\frac{19}{120}$.

(5) $\frac{7}{80}$.

(6) $\frac{1}{15}$.

(7) $\frac{7}{200}$.

(8) $\frac{13}{144}$.

(9) $\frac{4}{15}$.

(10) $\frac{1}{100}$.

15. (1) $3\frac{31}{40}$.

(2) $10\frac{8}{15}$.

(3) $3\frac{67}{60}$.

(4) $3\frac{1}{15}$.

(5) $5\frac{17}{12}$.

(6) $12\frac{23}{24}$.

(7) $5\frac{17}{20}$.

(8) $2\frac{37}{20}$.

(9) $8\frac{4}{15}$.

(10) $16\frac{9}{60}$.

16. (1) $9\frac{11}{14}$.

ANSWERS.

7

- | | | | |
|------------------------------|---------------------------|-------------------------------|--------------------------------|
| (2) $10\frac{2}{15}$. | (3) $61\frac{19}{20}$. | (4) $\frac{49}{192}$. | (5) $1343\frac{5}{12}$. |
| (3) $12\frac{7}{24}$. | (4) $47\frac{3}{4}$. | (5) $\frac{4}{35}$. | (6) $1984\frac{13}{20}$. |
| (4) $53\frac{83}{90}$. | (5) $60\frac{19}{84}$. | (6) $\frac{44}{135}$. | (7) $162\frac{1}{10}$. |
| (5) $70\frac{1}{12}$. | (6) $60\frac{23}{20}$. | (7) $\frac{91}{400}$. | (8) $886\frac{1}{5}$. |
| (6) $69\frac{1}{15}$. | (7) $119\frac{1}{20}$. | (8) $\frac{55}{216}$. | (9) $688\frac{11}{28}$. |
| (7) $6\frac{1}{8}$. | (8) $50\frac{1}{40}$. | (9) $\frac{1}{24}$. | (10) $540\frac{2}{25}$. |
| (8) $59\frac{59}{60}$. | (9) $7\frac{65}{72}$. | (10) $\frac{161}{756}$. | 29. (1) $21205\frac{1}{2}$. |
| (9) $17\frac{27}{28}$. | (10) $24\frac{11}{12}$. | 25. (1) $\frac{55}{108}$. | (2) $33075\frac{2}{3}$. |
| (10) $4\frac{1}{3}$. | 21. (1) $3\frac{1}{3}$. | (2) $\frac{3}{25}$. | (3) $42997\frac{1}{4}$. |
| 17. (1) $79\frac{1}{24}$. | (2) $3\frac{3}{4}$. | (3) $\frac{5}{18}$. | (4) $55462\frac{3}{4}$. |
| (2) $66\frac{13}{24}$. | (3) $4\frac{1}{6}$. | (4) $\frac{7}{180}$. | (5) $62770\frac{5}{8}$. |
| (3) $73\frac{1}{2}$. | (4) $4\frac{3}{8}$. | (5) $\frac{7}{245}$. | (6) $98099\frac{1}{6}$. |
| (4) $30\frac{17}{12}$. | (5) 4 . | (6) $\frac{11}{1000}$. | (7) $169,181\frac{7}{10}$. |
| (5) $9\frac{1}{8}$. | (6) $4\frac{1}{2}$. | (7) $\frac{117}{10000}$. | (8) $319,548\frac{1}{8}$. |
| (6) $3\frac{29}{40}$. | (7) $3\frac{1}{2}$. | (8) $\frac{5}{96}$. | (9) $566,915\frac{5}{8}$. |
| (7) $78\frac{3}{8}$. | (8) $4\frac{7}{12}$. | (9) $\frac{1}{45}$. | (10) $505,896\frac{2}{3}$. |
| (8) $22\frac{5}{8}$. | (9) $3\frac{4}{7}$. | (10) $\frac{437}{30000}$. | 30. (1) $\$ 1583\frac{1}{2}$. |
| (9) $58\frac{9}{40}$. | (10) $2\frac{1}{5}$. | 26. (1) $\frac{55}{162}$. | (2) $7083\frac{1}{2}$. |
| (10) $69\frac{3}{40}$. | 22. (1) $6\frac{2}{3}$. | (2) $\frac{11}{12}$. | (3) $10687\frac{1}{2}$. |
| 18. (1) $300\frac{2}{3}$. | (2) $3\frac{3}{4}$. | (3) $\frac{25}{108}$. | (4) 36992 . |
| (2) $465\frac{1}{3}$. | (3) $7\frac{1}{2}$. | (4) $\frac{49}{1440}$. | (5) 51450 . |
| (3) $620\frac{2}{3}$. | (4) $1\frac{1}{2}$. | (5) $\frac{33}{1225}$. | (6) 58176 . |
| (4) $763\frac{26}{45}$. | (5) $2\frac{2}{35}$. | (6) $\frac{22}{225}$. | (7) 11880 . |
| (5) $851\frac{1}{5}$. | (6) $3\frac{3}{5}$. | (7) $\frac{819}{100000}$. | (8) 57800 . |
| (6) $1373\frac{1}{5}$. | (7) $4\frac{8}{25}$. | (8) $\frac{55}{1152}$. | (9) $35245\frac{5}{8}$. |
| (7) $2508\frac{3}{8}$. | (8) $2\frac{1}{4}$. | (9) $\frac{1}{63}$. | (10) 25800 . |
| (8) $4697\frac{1}{8}$. | (9) 4 . | (10) $\frac{3059}{375000}$. | 31. (1) $\frac{2}{3}$. |
| (9) $8424\frac{2}{3}$. | (10) $4\frac{13}{28}$. | 27. (1) $1\frac{35}{108}$. | (2) $\frac{3}{8}$. |
| (10) $7524\frac{9}{10}$. | 23. (1) $15\frac{2}{3}$. | (2) $\frac{23}{24}$. | (3) $\frac{5}{18}$. |
| 19. (1) $14\frac{11}{12}$. | (2) $56\frac{2}{3}$. | (3) $1\frac{17}{36}$. | (4) $\frac{7}{32}$. |
| (2) $27\frac{23}{24}$. | (3) $64\frac{1}{8}$. | (4) $\frac{287}{1080}$. | (5) $\frac{4}{25}$. |
| (3) $20\frac{9}{12}$. | (4) $256\frac{3}{8}$. | (5) $\frac{621}{1745}$. | (6) $\frac{4}{27}$. |
| (4) $63\frac{67}{72}$. | (5) $428\frac{1}{2}$. | (6) $\frac{781}{8000}$. | (7) $\frac{1}{10}$. |
| (5) $85\frac{15}{28}$. | (6) $545\frac{2}{3}$. | (7) $\frac{2769}{50000}$. | (8) $\frac{11}{96}$. |
| (6) $90\frac{8}{15}$. | (7) $115\frac{1}{2}$. | (8) $\frac{1195}{2304}$. | (9) $\frac{5}{63}$. |
| (7) $16\frac{7}{40}$. | (8) 578 . | (9) $\frac{211}{945}$. | (10) $\frac{7}{125}$. |
| (8) $71\frac{35}{36}$. | (9) $330\frac{1}{7}$. | (10) $\frac{34523}{600000}$. | 32. (1) $1\frac{1}{5}$. |
| (9) $36\frac{68}{105}$. | (10) 258 . | 28. (1) $100\frac{15}{16}$. | (2) 7 . |
| (10) $25\frac{1}{2}$. | 24. (1) $1\frac{1}{8}$. | (2) $515\frac{2}{3}$. | (3) $4\frac{1}{4}$. |
| 20. (1) $61\frac{93}{120}$. | (2) $\frac{9}{32}$. | (3) $194\frac{5}{32}$. | (4) 30 . |
| (2) $2\frac{7}{15}$. | (3) $2\frac{10}{27}$. | (4) $661\frac{23}{48}$. | (5) $29\frac{1}{6}$. |

- (6) 20.
 (7) $19\frac{1}{4}$.
 (8) $42\frac{2}{3}$.
 (9) 27.
 (10) $26\frac{6}{19}$.
33. (1) $1\frac{1}{5}$.
 (2) $2\frac{5}{8}$.
 (3) $1\frac{1}{3}$.
 (4) $6\frac{9}{15}$.
 (5) $4\frac{2}{3}$.
 (6) $23\frac{6}{7}$.
 (7) $11\frac{7}{18}$.
 (8) $4\frac{5}{9}$.
 (9) $2\frac{1}{4}$.
 (10) $1\frac{9}{19}$.
34. (1) $24\frac{4}{9}$.
 (2) $20\frac{1}{9}$.
 (3) $113\frac{7}{10}$.
 (4) $23\frac{2}{3}$.
 (5) 46.
 (6) $24\frac{9}{14}$.
 (7) $143\frac{2}{3}$.
 (8) $351\frac{7}{10}$.
 (9) $152\frac{6}{7}$.
 (10) $10\frac{7}{25}$.
35. (1) $2\frac{47}{104}$.
 (2) $2\frac{3}{115}$.
 (3) $122\frac{7}{13}$.
 (4) $120\frac{3}{13}$.
 (5) $112\frac{2}{17}$.
 (6) $2\frac{28}{113}$.
 (7) $2\frac{49}{368}$.
 (8) $127\frac{7}{856}$.
 (9) $11\frac{9}{195}$.
 (10) $5\frac{7}{197}$.
36. (1) $2\frac{74}{153}$.
 (2) $115\frac{2}{73}$.
 (3) $2\frac{7}{218}$.
 (4) $6\frac{218}{537}$.
 (5) $5\frac{89}{188}$.
 (6) $41\frac{12}{55}$.
- (7) $17\frac{3}{4}$.
 (8) $5\frac{55}{736}$.
 (9) $15\frac{2}{25}$.
 (10) $17\frac{3}{4}$.
37. (1) $35\frac{1}{4}$.
 (2) $54\frac{23}{27}$.
 (3) $71\frac{11}{36}$.
 (4) $91\frac{4}{13}$.
 (5) $104\frac{7}{12}$.
 (6) $162\frac{37}{54}$.
 (7) $280\frac{17}{12}$.
 (8) $529\frac{87}{72}$.
 (9) $940\frac{19}{63}$.
 (10) $838\frac{29}{36}$.
38. (1) $41\frac{1}{10}$.
 (2) $61\frac{31}{25}$.
 (3) $81\frac{67}{30}$.
 (4) $113\frac{14}{75}$.
 (5) $125\frac{9}{20}$.
 (6) $194\frac{7}{10}$.
 (7) $331\frac{67}{50}$.
 (8) $637\frac{71}{26}$.
 (9) $112\frac{86}{105}$.
 (10) $1001\frac{58}{250}$.
39. (1) $\$38\frac{6}{13}$.
 (2) $\$321\frac{1}{23}$.
 (3) $\$941\frac{8}{13}$.
 (4) $\$841\frac{2}{11}$.
 (5) $\$602\frac{2}{13}$.
 (6) $\$72\frac{8}{71}$.
 (7) $\$152\frac{8}{11}$.
 (8) $\$80\frac{80}{239}$.
 (9) $\$04\frac{22}{107}$.
 (10) $\$253\frac{13}{19}$.
40. (1) $\$6\frac{6}{19}$.
 (2) $\$81\frac{1}{7}$.
 (3) $\$23\frac{47}{771}$.
 (4) $\$83\frac{8}{89}$.
 (5) $\$63\frac{42}{13}$.
 (6) $\$7\frac{37}{809}$.
 (7) $\$43\frac{1}{11}$.
- (8) $\$11\frac{21}{289}$.
 (9) $\$26\frac{38}{257}$.
 (10) $\$38\frac{88}{129}$.
41. (1) $\frac{55}{72}$.
 (2) $\frac{95}{112}$.
 (3) $11\frac{7}{12}$.
 (4) $\frac{207}{246}$.
 (5) $\frac{121}{140}$.
 (6) $1\frac{1}{10}$.
 (7) $\frac{359}{406}$.
 (8) $\frac{139}{144}$.
 (9) $1\frac{1}{10}$.
 (10) $\frac{449}{600}$.
42. (1) $61\frac{103}{120}$.
 (2) $27\frac{5}{15}$.
 (3) $61\frac{19}{120}$.
 (4) $47\frac{1}{45}$.
 (5) $60\frac{19}{84}$.
 (6) $60\frac{23}{120}$.
 (7) $119\frac{7}{20}$.
 (8) $50\frac{1}{40}$.
 (9) $7\frac{65}{84}$.
 (10) $\frac{11}{12}$.
43. (1) $131\frac{13}{12}$.
 (2) $\frac{78}{115}$.
 (3) $1\frac{81}{2544}$.
 (4) $\frac{206}{1025}$.
 (5) $\frac{94}{345}$.
 (6) $\frac{262}{355}$.
 (7) $\frac{3133}{2840}$.
 (8) $\frac{276}{1195}$.
 (9) $\frac{525}{856}$.
 (10) $2\frac{32}{595}$.
44. (1) $381\frac{11}{268}$.
 (2) $67\frac{2}{25}$.
 (3) $36\frac{237}{48}$.
 (4) $96\frac{494}{673}$.
 (5) $136\frac{239}{228}$.
 (6) $223\frac{21}{335}$.
 (7) $34\frac{81}{1136}$.
 (8) $881\frac{122}{1155}$.
- (9) $67\frac{5}{8}$.
 (10) $1361\frac{88}{125}$.
45. (1) $\frac{208}{425}$.
 (2) $1\frac{11}{104}$.
 (3) $\frac{424}{445}$.
 (4) $4\frac{583}{1648}$.
 (5) $2\frac{44}{17}$.
 (6) $1\frac{241}{1179}$.
 (7) $\frac{1988}{2153}$.
 (8) $332\frac{209}{312}$.
 (9) $112\frac{21}{755}$.
 (10) $\frac{1652}{5966}$.

Art. 145.

1. $\frac{5}{13}$.
2. $\$52.30\frac{1}{2}$.
3. $\$391$.
4. $\$445$.
5. $\$0.16\frac{496}{999}$.
6. $\$449\frac{1}{2}$.
7. $\$493\frac{8}{19}$.
8. $\$6$.
9. $\$19.01\frac{1}{2}$.
10. $312\frac{7}{21}$ m.

Art. 147.

I.

2. 500%; 20%.
3. 40%; 250%.
4. $66\frac{2}{3}\%$; 150%.
5. 25%; 400%.
6. 50%; 200%.
7. 1500%; $6\frac{2}{3}\%$.
8. 1%; 10000%.
9. 50%; 200%.
10. $\frac{1}{10}$ of 1%; 100,000%.

II.

1. 400%; 25%.
2. $33\frac{1}{3}\%$; 300%.

3. 250 %; 40 %.
4. 120 %; $83\frac{1}{3}$ %.
5. 600 %; $16\frac{2}{3}$ %.
6. 50 %; 200 %.
7. 125 %; 80 %.
8. 50 %; 200 %.
9. 60 %; $16\frac{2}{3}$ %.
10. $42\frac{6}{7}$ %; $233\frac{1}{3}$ %.

III.

1. 500 %; 20 %.
2. $6\frac{1}{4}$ %; 1600 %.
3. $26\frac{2}{3}$ %; $37\frac{1}{2}$ %.
4. 60 %; $16\frac{2}{3}$ %.
5. 500 %; 20 %.
6. $133\frac{1}{3}$ %; 75 %.
7. 200 %; 50 %.
8. 20 %; 500 %.
9. 100 %; 100 %.
10. $33\frac{1}{3}$ %; 300 %.

IV.

1. 2400 %; $4\frac{1}{6}$ %.
2. 700 %; $14\frac{2}{7}$ %.
3. 1200 %; $8\frac{1}{3}$ %.
4. 300 %; $33\frac{1}{3}$ %.
5. 500 %; 20 %.
6. $133\frac{1}{3}$ %; 75 %.
7. 300 %; $33\frac{1}{3}$ %.
8. $\frac{1}{2}$ of 1 %;
9. $66\frac{2}{3}$ %; 150 %.
10. $183\frac{1}{3}$ %; $54\frac{5}{11}$ %.

Art. 153.

1. $66\frac{2}{3}$ %.
2. $66\frac{2}{3}$ % Swedish;
3. $33\frac{1}{3}$ % other sources.

4. 8 % brass.

5. $6\frac{1}{4}$ %.

6. \$ 50.

7. $\frac{5}{8}$ or $55\frac{5}{8}$ %.

8. $\frac{3}{8}$ or 60 %.

9. 80 %; 5 %;
- 1 %; 12 %; 2 %.

10. $33\frac{1}{3}$ %; 6 %;
- $12\frac{1}{2}$ %; \$0.99.

Art. 157.

1. $7\frac{1}{2}$ A. at 10 ¢ a foot costs \$8963.20 more than 560 A. at 25 ¢ a sq. rd.

2. 1877 bu.; $49\frac{3}{4}$ rem.; \$99.95.

3. \$ 0.095.

4. 50 h.; 1680 m.

5. $29\frac{3}{4}$.

6. \$13.20.

7. $179\frac{1}{3}$ cans.

8. \$1166 $\frac{2}{3}$.

9. 27 lb.; 1 ¢ rem.

10. \$2227.98.

Art. 159.

1. \$9364.88.

2. $\frac{149}{378}$.

3. 10235 full steps.

4. 1440 revolutions.

5. $3\frac{5}{8}$.

6. $\frac{3}{10}$ m.

7. 77; 7; by 7,
- 11, 21, 77.

8. Mar. 14;

- May 1.

9. $\frac{1}{15}$.

10. $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{9}$, $\frac{1}{10}$, $\frac{1}{11}$, $\frac{1}{12}$, etc.

Art. 160.

1. \$15.52.

2. \$2001.81 $\frac{1}{12}$.

3. 3512 lb.

4. 263 lb.; $\frac{5760}{7000}$.

5. \$243.52.

6. \$1621.92.

7. \$698.76.

8. \$155.83.

9. \$15.44.

10. \$1573.72.

Art. 161.

1. \$2.98 $\frac{3}{4}$.

2. \$2.85.

3. \$13.

4. \$370.50.

5. \$1.57 $\frac{1}{2}$.

6. \$4.35 $\frac{5}{12}$.

7. \$64.

8. \$132.

9. \$43.

10. \$6.77 $\frac{1}{12}$.

11. \$0.12.

12. \$5 $\frac{1}{4}$.

13. \$6.075;

- \$0.04 $\frac{1}{6}$.

14. \$216.

15. 176 rails; $3\frac{3}{4}$ yd.

16. 24900 m.

17. $101\frac{1}{4}$ ft.

18. 30 sq. rd.;

- 907 $\frac{1}{2}$ sq. yd.;

- 4 squares;

- 36 squares;

- 32670 squares.

19. $89\frac{1}{2}$ in.; $7\frac{1}{4}$ in.

20. 12 in.

Art. 162.

1. \$58.80.

2. 8800 boxes.

3. \$2 $\frac{3}{4}$.

4. 17s.

5. £ $\frac{1}{6}$; $\frac{1}{10}$;

- 17s.; 204d.

6. A nautical mile is 73.3 ft.

- less than $1\frac{1}{2}$ × a common mile.

7. 33 yd. 25 in.

8. $\frac{5}{8}$ T., or

- 1890 $\frac{1}{11}$ lb.

9. $\frac{7}{8}$.

10. \$2.81 $\frac{9}{10}$.

Art. 163.

1. \$105.

2. $\frac{7}{16}$ bu.

3. 53 posts.

4. $4\frac{1}{8}$ d.

5. Mary, $\frac{1}{3}$;

- Sarah, $\frac{1}{4}$;

- both in $1\frac{1}{2}$ h.

6. John, $\frac{1}{4}$; his

- brother, $\frac{1}{6}$;

- both in $2\frac{3}{4}$ d.

7. \$8000.

8. 80 jars; 18750

- eggs.

9. $\frac{1}{145}$.

10. 20 bu.

Art. 165.

1. \$135.

2. Clifford, $\frac{1}{10}$;

- Clifford and

- Leonard, $\frac{1}{24}$;
 Leonard, $\frac{1}{40}$;
 Leonard can
 do the whole
 in 40 min.
 3. It is 10.54
 Thursday P.M.
 in New York.
 4. 3.34 Friday
 A.M. in Holy-
 head.
 5. \$8400.
 6. 36 meters.
 7. $\frac{5}{6}$ yd.
 8. \$11.25.
 9. The father.
 10. $2\frac{3}{4}$ d.

Art. 166.

1. 16 tables.
 2. $14\frac{318}{443}$.
 3. $299\frac{1}{4}$.
 4. $80\frac{31}{417}$.
 5. $\frac{165}{832}$.
 6. \$78.
 7. \$195.12.
 8. 39 bu.
 9. \$48.
 10. $\$596\frac{4}{11}$.

Art. 172.

4. About $\frac{93}{100}$ m.
 or 4910 ft.
 5. 1 meter.

Art. 177.

1. $\frac{7}{16}$.
 2. $\frac{9}{16}$.
 3. $\frac{5}{8}$.
 4. $\frac{11}{16}$.
 5. $\frac{13}{16}$.
 6. $\frac{5}{12}$.

7. $\frac{15}{16}$.
 8. $\frac{11}{12}$.
 9. $\frac{2}{3}$.
 10. $\frac{7}{24}$.
 11. $\frac{9}{16}$.
 12. $\frac{7}{16}$.
 13. $\frac{3}{8}$.
 14. $\frac{5}{16}$.
 15. $\frac{3}{16}$.
 16. $\frac{7}{12}$.
 17. $\frac{1}{16}$.
 18. $\frac{1}{12}$.
 19. $\frac{1}{6}$.
 20. $\frac{17}{24}$.

Art. 178.

4. $\frac{35}{100}$.
 5. $\frac{325}{1000}$.
 6. $\frac{65}{100}$.
 7. $\frac{3375}{10000}$.
 8. $\frac{375}{10000}$.
 9. $\frac{1375}{10000}$.
 10. $\frac{9375}{10000}$.

Art. 179.

7. $0.583\frac{1}{3}$.
 8. $0.833\frac{1}{3}$.
 9. $0.562\frac{1}{2}$.
 10. $0.266\frac{2}{3}$.
 11. $0.428\frac{4}{7}$.
 12. $0.555\frac{5}{9}$.
 13. $0.233\frac{1}{3}$.
 14. $0.777\frac{7}{9}$.
 15. $0.466\frac{2}{3}$.
 16. $1.062\frac{1}{2}$.
 17. $0.833\frac{1}{3}$.
 19. $0.916\frac{6}{7}$.
 20. $0.555\frac{5}{9}$.
 21. $0.545\frac{4}{9}$.
 22. $0.183\frac{3}{16}$.
 23. $0.466\frac{6}{13}$.
 24. 0.0016 .

25. 0.3888.
 26. 0.2307.
 27. 0.9230.

Art. 181.

1. 206.568.
 2. 335.722.
 3. 21.3725.
 4. 22.32589.
 5. 700.108.
 6. 680.40235.
 7. 234.695.
 8. 25.1429.
 9. 222.585.
 10. 123.27114.

Art. 182.

1. 226.472.
 2. 396.2494.
 3. 152.807.
 4. 3.193.
 5. 23.5237.

Art. 184.

1. 1.74.
 2. 1.5036.
 3. 31.13.
 4. 8.774.
 5. 3.749.
 6. 0.3976.
 7. 1.767.
 8. 1.5306.
 9. 27.926.
 10. 5.57.
 11. 5.893.
 12. 2.5416.
 13. 23.8352.
 14. 8.557.

Art. 185.

1. 182.62.
 2. 2.886.

3. 1.775.
 4. $0.16\frac{2}{3}$.
 5. $0.016\frac{2}{3}$.
 6. 7958.2.
 7. 0.235.
 8. 6.0.
 9. $1.6\frac{2}{3}$.
 10. $5.56\frac{2}{3}$.

Art. 186.

1. 2.975.
 2. 0.257.
 3. 99.75.
 4. 0.501.
 5. 2.262.

Art. 190.

1. 79.88904.
 2. \$8.064.
 3. 6.963744.
 4. 4.23.
 5. 0.46875.
 6. 0.675.
 7. 46.656.
 8. \$480.
 9. 51.20.
 10. 2.14 m.

Art. 191.

1. \$13.49.
 2. \$16.92.
 3. \$1050.
 4. 1372.5 T.
 5. \$8.55.
 6. 520.2.
 7. 750 lb.
 8. 0.5625.
 9. \$80.
 10. \$6000; \$120.

Art. 193.

1. 3600.
2. 0.289.
3. 78.4.
4. 0.39.
5. 70.5.
6. 113.5.
7. \$8.9125.
8. 640.0287.
9. 55.1286.
10. 3560.

Art. 196.

1. 14s.
2. $\frac{1}{2}$.
3. $\frac{16}{1000}$.
4. 1d. is 0.04d more than £0.004.
5. 50%.
6. 2 doz.
7. $\frac{3}{10}$ M.
8. 48 sheets.
9. 5%.
10. $6\frac{1}{4}$ %.
11. 75%.
12. 1.12.

Art. 199.

1. \$89.25.
2. \$89.25.
3. 10400,
1st ward;
12800,
2d ward;
8400,
3d ward;
8400,
4th ward.
4. 9159.2 ft.

5. 258 balls;
1032 oz.

6. 175.
7. 0.049.
8. 0.025.
9. 0.33333.
10. $0.187\frac{1}{2}$;
 $0.18\frac{3}{4}$; $\frac{3}{10}$.

Art. 200.

1. 2.005.
2. $0.083\frac{1}{3}$.
3. $0.08\frac{1}{3}$.
4. 0.10.
5. 0.68.
6. 0.073.
7. 0.0125 ; $\frac{1}{80}$.
8. 0.125 ; $\frac{1}{8}$.
9. 0.0025 ; $\frac{1}{400}$.
10. 0.2124.
11. 0.5.
12. 0.0625.
13. 0.015625.
14. 4 gi.
15. 3 gi.
16. 2.25 ft.
17. 13 ft.
18. 2 sq. ft.
19. 18 sq. ft.

Art. 201.

1. 208,876.8.
2. \$0.75.
3. 5500 lb.
4750 lb.
10334 lb.
15040 lb.
35624 lb.
4. \$7.20.
5. 21% of the
men receive

- 0.15 more than
\$1 a day; the
rest 0.38 more.

6. $0.08\frac{1}{3}$; 0.125 ;
 $0.0027\frac{2}{3}$.
7. $0.6383\frac{1}{3}$ yr.
8. Sept. 29.
9. \$1.
10. $13\frac{1}{2}$ loaves; $\frac{2}{27}$.

Art. 202.

1. $\frac{42}{128}$; $\frac{7\frac{1}{2}}{128}$;
 $\frac{52}{128}$.
2. 0.8089.
3. $0.521\frac{6}{7}$; $0.07\frac{1}{2}$;
 $0.05\frac{1}{3}$.
4. 0.3771.
5. 0.0075;
0.0023; 0.9990.
6. $\frac{15}{16}$; 0.15; 1.5.
7. 1.7663.
8. $\frac{177}{25}$; $\frac{177}{2500}$;
 $\frac{438}{25}$.
9. $10.18\frac{1}{3}$; $0.87\frac{5}{8}$;
 $0.18\frac{3}{4}$.
10. 0.80.

Art. 203.

1. $\frac{1}{3}$.
2. $8\frac{1}{3}$ %.
3. 8453.775 ft., or
3756.225 ft.
4. $\frac{1}{4}$ m.
5. \$180.60.
6. 124 bu.
7. 90 sq. in.
8. $\frac{1}{2}$ yr.
9. 25 m.
10. 64 cd.

Art. 204.

1. \$1840.
2. 27.2772.
3. 78.0195.
4. $10\frac{4}{85}$.
5. $1\frac{4}{11}$ of 1 sq. rd.
6. $1\frac{1}{100}$ A.;
 $1\frac{1}{11}$ sq. ft.
7. $886.661\frac{2}{3}$.
8. \$728.89.
9. \$115,520.977.
10. \$437 $\frac{1}{2}$.

Art. 205.

1. 252 d.
2. £ $2\frac{1}{40}$; 60 $\frac{3}{4}$ ¢.
3. \$38.88.
1 \$20-bill;
1 \$10-bill;
1 \$5-bill;
1 \$2-bill;
1 \$1-bill;
1 half-dollar;
1 quarter-dollar;
1 dime;
3 cents.
11 pieces.
4. £15;
8s. $27\frac{3}{4}$ d.
5. £10 3s. $1\frac{1}{2}$ d.
6. 795 gr.
7. 7 oz. 4 dr.
8. \$0.21 $\frac{1}{3}$.
9. \$41 $\frac{2}{3}$.
10. 4 oz. 1 pwt.
15 gr.

Art. 206.

2. $3216\frac{1}{4}$ m.
46 $\frac{1}{2}$ °.
3. $69\frac{1}{2}$ m.; $1\frac{1}{2}$ m.;
101 $\frac{1}{4}$ ft.

4. $17\frac{7}{8}$ m.
 5. 720 ft.; 690 m.
 6. 37 double eagles.
 7. 231 cu. in.;
 $57\frac{3}{4}$ cu. in.;
 $113\frac{1}{3}$ gal.
 8. $537.60\frac{1}{2}$ cu in.;
 41.85 + bags.
 9. 48 pt.; 80 gi.;
 2 pk. 4 qt.
 10. 20; 17 cwt. 50
 lb.; 2 pk.
 4 qt.; 1 pt.
 0.4 gi.
- Art. 208.
 6. \$147.
 7. 87.
 8. 64.
 9. 1000.
 10. 67.67.
- Art. 209.
 3. \$4.
 4. 1.88.
 5. 1.87.
 6. 3.72.
 7. 9.33.
 8. 33.26.
 9. 45.
 10. 11.63.
- Art. 210.
 6. \$26.55.
 7. 134.31.
 8. 127.80.
 9. 12.51.
 10. 90.71.
 11. 81.58.
12. \$9.67.
 13. 195.70.
 14. 21.08.
 15. 8.09.
 16. 188.83.
 17. 50.23.
 18. 16.67.
 19. 31.06.
 20. 51.79.
 21. 526.46.
- Art. 214.
 1. $\frac{1}{8}$ m.
 2. $32\frac{4}{5}$ ft.
 3. $24\frac{5}{11}$ m.
 4. $51\frac{301}{440}$ m.
 5. $6213\frac{377}{440}$ m.
 6. 1st rider, $\frac{2}{3}$ m.
 in 1 min.;
 2d rider, $\frac{1}{3}\frac{1}{4}$ m.
 in 1 min.
 7. $890\frac{151}{265}$ ft.
 8. 2 rd. $9\frac{1}{3}\frac{1}{4}$ ft.
 9. $61\frac{1}{2}\frac{1}{4}$ m.
 10. 37632 revolu-
 tions.
- Art. 217.
 8. 140 sq. rd.
 9. 640 A.; 64 A.;
 560 A.
 10. 36 sq. m.;
 24 m.
- Art. 218.
 1. $138\frac{3}{5}$ sq. ft.
 2. $45\frac{479}{1089}$ sq. rd.
 3. $78135\frac{3}{4}$ sq. ft.
 4. 864000 A.
 5. 43560 sq. ft.
6. \$3411.50.
 7. $332\frac{1}{3}$ A.
 8. $73\frac{503}{1089}$ sq. rd.
 9. 176 sq. ft. 35
 sq. in.
 10. \$5250.
- Art. 223.
 2. 225 sq. in.
 3. 324 sq. in.
 4. 288 sq. ft.
 5. 105 sq. ft.;
 260 sq. ft.;
 $272\frac{1}{4}$ sq. ft.;
 $30\frac{1}{4}$ sq. yd.
 6. $3\frac{1}{8}$ A.
 7. 1020 sq. in.
 8. $40\frac{1}{2}$ sq. yd.
 9. $8\frac{1}{2}$ sq. yd.
 10. 729 squares.
- Art. 224.
 1. \$17.50.
 2. 389 qt.
 3. 4 rd.
 4. 50¢.
 5. 576 tiles.
 6. \$12.50.
 9. 3840 pieces.
- Art. 225.
 2. Carpets 1 yd.
 wide could be
 used on floors
 12 ft., 15 ft.,
 27 ft., and 18
 ft. wide; car-
 pets $\frac{3}{4}$ yd. wide,
 on floors 22 $\frac{1}{2}$,
 13 $\frac{1}{2}$, 18, and
 27 ft. wide.
- Neither width
 could be used
 on floors 20
 ft. wide.
 3. 6 strips; 36
 yd.; $37\frac{1}{4}$ yd.
 4. 7 strips; 49
 yd.; \$61.25.
 5. \$25.
 6. \$70.
 7. \$26.67.
 8. \$28.35.
 9. \$258.90.
 10. \$24.83.
 11. \$90.92.
 12. 21 tiles.
 13. 2100 tiles.
 14. $32\frac{1}{3}\frac{2}{3}$ sq. ft.
 15. 1043 tiles.
 16. $21\frac{7}{8}$ sq. ft.
 17. 770 tiles.
 18. 14 rolls.
- Art. 226.
 1. 2850 slates.
 2. 26136 blocks.
 3. 70 plates.
 4. $1\frac{1}{2}$ sq. ft. more.
 5. \$147.
 6. 48 sq. ft.;
 200 sq. ft.;
 64 sq. ft.;
 192 sq. ft.;
 140 sq. ft.
 7. $71\frac{5}{8}$ sq. yd.
- Art. 228.
 1. \$4098.60.
 2. 300.
 3. 937.50.

4. \$ 1350.
5. 2550.38.
6. 1728.
7. 2245.32.
8. 1975.59.
9. 2602.33.
10. 1447.88.
11. 937.20.
12. 1929.60.
13. 272.68.
14. 604.08.
15. 407.68.
16. 94.08.
17. 391.
18. 67.50.
19. 124.
20. 320.

Art. 229.

7. $93\frac{3}{4}$ sq. ft.
8. 10560 sq. ft.
9. $11\frac{1}{2}$ sq. ft.
10. $3\frac{3}{8}$ sq. ft.

Art. 231.

7. 260 sq. ft. ;
110 $\frac{1}{4}$ sq. in.
9. $16\frac{1}{2}$ sq. ft.
10. $10\frac{5}{16}$ sq. ft.

Art. 232.

5. $1801\frac{3}{4}$ sq. ft.
6. \$ 25.

Art. 233.

10. 360.
11. 750.
12. $1\frac{1}{8}$ sq. ft.
13. 231 sq. ft.
14. $73\frac{1}{2}$ sq. ft.
15. $6975\frac{3}{8}$ sq. ft.

Art. 234.

5. 396 sq. in.
6. 7500 sq. ft.

Art. 235.

11. 62.832 ft.
12. 65.25355 ft.
13. 565.488 ft.
14. $1.48544\frac{2}{3}$ ft.
15. 51.051 in.
16. $1.19366\frac{1}{4}$ ft.

Art. 236.

10. 50.
11. 113.0976 sq. ft.
12. 78.54 sq. ft.
13. 795.775 sq. ft.
14. 198.94375 sq. ft.

Art. 237.

6. 19.635 sq. ft.
7. $\frac{1}{4}$.
8. 113.0976 sq. in.
9. \$ 8.48.
10. 31416 sq. ft.

Art. 238.

6. 1256.64.
7. 509.296.
8. 1809.5616.
9. 1017.8784.
10. 3183.1.
11. 1963.5.

Art. 241.

1. 1.5 + in.
2. 7.4613 in.
3. 4.00009 + rd.

Art. 242.

4. Diameter of
hhd. = 3.978
+ ft. ;
Diameter of
door = 3.833
+ ft.
5. 1.8169 m.
6. Curved edge
= 60 ;
Straight edge
= 57.2958.
7. 18.8496 in.
8. 496.24 ft.
9. 9685.84 sq. ft.

Art. 243.

7. 216 cu. in.
8. 512 cu. in.
9. 125 cu. ft.
10. 1728 cu. ft.
11. 1000 cu. yd.
12. 8000 cu. in.
13. 125 ; 343 ;
729 ; 1728 ;
216 ; 1000 ;
133 ; 8000.
14. 4 ; 6 ; 12 ; 8 ;
11 ; 9 ; 30.

Art. 244.

2. 180 cu. in.
3. 400 cu. in.
4. $7\frac{1}{2}$ cu. ft.
5. 1280 cu. ft.
6. $4\frac{1}{2}$ cu. ft.
7. 4608 cu. ft.
8. $11333\frac{1}{3}$ cu. ft.
9. 500 cu. ft.
10. $177\frac{2}{3}$ cu. ft.

Art. 245.

10. \$ 18.
11. 100.
12. 112.50.
13. 19.69.
14. 29.30.
15. 5.36.

Art. 246.

3. 10 sq. ft.
4. 120 sq. ft.
5. 12 boards.
6. 20 bd. ft. ;
15 bd. ft. ;
12 $\frac{1}{2}$ bd. ft. ;
25 bd. ft. ;
 $a \times 10$ bd. ft.
7. $37\frac{1}{2}$ bd. ft.
8. 256 bd. ft.
9. $13\frac{1}{3}$ bd. ft. ;
16 $\frac{2}{3}$ bd. ft. ;
20 bd. ft. ;
21 $\frac{2}{3}$ bd. ft. ;
23 $\frac{1}{3}$ bd. ft. ;
15 bd. ft.
10. 337 $\frac{1}{2}$ bd. ft.
11. 216 bd. ft.

Art. 247.

5. 486 cu. in.
6. 600 cu. in.
7. 1536 cu. in.
8. 8 in.
9. 10 ft.
10. 7 in.

Art. 248.

3. 114 sq. ft.
4. 88 sq. ft.

ANSWERS.

7. 248 sq. in.
8. 468 sq. ft.
9. 1240 sq. ft.
10. 848; 82.
11. 552; $94\frac{2}{3}$.
12. 576; $22\frac{1}{4}$.

Art. 249.

3. 0.2146 shav-
ing;
0.7854 cylin-
der.
4. 314.16 cu. in.
5. $\frac{7854}{10000}$; $\frac{7854}{10000}$.
6. 31.416 cu. ft.
7. 330.2928 cu. in.
8. 502.656 cu. ft.
9. 628.32 cu. ft.
10. 7 in.

Art. 250.

2. 12.5664 in.
3. 2.54648 in.
4. 25.1328 sq. in.
8. 314.16 sq. in.
9. 353.43 sq. in.
10. 224.3456 sq. in.

Art. 252.

1. 12 in. \times 3 in.;
perimeter 30
in.;
9 in. \times 4 in.;
perimeter 26
in.;
6 in. \times 6 in.;
perimeter 24
in.
2. 27.225 ft.

3. 12 in. average
width;
200 running
feet.
4. $13\frac{1}{2}$ ft.
5. 1.273 +.
7. $\frac{3}{4}$ in.
8. 128 oranges.
10. 800 A.

Art. 253.

1. $24 \times 6 = 144$;
 $18 \times 11 = 198$.
2. 16.
3. 6.
6. 78 sq. ft.
7. 12 in.
8. 120 ft.
9. 9 ft.
10. $544\frac{1}{2}$ ft.
11. 26 in.
12. 18 ft.
13. 9 in.
14. 96 rd.
15. 20 in.
16. 62500 sq. ft.
17. 31.416 ft.
18. 7854 sq. ft.
19. 100 ft.
20. 6 in.

Art. 254.

1. \$ 10.
2. 4 d.
3. 6 in.
4. 12 ft.
5. 12 in.
6. 4 ft.
7. 1089 ft.;
 $5\frac{9}{11}$ ft.

8. 30 in.
9. $1\frac{1}{3}$ ft.
10. 2.8 + ft.
11. $\frac{1}{2}$ ft.
12. 660 ft.

Art. 255.

1. 3 ft.
2. 184078 $\frac{1}{2}$ gal.
3. \$48595 gain.
4. \$133650.
5. 7 ft.
6. 9 h.
7. \$2.50.
8. 960; 6.
9. \$135.41.
10. \$9.
11. \$2242.50.

Art. 257.

1. \$50.63.
2. 10939 $\frac{1}{2}$ lb.
3. \$26.98.
4. \$1565.44.
5. 37.0992 bbl.
6. 603.1872 sq. in.
7. \$276.67.
8. \$5760.
9. \$476.80.
10. Perimeter of
6-inch square
= 24 in.;
of 9×4 rec-
tangle = 26 in.;
of 3×12 rec-
tangle = 30 in.

Art. 258.

1. 0.2146.
2. 909.
3. \$6106.88.

4. $170\frac{5}{8}$ sq. ft.
5. $1\frac{1}{3}\%$ loss.
6. \$0.13.
7. Once in 1000
times.
8. \$17.82.
9. 630.
10. $8\frac{16}{45}$ min.

Art. 262.

7. \$3375.
8. 4301 bu. corn;
3542 bu. oats;
4807 bu. wheat.
9. 935 $\frac{1}{2}$ T.
10. 58.8 m.
11. 955 $\frac{1}{2}$ A.
12. \$2.44.
13. \$209.25.
14. \$51.19.
15. \$100 to 1.
17. \$56000.
18. \$560.
19. \$434.
20. 46946.

Art. 265.

2. 3326.4 T.
3. 6300 volumes
in library;
1764 works of
fiction.
4. 71 $\%$
5. 96 $\%$
6. 30000 T.
7. \$17634.37, or
 $45\frac{3}{4}\%$
8. 32 $\%$
9. \$106.25.
10. $44\frac{1}{3}\%$

Art. 266.

1. All are equally profitable.
2. $\frac{2}{3}$ of something is $100 \times \frac{2}{3} \%$ of it.
3. 25%.
4. $58\frac{1}{3} \%$.
5. 45%.
6. 95%.
7. $16\frac{2}{3} \%$.
8. 4; 300; $\frac{2}{3} \%$.
9. 6; 1600; $\frac{1}{4} \%$.
10. 112; $83\frac{1}{3} \%$; $16\frac{1}{2} \%$.
11. $1\frac{1}{2}$; 140; $\frac{1}{5}$.
12. $33\frac{1}{3} \%$; $3\frac{1}{3}$; 25%.
13. 625; 2000; 1000.
14. \$1241.67.
15. \$12000.
16. $36\frac{1}{4} \%$.

Art. 267.

1. 12%.
2. 960.
3. $28\frac{16}{5} \%$.
4. 40 A.
5. 92%.
6. \$1728.
7. $56\frac{1}{4} \%$.
8. 225 cd.
9. $\frac{2}{5} \%$.
10. 7800 T.
11. 4%.
12. 18%.
13. $13\frac{8}{9} \%$.
14. \$9600.
15. $5\frac{2}{5} \%$.

16. 3456.

17. $3.8+ \%$.

18. \$4500.

19. \$72.90.

20. $1\frac{2}{1000}$.

Art. 268.

1. \$6500.
2. \$24640.
3. \$60; \$72.
4. $33\frac{1}{3} \%$.
5. $16\frac{2}{3} \%$.
6. 25%.
7. $33\frac{1}{3} \%$.
8. $3\frac{1}{4} \%$ more is gained by buying for \$4 and selling for \$4.80.
9. \$9.
10. $8\frac{1}{3} \%$.

Art. 269.

1. \$4.80.
2. \$72.
4. \$6000.
5. 71.
6. 300 A.
7. \$47000.
8. \$2000.
9. 60.
10. 35%.

Art. 273.

1. 80%.
2. \$60000.
3. \$50000.
4. \$7.81.
5. \$5000.

Art. 274.

1. \$96.
2. \$48.
3. \$32.
4. \$30.
5. \$30.
6. \$35.
7. 25%.
8. \$20 or 25%.
9. 20% import-er's gain; $16\frac{2}{3} \%$ retail-er's gain.
10. $1\frac{1}{5} \%$.
11. \$870.

Art. 275.

1. $18\frac{3}{4} \%$ gain on meat; $14\frac{7}{8} \%$ gain on pota-toes.
2. \$10.
3. 20%.
4. 1¢ saved by buying to-day.
5. \$48000; \$50400.
6. \$140.
7. $15\frac{1}{8} \%$.
8. $4\frac{1}{6} \%$.
9. $8\frac{1}{3} \%$.
10. 50%.
11. $14\frac{7}{8} \%$.
12. 8%.
13. \$5200.

Art. 277.

3. \$44.80.
4. 137.60.
5. 28.03.

6. \$5.88.

7. 298.54.

8. 375.

9. 104.99.

10. 125.16.

Art. 278.

7. \$8.71.
8. 9.92.
9. 1.56.
10. 1.71.
11. 1.36.
12. 3.13.
13. 20.83.
14. 11.68.
15. 38.65.
16. 0.05.

Art. 279.

9. \$4.48.
11. 10.67.
12. 15.53.
13. 13.27.
14. 14.21.
15. 6.43.
16. 1.90.
17. 4.09.
18. 9.06.
19. 1.01.
20. 8.63.
21. 196.67.
22. 1.33.
23. 16.15.

Art. 281.

3. \$546.86.
4. 7920.11.
5. 1909.96.
6. 95.06.

7. \$1135.24.
8. 214.28.
9. 92.52.
10. 1187.25.
11. 483.69.

Art. 282.

7. 3 yr. 21 d.
8. 2 yr. 2 mo. 6 d.
9. 2 yr. 10 mo. 21 d.

Art. 283.

1. \$14.49.
2. 95.
3. 17.09.
4. 7.99.
5. 7.52.
6. 37.50.
7. 13.24.
8. 4.61.
9. 21.85.
10. 166.45.
11. 1.89.
12. 1.67.
13. 40.80.
19. 88.

Art. 285.

1. \$26.79.
2. 759.54.
3. 540.11.
4. 15.96.
5. 900.
6. 540.02.
7. 234.
8. 60.67.
9. 25.20.
10. 3.15.
11. 3.47.
12. 337.87.

13. \$147.52.

14. 4.41.
15. 19.44.
16. 10.
17. 58.90.
18. 227.50.
19. 746.67.
20. 105.22.
21. 333.33.
22. 1.29.
23. 4.25.
24. 48.95.
25. 1312.50.

Art. 286.

4. \$144.
6. 4.93.
7. 9.70.
8. 7.76.
9. 11.05.
10. 16.77.
11. 29.59.
12. 160.93.
13. 897.53.

Art. 289.

5. \$1900.
6. 10%.
7. \$500.
8. 4%.
9. \$1.50.
10. \$500.

Art. 290.

1. 27; 73%.
2. 50%.
3. \$0.40.
4. \$0.39 $\frac{3}{4}$.
5. 50% of \$1 +;
20% of 50¢ +;
1% of 40¢.

6. \$11.70.
7. \$24.26.
8. \$320.10.
9. \$5.24.
10. \$240.88.
11. \$2390.08.

Art. 291.

1. \$6500.
2. 1%.
3. 4%.
4. Cash discount
is 3%.
5. \$1266.67.
6. \$2375.
7. \$1400; \$1425.
8. 20%.
9. \$10590.
10. \$3920.
11. 73%.
12. 15%.
13. 20%.
14. \$313.63.
15. \$10423.88.
16. \$598.50.

Art. 293.

1. \$10000.
\$10000.
\$2000.
2. $\frac{1}{2}$ %.
3. $1\frac{1}{2}$ %.
4. \$4000.
5. \$62.50.
6. $1\frac{1}{4}$ %.
7. \$31.60.
8. \$5000.
9. $\frac{3}{4}$ %; $1\frac{7}{10}$ %.
10. \$5625.
11. \$375.

12. 2d risk is $\frac{1}{4}$ %
cheaper.
13. \$230.09;
98 $\frac{2}{3}$ %.
14. \$2040.82.
15. \$7000.
16. \$120; \$3333 $\frac{1}{3}$.

Art. 294.

6. 80%.
7. \$110; 4%.
8. \$8.10.
9. \$6.91.
10. Of gross re-
ceipts.

Art. 296.

1. \$120.
2. 102 $\frac{1}{2}$ %; \$20;
\$2000.
3. \$37 $\frac{1}{2}$.
4. \$1600; \$1620.
5. $\frac{400}{107}$.

Art. 297.

1. 103; \$27.
2. \$1200 amt. in-
vested; \$1242
remittance.
3. \$34.95.
4. \$2.50; 15%.
5. \$293.06.
6. \$8.74;
\$428.26.
7. \$2750; \$2675.
8. \$25; 10%.
9. \$1000; \$20.
10. $1\frac{1}{2}$ %; \$1082.
11. \$1552.80;
\$67.20.
12. \$450.70;
3%.

Art. 298.	Art. 302.	13. \$17 per \$1000;	Art. 312.
1. \$421.05.	9. \$355.80.	\$263,372.50.	1. \$790.70
2. 526.67.	10. 600.60.	14. $1\frac{3}{4}\%$.	(grace);
3. 200.00	11. 370.	15. $1\frac{3}{16}\%$;	\$791.
9.63	Art. 304.	\$1,930,769.23.	2. \$713.70
4.37	5. \$780.	16. \$244.	(grace);
<u>\$214.00</u>	6. 3180.	17. $1\frac{1}{16}\%$;	\$714.
4. 20000.	7. 576.31.	\$120879.	3. \$8.97 (grace);
5. 589.20;	Art. 305.	Art. 308.	\$8.75.
1.12% +.	1. \$156.10.	6. \$78.81.	4. \$594.75
6. 3.52;	2. 865.49.	7. 206.46.	(grace);
2.37% +.	3. 2280.01.	8. 449.95.	\$595.
7. 777.65; $3\frac{3}{4}\%$.	4. 750.97.	9. 1030.30.	5. \$523.56
8. 207.90;	5. 197.74.	10. 38.14.	(grace);
198.	6. 806.99.	Art. 309.	\$523.69.
9. 1250;	Art. 306.	4. \$366.94 +.	6. \$906.34
43.75.	2. \$45.	5. 424.48.	(grace);
10. 787.49;	4. \$45.	6. 270.61.	\$906.68.
15.39.	5. \$123.65.	7. 812.06.	7. \$310.82
Art. 299.	8. Specific duty	8. 428.83.	(grace);
1. \$314.93.	9. \$60.75; $3\frac{3}{4}\%$.	Art. 310.	\$784.50
2. 12000.	10. $122\frac{1}{2}\%$; \$150.	5. Mar. 3.	(grace);
3. 4123.38.	11. \$424.59.	6. Sept. 8.	\$953.28
4. 7600; \$400.	12. 20%.	7. May 31.	(grace);
5. 16.87 $\frac{1}{2}$.	13. 15%.	8. Mar. 3.	\$953.60.
6. 9250000.	14. \$0.32 gain.	9. Nov. 30.	10. \$715.53
7. 4006.	Art. 307.	10. Feb. 22.	(grace);
8. 30000.	8. \$300,000.	11. Feb. 28.	\$715.74.
9. 6297.63.	9. \$50,000;	12. Apr. 18.	11. \$836.25
10. 58.87.	10,000.	13. Nov. 15.	(grace);
Art. 301.	10. \$15 on \$1000;	14. July 7.	12. \$870.90
1. \$261.52.	$1\frac{1}{2}\%$ on \$1;	15. Mar. 3.	(grace);
2. 987.60.	\$8000.	16. Apr. 29.	13. \$871.05.
3. 737.39.	11. $1\frac{1}{2}\%$; \$0.012;	Art. 311.	\$91.32
4. 2035.20.	$1\frac{1}{3}\%$.	13. \$590.	(grace);
5. 753.15.	12. $1\frac{1}{10}\%$; \$1.48.	14. 1187.60.	14. \$5907.75
6. 310.17.			(grace);
7. 76.58.			\$5910.
9. 1356.53.			
10. 1847.71.			

15. \$4241.50

(grace);

\$4243.29.

16. \$814.93

(grace);

\$815.43.

Art. 313.

1. July 7/10;

27 d. or 30 d.

2. Feb. 15/18;

63 d. or 66 d.

3. Oct. 5/8;

37 d. or 40 d.

4. June 17/20;

47 d. or 50 d.

5. Aug. 4/7;

57 d. or 60 d.

6. Feb. 24/27;

26 d. or 29 d.

7. Jan. 19/22;

70 d. or 73 d.;

2 mo. 9 d. or

2 mo. 12 d.

8. May 11/14;

66 d. or 69 d.;

2 mo. 5 d. or

2 mo. 8 d.

Art. 314.

1. 63 d. or 66 d.

2. 24 d.

3. \$433.

4. \$994.86.

5. \$795.33.

6. \$2980.50.

7. \$447.65

(grace);

\$447.80.

8. \$705.06.

9. \$794.22

(grace);

\$794.55.

10. \$1195.50

(grace);

\$1196.45.

\$1195.97

(grace);

\$1196.69.

11. \$247.60

(grace);

\$247.69.

12. \$1716.07.

13. \$188.74

(grace);

\$188.80.

14. \$273.24

(grace);

\$273.43.

\$273.18

(grace);

\$273.37.

15. \$343.44

(grace);

\$343.70.

16. \$752.65

(grace);

\$752.85.

\$752.78

(grace);

\$752.97.

17. \$4988.89

(grace);

\$4989.93.

Art. 315.

7. \$1223.64.

8. \$502.24

(grace);

\$502.15.

9. \$726.92

(grace);

\$726.81 (time

in exact days).

Art. 316.

5. $\frac{1}{2}$.

6. \$60; \$8.

Art. 317.

1. \$5; 5%.

2. \$5.

3. \$1125.

4. \$20.

5. 60 shares.

6. \$101.50;

\$1.50.

7. \$370.

8. \$100.

9. 6%.

10. 6%.

11. \$3450; \$138;

4%.

15. \$5; \$200.

Art. 318.

1. \$500.

4. \$700.

5. 5%.

6. \$4000.

7. 150.

8. 110 $\frac{1}{2}$.

9. \$10100.

10. \$4906.25.

11. \$15 nearly on

a 100-dollar

bond.

Art. 319.

4. \$44,166,666.67

5. 90%.

6. 130%.

8. \$2838.38.

9. \$9712.50.

10. \$4015.

11. \$4280.

12. \$3940.

14. 3.84%.

15. 5.49%.

Art. 320.

8. \$476.19.

9. 694.44.

10. 1043.48.

11. 697.12.

12. 83.50.

13. 733.33.

Art. 321.

8. 2 yr.

9. 5%.

10. \$200.

11. 4 yr. 6 mo.

12. \$840.

13. 5%.

14. \$960.

15. 1 yr. 8 mo.

16. 4 $\frac{1}{2}$ %.

17. \$179.28.

Art. 322.

3. \$175.53.

4. \$25.53.

5. \$20.76; 30%.

Art. 325.

4. Aug. 11.

5. \$6.25;

\$1494.25.

6. \$1485.50.

8. 90 d. after

date.

- 9 \$493.50.
 10. \$490; \$489.75
 (grace).
 11. \$3938 (grace);
 \$3940.
 12. \$10.25.

Art. 326.

2. \$3920.
 3. £2010.
 4. £860 10s.
 5. M. 3640.
 6. \$387.60.

Art. 327.

1. 2872.50.
 2. \$21.56.
 3. 217.01 $\frac{1}{4}$.
 4. 2085.71.
 5. \$73.46.
 6. \$6185.
 7. \$1265.25.
 8. \$12.09.
 9. \$5.19.
 10. \$40.41.
 11. \$87.70;
 \$89.24
 (grace).
 12. \$49.61;
 \$51.14
 (grace).
 13. \$193.95
 (grace);
 \$194.74.
 14. \$131.99.
 15. \$58.80.
 16. \$256.10.
 17. \$845.56.
 18. \$941.11.

19. \$1886.70.
 20. \$1213.21.

Art. 328.

1. \$250.
 2. \$238.10.
 3. $\frac{121}{100}\%$ or
 0.605%.
 4. \$156.25.
 5. \$33.39 loss.
 6. 6535.95 — bu.
 7. \$4189.47.
 8. \$1590.
 9. Gain \$14.25.
 10. \$1960.78.

Art. 329.

2. \$402.38.
 3. \$751.82.
 4. \$0.93.
 5. \$563.20.
 6. \$3.33.
 7. \$820.
 8. 2 $\frac{1}{2}$ yr.
 9. 8%.
 10. \$905.66.
 11. \$690.08.
 12. \$497.78;
 \$497.57
 (grace).
 13. \$212.24.
 14. \$48; \$47.36;
 \$48.72;
 \$48.00
 (grace);
 \$48.30;
 \$44.44.
 15. \$20.

Art. 330.

4. $\frac{1}{3}$; 3.
 20. 300.
 21. 33 $\frac{1}{3}$.
 22. 80.
 23. 9.
 24. $\frac{1}{6}$.
 25. 8 yd.
 26. \$20.
 27. 75.
 28. 240.
 29. 1536.
 30. 65.
 31. 43.5.

Art. 331.

15. \$15.
 16. \$378.
 17. \$126.
 18. 5 $\frac{1}{4}$ h.
 19. 22 $\frac{1}{2}$ d.
 20. 4285.71 m.
 21. \$133.35.
 22. 24 A.
 23. 6 $\frac{2}{3}$ oz.
 24. \$0.80.

Art. 332.

1. $\frac{4}{5}$; $\frac{1}{5}$.
 2. S. \$1200;
 B. \$300.
 3. S. \$560;
 B. \$140.
 5. \$2000;
 \$3000;
 \$4000.
 6. $\frac{5}{12}$.
 7. $\frac{1}{3}$ or \$1.33.
 8. \$300; \$450.
 9. \$10,000.

10. \$0.84 $\frac{8}{15}$;
 \$507.70.
 11. \$7411.76.
 12. \$6000;
 \$4000;
 \$3000.
 13. \$20,000.

Art. 335.

17. 23.
 18. 26.
 19. 34.
 20. 42.
 21. 45.
 22. 54.
 23. 63.
 24. 68.
 25. 75.
 26. 84.
 27. 64.
 28. 73.
 29. 83.
 30. 92.
 31. 58.
 32. 99.

Art. 336.

1. 532.
 2. 547.
 3. 636.
 4. 746.
 5. 869.
 6. 1462.
 7. 0.75.
 8. 0.96.
 9. 6.5.
 10. 0.8848+.
 11. 0.9433+.
 12. 4.4121+.
 13. 28.7210+.
 14. 4.1273+.
 15. 0.8.

16. 0.2529+.
17. 43.9590+.
18. 27.9991+.
19. 15.03.
20. 1430.
21. 8.74.
22. 0.8944+.
23. $\frac{7}{12}$.
24. 0.559+.
25. 23.
26. 2.380+.
27. 9.099+.
28. 0.5422+.
29. 12.3237+.
30. 1.4790+.

Art. 337.

1. 307.
2. 0.8538+.
3. 3.7249+.
4. 1004.
5. 3136.
6. 7921.
7. 45.09.
8. 0.9682+.
9. $1\frac{1}{2}$.
10. 999.
11. $9\frac{3}{4}$.
12. 64.
13. 1.4142+.
14. 0.4472+.
15. 0.19364+.
16. 11.2743+.
17. 28.0178+.
18. 1856.
19. 30.2324+.
20. 0.860813+.

Art. 338.

12. 44.
13. 21.909+.
14. 20.933+.

15. 61.846+.
16. 21.354+.
17. 146.09+.
18. 47.434+.
19. 186.051+.
20. 116.275+.
21. 5.477+.

Art. 339.

1. 2 ft.
2. 40 rd.; 160 rd.
3. 24.166+ ft.
4. 21.931+ ft.
5. 73.32+ sq. ft.
6. 4 ft. 2.9+ in.
7. 20.784+ ft.
8. \$141.42.
9. 15.866+ ft.
10. 108.1662+ sq. ft.

Art. 340.

1. 6.9282+ rd.
2. 43.863 ft.
3. 5:4.
4. 21.089 rd.
5. 208.71+ ft.
6. 97.616+ ft.
7. 63.639 ft.
8. 176.568+ rd.
9. 14.27 rd., or 235.5+ ft.
10. 101.98+ ft.

Art. 341.

5. 1178.10.
6. 670.208.
7. 942.48.
8. 11.313+ ; 189.55+.

Art. 342.

1. 36 cu. ft.
2. $29\frac{1}{2}$ cu. ft., or 29.629+ cu. ft.
4. 471.5 cu. in.
5. 500 cu. in.

Art. 343.

3. 384 sq. in.
4. 412.300 sq. in.
5. 4 sq. ft.
10. 105 sq. in.
11. 375 sq. in.
12. 235.62 sq. in.
13. 301.593+ sq. in.
14. 463.62+ sq. in.
15. Contents.
16. 0.2146.

Art. 344.

7. 452.3904.
8. 12,566,400.
9. 154 sq. in.
10. 200,000,000.

Art. 345.

7. 113 cu. in.
10. 4.1888 cu. ft.
11. 4,188,800,000.
12. 0.4764 ; 0.5236.

Art. 346.

9. \$2700.
10. 4:9.
11. \$18.56.
12. $19\frac{1}{2}$.
13. 5.
14. \$78.75.
15. \$21.09.

Art. 347.

5. 64; 8.
6. 27 lb.
7. 1171.87 $\frac{1}{2}$ bu.
8. 96 h.
9. 162 lb.
10. 1; 373,248.

Art. 348.

1. \$9.25.
2. 681.7925+ sq. ft.
3. 16 rd.
5. $42\frac{5}{8}$ sq. ft.
6. 315 sq. ft.
7. \$32.26.
8. 124.686.
9. 875.
10. 1.27324 ft.
11. 3 ft.
12. 768 sq. ft.
13. 4.1888 cu. ft.
14. 14.499 sq. ft.

Art. 349.

1. 1080.
2. 6.2832 cu. in.
3. 12.65- in.
4. 530.145 sq. in.
5. 24902.18142+.
6. 180 rd.; 169.68+ rd.; 150.40- rd.
7. 94.81+.
8. 26.529+.
9. 4: $3\frac{1}{6}$.
10. 1607.8125 lb.
11. 14.9334+ ft.
12. 28.2744.
13. 18,000 lb.

14. 706.86 sq. ft.
15. 70.71 + ft.
16. 2108.

Art. 365.

1. $42\frac{5}{2}$.
2. $111\frac{5}{10}$.
3. \$63.28;
\$2763.88.
4. \$59.42.
5. $1\frac{2}{3}$.
6. \$889.12 $\frac{1}{2}$.
7. 0.679375.
8. $1\frac{1}{2}$ d.
9. 104 $\frac{1}{3}$ rd.
10. 1349 cu. in.
11. 27 $\frac{1}{2}$ yd.
12. \$2.24.
13. 4098 $\frac{1}{2}$.
14. $\frac{2}{3}$.

Art. 366.

1. \$1504.29.
2. \$2745.76.
3. Neither.
4. 6750.
5. \$320.
6. 16.96 %;
\$16.11.
7. Loss \$13.33 $\frac{1}{2}$.
8. Cash.
9. 11 $\frac{1}{2}$.
10. 38 $\frac{2}{3}$ T.

Art. 367.

1. 120° E. of
starting point.
2. 35 $\frac{1}{4}$.
3. 88.
4. $\frac{821}{1372}$.

5. 2 min. 39 $\frac{1}{2}$ $\frac{1}{3}$
sec.
6. $1\frac{9}{16}$ T.
7. 1072 $\frac{2}{3}$ +.
8. 300 lb.
9. 16.
10. 8.24.

Art. 368.

1. \$552.49.
2. \$2687.44.
3. \$20.
4. \$27.61.
5. \$4378.79.
6. 1413.72 sq. ft.
7. 460.195 + lb.
8. 1583.36 +.
9. \$32.35.
10. \$337.27.

Art. 369.

1. \$4372.60.
2. 17 $\frac{1}{2}$ %.
3. 10.63 + in.
4. 35 %.
5. \$141.20.
6. \$421.85.
7. 97 $\frac{1}{12}$ %.
8. \$108,800,000.
9. 63.8 : 36.2.
10. \$0.01375 +.
11. 752.52 - ft.;
twice as much
more.
12. 5.32 - in.
13. \$618.

Art. 370.

1. \$217.50.
2. \$11.55 +.
3. \$5.91.

4. 20°.
5. \$527.15 gain.
6. \$3.24.
7. 20¢.
8. \$12000.
9. \$6.75.
10. The latter by
 $\frac{3}{18}$.
11. $\frac{10}{77}$.
12. \$1045.45 +.
13. 18.8 %.

Art. 371.

1. 70.71 - m.
2. Int. for $\frac{1}{4}$ yr.
= 1 %.
3. 11.30 P.M.
4. 389.70 + sq. ft.
5. \$100.
6. 2.652.
7. 6.6338 + lb.
8. \$450.
9. \$107.62 $\frac{1}{2}$.
10. \$789.78.

Art. 372.

1. 92¢.
2. 27 $\frac{1}{3}$ $\frac{1}{2}$.
3. \$1980.
4. 41 $\frac{1}{3}$ %.
5. \$2924.10.
6. \$400.
7. \$145.90 +.
8. 17.72 + ft.
9. 1232 cu. in.
10. A \$10800;
B \$7200.
11. \$851.27 + or
\$851.59 +.
12. \$83 $\frac{1}{3}$.
13. 9.36 + %.

14. 25 %.
15. 1 in.

Art. 373.

1. \$26.25.
2. \$25000.
3. 36-shares;
\$50 rem.
4. 6 $\frac{2}{3}$ %.
5. \$3668.75.
6. 5 %.
7. \$178.75.
8. \$20937.50.
9. Increase \$15.
10. The former by
 $\frac{1}{6}$ %.
11. \$10250.
12. \$6300.
13. 75° East.
14. 10.39 + in.
15. 93.53 - sq. in.
84.82 + sq. in.

Art. 375.

1. \$9 $\frac{7}{8}$ T.
2. 87 $\frac{1}{2}$ ¢.
3. \$2400.
4. 160 bbl.
5. \$58500.
6. \$1930.43.
7. A \$12; B \$9;
C \$24.
8. 62 $\frac{1}{2}$ A.
9. 44 $\frac{1}{3}$ ft.
10. \$9.21.
11. 87.68 ft.
12. \$1440.46.

Art. 376.

1. 50 %.
2. 12 ft.

3. $6\frac{1}{4}\%$.
4. 7360 bu.
5. \$160; \$208.
6. $\frac{1}{3}$.
7. $5\frac{2}{3}\%$.
8. $8\frac{1}{3}$ yd.
9. 9 A.
10. Bonds $2\frac{5}{7}\%$.
11. $26\frac{1}{2}$ rd.
12. $16\frac{2}{3}$ sq. ft.
13. 116 ft. $8\frac{1}{8}$ in.
14. 413013 $\frac{1}{3}$ T.

Art. 377.

1. 0.02.
2. \$90.56.
3. $2\frac{2}{3}\%$.
4. \$13.82.
5. $17\frac{2}{3}$ yd.
6. 93.81—rd.
7. $7\frac{1}{7}\%$.
8. 1963 $\frac{1}{2}$ sq. in.
9. 9 in. \times 9 in.
10. \$140.62 $\frac{1}{2}$.
11. \$1371.09.
12. $16\frac{2}{3}\%$.
13. A \$3240;
B \$1260;
C \$1260.
14. 399.999975.
15. \$48.

Art. 378.

1. 125%.
2. $10\frac{1}{18}$ sq. ft.
3. \$12.
4. \$11,880.
5. \$20,000.
6. 125 ft.
7. \$3696.
8. 500 men.

9. 40¢.
10. \$105.80.
11. 2332.64.
12. \$371.20.
13. $22\frac{2}{7}\%$.
14. $3263\frac{2}{3}$ m.

Art. 379.

1. \$187.20.
2. \$220.
3. \$10.50.
4. \$866 $\frac{2}{3}$.
5. 10%.
6. $32\frac{1}{3}\%$.
7. \$48 gain.
8. \$9774.72;
9. \$29.75.
10. \$212.61.
11. \$6.25.

Art. 380.

1. $16\frac{1}{2}$ d.
2. \$6150.
3. \$8240.
4. \$376.125.
5. 616.11 + sq. ft.
6. 35.99—gal.
7. $256\frac{1}{2}$ bd. ft.
8. \$6400.
9. \$22,300.
10. 75 bbl.
11. 1430.

Art. 381.

1. $1\frac{1}{3}$ d. more.
2. $5\frac{1}{2}$ ft.
3. \$1031.25.
4. $5\frac{1}{3}$ h.

5. \$538.77;
\$538.45
(grace).
6. $\frac{1}{25}$.
7. \$375.
8. $140.06 +$ sq. in.
9. Nothing.
10. \$285.
11. Rich \$800;
Foster \$1200.
12. $336.13 +$.
13. $52\frac{1}{2}$ sq. ft.

Art. 382.

1. 5 h.
2. \$18,000.
3. \$625.
4. \$5023.57.
5. \$3010.
6. 3125.
7. \$817.16.
8. $42.1 +$ yd.
9. \$12.10.
10. \$17.36—.
11. \$26,000.

Art. 383.

1. $15\frac{1}{3}$ T.
2. A \$36; B \$38;
C \$26.
3. \$3872.12 $\frac{1}{2}$.
4. 69¢.
5. $1\frac{3}{4}$ yr.
6. \$15 $\frac{3}{4}$.
7. \$72.
8. 1765.17 gal.
9. 9929.4.
10. 12.44 ft.
11. 1:10.
12. \$16.80.
13. $56\frac{1}{2}$ sq. ft.

Art. 384.

1. \$33,333 $\frac{1}{3}$.
2. $7\frac{1}{2}\%$.
3. \$266.48.
4. 50 m.
5. 12 d.
6. \$115.20.
7. 12 rd.
8. 2 ft. 4 in.
9. 32.725 lb.
10. \$3.39+.
11. The former by
\$70.30.
12. 64 men.
13. 9 oz.

Art. 385.

1. 14 bu. 2 pk.
4 qt.
2. \$37.70.
3. \$620.12 $\frac{1}{2}$.
4. \$4504.50.
5. \$249.64;
\$249.64
(grace).
6. 2.6—cu. ft.
7. 5%.
8. \$55.
9. A \$91.30;
B \$114.13;
C \$144.57.
10. \$55,000.
11. \$1030.64.
12. $44.74 - \%$.

Art. 386.

1. \$72,000.
2. \$76.
3. A $\frac{5}{24}$; B $\frac{5}{16}$;
C $\frac{5}{18}$; D $\frac{5}{12}$.
4. \$577.40.

5. \$386; 600 fr.	Art. 387.	7. 20.01+ m.	2. \$2007.50.
6. \$32,137.50.	1. \$1.54.	8. $21\frac{3}{4}$.	3. \$2.91 $\frac{2}{3}$;
7. 116 ft.	2. \$7.20.	9. \$16.80.	\$912.91 $\frac{2}{3}$.
8. \$1059.87+;	3. \$5700.	10. \$80,960.	4. \$9956.86—.
\$1059.67	4. 6.	11. \$10,552.	5. \$1337.
(grace).	5. 504 bu.;	12. \$147.	6. Makes \$6.50.
9. \$2123.33 $\frac{1}{3}$.	\$290.64.	Art. 388.	7. \$3.54.
10. Of B; \$1.39.	6. 0.02078125.	1. 38.	8. 61%.
11. 159,155 cu. ft.			

APPENDIX I.

Art. 19.	Art. 21.	10. \$843.36;	9. 72.
7. 847200 ^g ;	2. \$356.40.	73 d.;	10. 78.
1867.75+ lb.	3. 567.	Aug. 20.	11. 85.
8. 26.43 ^{III} .	4. 554.40.	11. Jan. 4.	12. 96.
9. 16.72 ^{sq. m.}	5. 198.39.	12. Aug. 23.	13. 75 in.
10. 1312.359 yd.	6. 774.90.	Art. 26.	14. 15.11+ in.
11. 395.38 sq. rd.	7. 126.30.	2. May 31.	16. 156.
12. 1000 ^l .	8. 86.87.	3. Oct. 24.	17. 235.
13. \$10.63 gain.	9. 454.78.	4. May 23.	18. 3.34.
14. 960 ^{Kg.}	Art. 23.	5. Nov. 13.	19. 0.4644+.
15. 88.9056 ^{Kg.}	2. \$603.02.	6. Jan. 5.	20. 0.444+.
16. 2.845184 ^{Mm.}	3. \$1175.77.	Art. 28.	21. 2.924+.
17. \$3.3108.	Art. 25.	1. 23.	22. $1\frac{2}{3}$.
18. 56 ^{Hu.} ; 138.384.	3. 3 mo. 10 d.	2. 32.	23. $\frac{27}{9}$.
19. 264.17 gal.	4. 1 mo. 24 d.	3. 36.	24. $1\frac{1}{3}$.
20. 800.	6. Apr. 29.	4. 42.	25. $1\frac{1}{3}$.
21. \$3.21.	7. July 26.	5. 47.	26. 4.175+.
22. 196.875 ^{Kg.}	8. June 11.	6. 54.	27. 7 $\frac{2}{3}$.
23. 7500 ^{m.}	9. Dec. 21.	7. 56.	28. 66 $\frac{2}{3}$ sq. ft.
24. 13.9286947.		8. 62.	29. 24 ft.

APPENDIX II.

Art. 8.

1. 25.
2. 50.
3. 74 bbl.
4. \$100.
5. 8.
6. \$1000.
7. 20.
8. $2x = 2 \times 6$.
9. 6.
10. 11×10 ;
 9×10 .

Art. 9.

1. $x = 90 - (30 + 35)$.
2. $x = \frac{20 \times 8}{2}$.
3. $x = \frac{150}{100 - 25}$.
4. $x = \frac{150}{3}$.
5. $x = 7 - 2$.
6. $x = \frac{1}{2} \times \frac{1}{2}$ of $\frac{160}{2} \div 2$.
7. $x = \frac{200}{400} \div 2$;
or $x = 400 \times 2$.
8. $x = .01 \times 24$;
or $x = \frac{24 \times 144}{144}$.

Art. 10.

3. $m(a + b)$.
4. $\frac{d}{n}$.
5. $y = 3(b + 1)$.
7. $\frac{1}{a} + \frac{1}{c}$.

8. $\frac{1}{x}; \frac{m}{x}$

9. bn .

10. $\frac{d}{c}$

Art. 15.

2. 3.
3. 3.
4. 10.
5. \$4.01.
6. 64.
7. 135.
8. \$4000.
9. 8.
10. \$10.

Art. 16.

2. 72.
3. 12.
4. 24.
5. \$300.
6. \$56.
7. 36 m.
8. 120.
9. \$31.8852.
10. \$84.

Art. 17.

2. 10.
3. 5.
4. 8.
5. $11\frac{1}{11}$.
6. 1.
7. 18.
8. 180.
9. 6.
10. 20.
11. 30.

12. $14\frac{1}{2}$.

13. 12.

14. 4.

15. 5.

Art. 18.

1. \$210; \$70.
2. 20 qt.; 25 qt.
3. 9; 14.
4. 48; 39.
5. \$0.40; \$0.80.
6. A 2180; B 800;
C 320.
7. 168. 9. 150.
8. 60. 10. 7.

Art. 19.

1. \$10,000; \$5000.
2. 12.
3. F \$94; G \$122;
H \$56.
4. 200 gal. each.
5. 18; 12.
6. 8.
7. 11; 13; 15.
8. 84; 91; 98.
9. 60; 20.
10. \$87\frac{1}{2}

Art. 20.

4. 0.
5. $10x^2$.
6. $7(b + c)$.
7. $-12x$.

Art. 21.

1. $+7m$.
2. $1m$.
4. 24 rec'd.

Art. 24.

10. 4.
11. $c - b - d$.
12. 4.
13. $2x - a - y$.
14. $y - 2ax$.
15. 0.
16. $5 - (z + 1) + 2$.
17. $a + (b - c) - e$.
18. $x - (y - z) + a$.

Art. 25.

1. 10 abc .
2. 18 abv .
3. 36 axy .
4. 48 bcy .
5. 228 $abxy$.
6. 63 $bcdz$.
10. $2(abcx)^2$.
11. $4a^4c^2mx^4$.
12. $8a^2b^2$.
13. $6abc^2x^2y$.
14. a^3 .
15. $(m + n)^2$.
16. a^3bcx^2 .
17. $6(a - b)$.
18. $6(a - b)^2$.
19. $(m + n)^3$.
20. $12(x + y)^3$.
24. $3x^3 + 3xy^2 + 3xz^2$.
25. $x^3yz + xy^2z + xyz^3$.
26. $6am$.
27. $10ax^4y^2 + 15ax^3y^3 + 20ax^2y^2z$.
28. $14a + 21b + 7c$.
31. (Combine before multiplying.)

Art. 29.

2. $\frac{2}{y}$.
3. $\frac{a^2}{4x^3}$.
4. $\frac{a - c}{ac}$.
5. $\frac{4ade}{5}$.
6. $\frac{3a^2xy^2}{4}$.
7. $\frac{4ax^2}{5m^2}$.
8. $\frac{4x}{5m}$.
9. $\frac{7a}{11b}$.
10. $\frac{3bdc}{4ax}$.

Art. 30.

3. $\frac{15ay}{10a^2b}; \frac{bx}{10a^2b}$.
4. $\frac{3x}{6x^2y}; \frac{2am}{6x^2y}$.
5. $\frac{16bx}{48ab^2m^2}; \frac{8amy}{48ab^2m^2}; \frac{3b^2m^2z}{48ab^2m^2}$.

Art. 31.

1. $\frac{4a + 2b^2}{xy}$.
2. $\frac{ayz + axz + axy}{xyz}$ or $\frac{a(yz + xz + xy)}{xyz}$.
3. $\frac{2xn - 3am}{mn}$.

4. $\frac{2xyz^2 + 12z^2 + 15x^2y^2}{3x^2y^2z^2}$.
5. $\frac{4c - 2ad^3}{c^2d^3y}$.
6. $\frac{3y(a - b) + 3x(a + d)}{xy}$.
7. $\frac{adn + bcn - bdm}{bdn}$.
8. $\frac{x(cmy + d'n)}{c^2d^3y^3}$.
9. $\frac{a(b^2 + x^2) - y}{a}$.

Art. 32.

2. $\frac{2c}{y}$.
3. $\frac{b}{4ay}$.
4. $\frac{2d^2mnxy}{3a^2z}$.
5. $\frac{2a^2c}{b^3d^4}$.
6. $\frac{ax}{2bcd}$.
7. $(a + b)(x + y)$.

Art. 33.

2. $\frac{2e}{x}$.
3. $\frac{z^2}{5m^2}$.
4. $\frac{y}{x^2}$.
5. $\frac{a^2b}{5xz}$.
6. $\frac{2(2a + b^2)}{5(2a + b)xy^2}$.
7. $\frac{7cd}{2bz^2}$.

$$8. \frac{2cfy}{3ab}$$

$$9. \frac{x(y+d)}{3cd^3y}$$

$$10. \frac{2cd}{ab^2}$$

Art. 34.

7. 5; 11.
8. 8; 3.
9. 15; 4.
10. \$75; \$100.

Art. 35.

2. 8; 5.
3. 11; 7.
4. 12; 2.
5. 6; 1.
6. 9; 3.
7. 12; 5.
8. 9; 4.
9. 7; 7.
10. 12; 2.
11. 15; 10.
12. 16; 5.

13. 13; 8.
14. 30; 24.
15. 18; 12.
16. 20; 12.
17. 10; 8.
18. 8; 6.
19. 27; $7\frac{1}{2}$.

Art. 36.

1. A \$50;
B \$40.

2. \$550; \$300.
3. 64; 36.
4. 40; 24.
5. Tea 75¢;
Coffee 40¢.
6. 70; 30.
7. 63; 27.
8. 20; 12.
9. A \$270;
B \$252.
10. $\frac{1}{3}$.